Hotelling Rule for Competitive Market

1. Notation

P_t	price of extracted mineral in year t.
C	unit extraction cost (assume constant).
r	interest rate (assume constant).
A	choke price, price at which quantity demanded goes to zero.
T	date when exhaustion occurs, i.e., year when last unit is consumed.
P_t–C	value of a unit of the mineral in the ground.

2. Conditions for Equilibrium

(i) $\dfrac{(P_t - C)}{(1+r)^t}$ is equal in all periods.

(Production in any period yields same present value profit per unit.)

(ii) $P_T = A$. (Last unit sold sells at choke price.)

3. Rationale for conditions

Condition (i):

Consider one unit of the mineral in the ground.

PV profit from selling:

in present is $P_0 - C$, in year 1 is $\frac{(P_1 - C)}{(1+r)}$, in year 2 is $\frac{(P_2 - C)}{(1+r)^2}$, etc.

If positive amounts are sold for consumption in all periods, then PV profit must be the same in all periods. Therefore:

$$\frac{(P_t - C)}{(1+r)^t} = P_0 - C, \quad \text{for } t = 1, 2, 3, \ldots, \text{or}$$

$$\boxed{P_t - C = (P_0 - C)(1+r)^t}$$

Note: $(P_0 - C)(1+r) = (P_1 - C)$, so $(P_1 - C)-(P_0 - C)=r(P_0 - C)$. In words, this says that price minus marginal cost grows at the rate of interest.

Condition (ii):

If the last unit sold at a price below A, any mineral owner who anticipated this would have earned a capital gain exceeding r% per year by withholding a unit of the resource until all others had exhausted their deposits, and then selling it at price A in the next instant.

If the price rose to A before all deposits were exhausted, then those holding deposits at that point would earn a zero rate of return on them.

Nonrenewable Resources Sample Problem

Demand: $Q_t = 100 - P_t$ (Q is in tons, P is in $/ton).

Cost: C = 10

Interest rate: r = .10

Reserve: R = 153 tons.

Questions: During how many periods does extraction take place?
What is price in initial period?

(Assume exactly 1 unit is sold in period T.)

Solution: year	P_t	P_t - C	Q_t	Sum(Q_t)
T	99	89	1	1
T-1	91	81	9	10
T-2	84	74	16	26
T-3	77	67	23	49
T-4	71	61	29	78
T-5	65	55	35	113
T-6	60	50	40	153

T-6 is initial year of extraction; price is $60. Extraction occurs over 7 years.

Note: $(P_{T-1} - C) = (P_T - C)/(1 + r)$; $(P_{T-2} - C) = (P_T - C)/(1 + r)^2$; etc.

Hotelling Rule for a Monopoly

1. Notation

P_t	price of extracted mineral in year t.
MR_t	marginal revenue from sales in t.
C	unit extraction cost.
r	interest rate.
A	choke price.
T_M	year when last unit is consumed.

2. Conditions for Equilibrium

(i) $\dfrac{(MR_t - C)}{(1+r)^t}$ is equal in all periods.

(ii) $P(T_M) = A$.

3. Rationale for Conditions

Condition (i):

PV profit from selling one unit:

in present is $MR_0 - C$, in year 1 is $\dfrac{(MR_1 - C)}{(1+r)}$, in year 2 is $\dfrac{(MR_2 - C)}{(1+r)^2}$, etc.

In equilibrium the monopolist is indifferent between selling that unit in the present or in any future period, so the PV profit must be the same in all periods:

$$\frac{(MR_t - C)}{(1+r)^t} = MR_0 - C \quad \text{for all t, or}$$

$$\boxed{MR_t - C = (MR_0 - C)(1+r)^t}$$

Condition (ii):

Same rationale as for competitive case.