

Economics 245B
Exercise 2

Consider the regression model

$$Y_t = X_t' \beta + U_t, \quad t = 1, \dots, n$$

in which X_t and β are $K \times 1$ vectors. While the error is mean zero and i.i.d. unconditionally, U_t exhibits conditional heteroskedasticity of the ARCH form

$$U_t = \sigma_t V_t,$$

with $V_t \sim \text{iid } N(0, 1)$ and

$$\sigma_t^2 = \alpha_0 + \alpha_1 U_{t-1}^2$$

To ensure the conditional variance is positive, we impose the sufficient condition $\alpha_0 > 0$ and $\alpha_1 \geq 0$.

a) Construct the likelihood function $L(\beta, \alpha)$. How do we impose the constraints?

b) To avoid constrained optimization, one could replace (α_0, α_1) with (γ_0^2, γ_1^2) . (As γ_i need not be positive, unconstrained optimization can be used. The invariance property of ML estimators allows one to then construct the estimate of α_i by squaring the estimate of γ_i .) Construct $L^*(\beta, \gamma)$. What is the relation between the MLE of γ from L^* and the MLE of α from L ?

c) Show that $\gamma_i = 0$ solves the first-order condition for L^* for each i and any data.

d) What do you conclude about the usefulness of reparameterizations of the likelihood as in b)?