Econ 260A
Homework problems

General renewable resources

1. An exotic plant was introduced onto the island of Hawaii and began spreading through the forest. The plant's natural growth function is:

   $$\frac{dx(t)}{dt} = \theta x(t) \text{, where } x \text{ is the stock and } 0<\theta<1.$$

   The plant can be harvested from the environment at cost, *c(y)*, where *y(t)* is the rate of harvest. Consumers can either consume or sell what they harvest at a positive price and thus have an incentive to remove it from the environment. Harvest reduces the stock commensurately. Unfortunately, the plant has an undesirable effect on wild animals, so a larger *stock* in the environment implies lower utility. This damage, *D(x)*, is a "public bad" experienced by all consumers identically as a function of the total stock in the environment. Assume θ is less than *r*, the relevant interest rate.
   i. Set up the optimal policy problem for harvesting this plant and state any assumptions you believe are appropriate. Write out the necessary conditions for an optimum and interpret them in a steady state equilibrium. (You may express your model in partial or general equilibrium, discrete or continuous time.)
   ii. Is the steady state rate of removal that would be chosen by citizens acting individually Pareto efficient?

2. A stock, *x*, of DDT accumulates in the environment according to the equation

   $$\frac{dx(t)}{dt} = y(t) - \theta x(t),$$

   where *y* is the rate of emissions, i.e., the rate at which it is applied by competitive farmers, and θ is the rate of decay in the environment. Each period there is an environmental damage, *D(x(t))*, which depends on the size of the stock. A social planner wishes to control the use of DDT to maximize the present value of net agricultural benefits from DDT use minus environmental damage. The net agricultural benefit is $ay - (b/2)y^2$ and the environmental damage function is $D(x) = (c/2)x^2$, where *a*, *b* and *c* are positive constants.
   i. Characterize an optimal time path for DDT emissions and interpret the steady state equilibrium conditions.
   ii. How would the industry use DDT in the absence of any policy to control it?
   iii. Characterize a policy that would cause competitive farmers to implement the welfare maximizing outcome.

3. Each year forests in Zooland produce a crop of tropical fruit that hikers enjoy eating. As the fruit ripens its value to consumers increases according to the function V(t), where V(˙) is the current value of a single piece of fruit and t is the date during the year when it is picked. Assume V is increasing, concave, and has a unique interior maximum during the year and consumers discount current utility at rate r when making intertemporal consumption decision. Also, unripe fruit is bitter, so V can be negative for very small values of t.

i. Compare the 'equilibrium ripeness' (date during the year when the fruit is picked) for fruit picked from privately owned forests versus forests that are essentially treated as free access 'commons.' Explain your answer.
ii. Suppose one forest is privately owned, but subject to theft. The owner believes the probability that a given piece of fruit will be stolen in any instant of time is equal to π. How long would the owner allow the fruit to grow before picking?

Nonrenewable resources

4. A competitive industry extracts a nonrenewable resource with an extraction technology that exhibits constant cost, c, per unit. (Costs are constant at the industry level, although individual extraction firms may have u-shaped average cost curves. What is assumed here is that the supply of extraction effort is perfectly elastic at the industry level at a unit cost of c.) (Inverse) demand for the resource is p = a-by, where y is the rate of extraction and a>c. The interest rate is r and the initial stock available to extract is R_0.
   i. Describe the competitive equilibrium price path and explain why it is an equilibrium.
   ii. Suppose at time t_0, when the price is $p(t_0)$ and the remaining reserve is $R(t_0)>0$, the government announces, unexpectedly, that price will not be allowed to rise above its current level. Characterize the 'post policy' equilibrium price path.
   iii. Re-answer part ii under the assumption that the industry is a monopoly.

5. A nonrenewable resource exists in two grades, 1 and 2. Unit extraction costs for the two deposits are $c_1=3$ and $c_2=6$ and initial reserves are $R_1(0)$ and $R_2(0)$, respectively. Demand for the extracted resource is $p=10-y$, where p is price and y is quantity. Use r to denote the interest rate. The extracting industry is perfectly competitive.
   i. Describe in as much detail as possible the competitive time profile of price and extraction.
   ii. Suppose it is learned that extraction causes environmental damage of $5y$ per period. Describe the time profile of extraction that would maximize the present value of benefits from consumption (consumer surplus) net of extraction cost and environmental damage and propose a policy that would cause the industry to implement this outcome.

6. A exhaustible resource stock (coal) is extracted by a competitive industry at constant unit cost. Consumption of it generates a stock pollutant (CO_2), the costs of which are external to the industry and increasing in the stock in the environment. Demand for the resource is stationary and has a choke price.
   i. Describe the equilibrium price and extraction paths of the conventional resource.
   ii. There is a clean "backstop" technology (solar), but its marginal cost exceeds the conventional resource's choke price. Evaluate a policy of subsidizing the cost of the backstop so that its price falls below the resource's choke price, thus facilitating its adoption. How would this affect the present value of environmental damages?

Groundwater

7. Groundwater can be extracted at a cost *cwD*, where *w* is the rate of withdrawal, *D* is the distance water must be pumped to the surface, *c* is a constant. *cwD* is the aggregate cost function for all users. Depth changes over time according the equation $\frac{dD(t)}{dt} = w(t) - R$ where *R* is natural recharg. The aggregate demand function for water is *p=a-bw*, where *p* is price and *a* and *b* are positive constants. The interest rate is *r*>0.
   i. Suppose the resource is managed to maximize net social welfare, defined as willingness to pay for water minus pumping cost,l *cwD*. Write out the objective function for a manager who seeks to maximize the present value of net social benefits (willingness to pay minus cost) and characterize the steady state levels of *w* and *D* in terms of *a*, *b*, *c*, *r* and *R*.
   ii. Imagine that a management agency attempts to implement the solution in (i) by creating 'rights' to water in the aquifer. Specifically, a 'water account' is created for each user. Each user is initially allocated a pro rata share of water then in storage, a pro rata share of recharge is credited (added) to the individual's account each year, and the user's actual withdrawal is deducted from the individual's account each year. Will this policy achieve the optimal outcome?
   iii. Suppose instead that the resource is un-owned, there is no central management, and the number of individuals who withdraw water is sufficiently large that each regards *D* as given, independent of his or her own rate of extraction. Characterize the steady state level of withdrawal, depth, and net welfare in terms of *a*, *b*, *c*, *r* and *R*. Explain. (Don't try to characterize the behavior of individual users; you don't have enough information for this. Just characterize aggregate withdrawals, where *cwD* is the aggregate cost function and *p=a-bw* is the aggregate demand function.)
   iv. In this free access situation, suppose society can make a one-time investment of ε>0, where ε is small, that will cut the pumping cost in half, to .5*cwD*. How will this affect steady state withdrawal, depth, and welfare?

Fisheries

8. Abalone grow according to the growth function *w(T)*, where *w* is weight, *T* is age and *w(T)* is concave in the relevant range. The cost of harvesting one abalone, *c*, is independent of the animal's weight and independent of the number of abalone in the stock (so there are no search costs.) The value of an abalone of age *T* is *pw(T)*, where *p* is price per pound. The relevant interest rate is *r*. While growing in the wild, abalone suffer an instantaneous mortality rate *s*, so the fraction surviving to age *T* from an initial population is e^{-sT}.
   i. Assume there is no regulatory limit on harvest age, the unit cost of harvesting abalone is constant, and price is constant. Assume the number of harvesters is 'large'. What will be the oldest age observed for abalone in the wild? Explain.
   ii. What harvest age would maximize the present value of net harvest receipts for a *single cohort* of abalone? Explain. (By looking at only at a single cohort, we

purposely ignore the possible dependence of future stocks on current harvesting.) How would the optimal harvest age, T^*, be affected by a higher natural mortality rate, s?

9. The natural growth rate for a stock, x, of commercially valuable fish is $\frac{dx(t)}{dt} = \alpha(K - x(t))x(t)$, where α and K are constants. The technology for harvest, y, is $y=\gamma xE$, where E is effort and γ is a parameter. Let p and w represent the price of catch and cost per unit effort, respectively. Assume the demand for y and supply of E are both perfectly elastic, so p and w are constant.
   i. What are the long run equilibrium level of stock in the absence of any harvest and the maximum sustainable yield?
   ii. Express harvest cost as a function of the rate of x and y and characterize the long run equilibrium stock that would be observed under free access (assuming each harvester takes x, as given.)
   iii. Describe the steady state solution to this planner's problem, to maximize the present value of benefits from consumption (revenue minus harvest cost.)

Forests

10. Biomass in an acre forest grows according to $F(T)$, where T is the age of the trees. The stumpage price, p, and interest rate, r, are expected to remain constant. Replanting is possible at a constant fixed cost per rotation, C.
   i. Characterize a harvest policy that maximizes the present value of harvest profits.
   ii. The government is considering a requirement that firms pay for environmental remediation, costing E per acre at the end of each harvest. How would this affect (qualitatively) the harvest age and sustained yield?
   iii. Suppose this forest generates a stream of services $S(T)$, where $S'(T)>0$ and $S(0)=0$, while it is growing and these services are external to the forest owner. Characterize the harvest age that would maximize the present value of net revenues plus services. Describe a policy that would correct for this externality.