All-Pay Auctions

- In an all-pay auction, every bidder pays what they bid regardless of whether or not they win.
- Examples:
  - Elections
  - Almost any kind of contest or sports event
  - Research and Development
  - Wars
  - Lobbying
- Since bids are wasted if you don’t win, bidders have a strong incentive to bid aggressively if they bid at all.

All-Pay Auctions

- A simple all-pay auction:
  - The object for sale is worth 1 to all of *n* identical bidders and all of them know this valuation exactly.
  - Clearly no one will bid more than 1.
  - Each bidder's bid is an amount *x* in the interval [0,1].
  - The winner is the highest bidder and gets a payoff of 1 – *x*
  - The payoff of all other bidders is –*x*
- There is no pure strategy equilibrium of this game.
- A mixed strategy of a player is a *probability distribution* over the possible bids between 0 and 1.
- We represent a mixed strategy by a probability distribution function P, where *P*(*x*) which gives the probability that a player bids *less* than some amount *x* between 0 and 1.

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- We look for a symmetric equilibrium.
  - Suppose bidders 2,…,n used the mixed strategy P and consider bidder 1.
  - Suppose bidder 1 bids x.
  - Her expected payoff is given by

$$P(x)^{n-1}(1-x) + (1- P(x)^{n-1})(-x)$$

$$= P(x)^{n-1} - x$$

  - In order for bidder 1 to be willing to randomize we require $P(x)^{n-1} - x = c$.

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– Moreover, since P is a probability distribution function we require $P(0)=0$

– Hence, $c=0$

– Hence, $P(x) = x^{1/(n-1)}$

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- Equilibrium mixed strategies in this example: