

Chapter 4

Bidding Behavior

We assume people attempt to maximize their payoff from participating in an auction. Hence, we are in a sense trying to determine their optimal bids. However, an auction is a game in which the payoff an individual earns from any given bid depends on the bids placed by others. Hence, the notion of an optimal or “best” bid cannot usually be defined in isolation. We need some conjecture about how others will bid.

Our prediction for bidding behavior is based on the idea that people bid optimally based on their predictions about how others will bid and that these predictions are correct. How everyone bids depends, of course, on their use-values. It is assumed that each bidder knows her own use-value, but only knows the probability distribution function that determines other bidders’ use-values. Hence, bidders must best respond to their beliefs regarding the *bid functions* used by other bidders and the likelihood of the use-values that will enter into these bid functions. In other words, our predictions for bidding behavior are the *Bayes Nash equilibrium* bid functions. A Bayes Nash equilibrium for an auction is a bid-function profile $b = (b_1(\cdot), \dots, b_n(\cdot))$ such that for each bidder i and each possible use-value v_i for bidder i , the bid $b_i(v_i)$ maximizes bidder i ’s expected payoff given the vector $b_{-i} = (b_1(\cdot), \dots, b_{i-1}(\cdot), b_{i+1}(\cdot), \dots, b_n(\cdot))$ of bid functions for the other $n - 1$ bidders. This definition is not quite operational yet, since we do not explain how to compute the expected payoff to a bid. We will do this explicitly in the following sections, using the particulars of the different auction formats.

We start off by considering the second price auction. You may wonder why we don’t start with the first-price auction. The answer is that the theoretical predictions for the second-price auction are much easier to derive. This doesn’t mean that second-price auctions are less interesting, however. Reconciling observed behavior with the theory can be equally challenging, with just as many opportunities to gain insight into how people perceive auctions.

4.1 Second-Price Auctions

In the second-price auction the bidder who places the highest bid wins and she pays the amount of the second-highest bid. In these auctions, your bid

determines whether or not you are the auction winner, but it does not determine how much you pay. In the event you are the high bidder and win the auction you pay the amount of the second-highest bid. You might think that this suggests you should bid very high to ensure that you win the auction. And, in experiments this occurs quite often.¹ However, understand that whenever you bid above your use-value there is a chance that you will win the auction and pay a price that is above your value. That is, you might lose money! This happens when the second-highest bid is below your bid, but above your use-value.

Let's consider a 2-bidder auction where use-values range from 0 to \$100. Suppose, for example, that your use-value is \$52, you bid \$90, and the only other player bids \$70. In this case, because you have the highest bid you win the auction. However, your earnings from the auction would be $\$52 - \$70 = \$-18$. That is, you lose \$18. In fact, whenever you bid above your use-value you run the risk of losing money in this way. Moreover, it is important to understand that you cannot improve your payoff by bidding more than your value! If you need to bid more than your use-value to win the auction, it means the price you will have to pay is more than your value, in which case you will lose money by winning the auction. If the second-highest bid is below your use-value, then a bid equal to your use-value will win the auction anyway and earn the same payoff as a higher bid.

It is also true that you cannot increase your payoff by bidding less than your use-value and, in fact, you might miss out on possible gains. Suppose your use-value is \$52, you bid \$35, and the only other player bids \$40. In this case, because you do not have the highest bid you do not win the auction. Your earnings from the auction would be 0. Any bid above \$40 would have caused you to win the auction and earned you a payoff of $\$52 - \$40 = \$12$. It does not cost you anything to bid as high as your value and in fact you might gain by doing so if the other player's bid is higher than \$40 but below \$52. If you bid above your value, however, you run the risk of losing money if the other player bids more than your use-value.

Theorem 1. The unique equilibrium in weakly-dominant strategies of the second-price auction consists of each bidder following the strategy

$$b^{II}(v_i) = v_i.$$

Proof. To be added later.

Other asymmetric equilibria in weakly dominated strategies exist. Suppose the highest use value is \$100. Then, $b_1 = \$100, b_2, \dots, b_n = 0$ is also an equilibrium. However, $b_1 = \$100$ is weakly dominated by $b_1 = v_1$ and $b_i = 0$ is weakly dominated by $b_i = v_i$, for $i = 2, \dots, n$. See Blume and Heidhues (JET, 2005).

Report on Experiments 1 and 2: SPA with no entry fee and no reserve for 2- and 5-bidder auctions.

¹See Kagel and Levin, 1993.

More generally, assume that the second-price auction has an entry fee, c , and a reserve, r , where (in terms of normalized values) $0 \leq c \leq 1 - r \leq 1$. Since the entry fee is paid regardless of whether or not a bidder wins the auction, it is no longer a weakly dominant strategy for bidders to bid their value. In particular, bidders with low values will choose not to participate in the auction because it is unlikely they will earn enough profit in the auction to cover the entry fee (or even impossible if their value is less than c). The implication is that only bidders with high enough values will enter the auction. How high is high enough? Let's think about an equilibrium in which only bidders with a value above a certain cutoff, v_0 , choose to participate. As always, we let \tilde{y} denote the random variable whose realization y is the maximum of $n - 1$ independent draws from F . Recall that the distribution function of \tilde{y} is $G(y)$. The payoff to a bidder of type v_0 to participating and bidding v_0 is $(v_0 - r)G(v_0)$. In order for this bidder to be indifferent between participating and not participating we require

$$(v_0 - r)G(v_0) = c.$$

This equation defines our critical value v_0 . A bidder with a value greater than v_0 has a payoff at least as large this, because she still wins in the scenarios where y is less than v_0 , and now she also win in scenarios where y is greater than v_0 but less than her value. Hence any bidder with a value greater than v_0 will participate in the auction. Given that all bidders with values greater than v_0 participate we can use the same reasoning as we did in the case of no entry fee to show that value-bidding is an equilibrium. Thus, we have established that an equilibrium of second-price auction with reserve r and entry fee c consists of each bidder following the strategy

$$b^{II}(v_i) = \begin{cases} v_i & \text{if } v_i \geq v_0 \\ No & \text{if } v_i < v_0 \end{cases}$$

where v_0 is the solution to $(v_0 - r)G(v_0) = c$.

This symmetric equilibrium is in weakly dominant strategies only if $c = 0$.

4.1.1 Sunk Cost Fallacy

The theory predicts that once you have paid the entry fee you should bid the same way as you would if you had not entered. This is not surprising if you think about it. The argument we gave to show that value bidding is a weakly dominant strategy does not depend on the support of values for other bidders. Yet, it has been demonstrated in laboratory experiments that subjects often bid different after paying an entry fee. In particular experiment evidence reported in Charness, Garratt and Hartman (2008) shows that people often deduct a fraction of their entry fee from their bid. Once a subject has paid to enter the auction, any entry fee that they might have paid should have no bearing on their bidding decision. This is a nice instance of the sunk cost fallacy.

Report on Experiments 3 and 4: 2-bidder SPA with no entry fee and reserve $r = \$50$, 2-bidder SPA with entry fee $c = \$25$ and no reserve.

4.2 First-Price Auctions

In the first-price auction the bidder who places the highest bid wins and she pays the amount of her bid. In these auctions, your bid determines whether or not you are the auction winner *and* the price you pay. Since you pay your own bid it is not optimal to bid your value, as was the case for the second-price auction. This would ensure you a payoff of 0. By lowering your bid below your use-value you make a positive payoff when you do win. However, lowering your bid reduces the probability that you win the auction. In order to win the auction your bid must be higher than everyone else's. Assuming other bidders also bid some fraction of their use-values, their bids will be random variables. Hence, the lower your bid the lower the chance that all bids are below yours. The optimum bid must therefore balance the gain in payoff from bidding lower, and hence earning a higher profit when you win the auction, with the loss in probability of winning.

Theorem 2. The unique, symmetric equilibrium of the first-price auction consists of each bidder following the strategy

$$b^I(v_i) = \int_0^{v_i} y \frac{g(y)}{G(v_i)} dy$$

Proof. Note that any bid greater than $b^I(1)$ is dominated by $b^I(1)$. The payoff to a single deviator from bidding $b^I(z)$, $z \in [0, 1]$, when value is v (ignore subscript i) is

$$\begin{aligned} \Pi(b^I(z), v) &= G(z)[v - b^I(z)] \\ &= G(z)v - \int_0^z yg(y)dy \\ \text{(integration by parts)} &= G(z)v - G(z)z + \int_0^z G(y)dy \\ &= G(z)(v - z) + \int_0^z G(y)dy \end{aligned}$$

Likewise,

$$\begin{aligned} \Pi(b^I(v), v) &= G(v)[v - b^I(v)] \\ &= G(v)v - \int_0^v yg(y)dy \\ \text{(integration by parts)} &= G(v)v - G(v)v + \int_0^v G(y)dy \\ &= \int_0^v G(y)dy \end{aligned}$$

Hence, the loss to deviating is

$$\Pi(b^I(v), v) - \Pi(b^I(z), v) = \int_0^v G(y)dy - G(z)(v - z) - \int_0^z G(y)dy \quad (*)$$

Consider $z > v$

$$\begin{aligned}
 (*) &= G(z)(z - v) - \int_v^z G(y)dy \\
 &= \int_v^z G(z)dy - \int_v^z G(y)dy \\
 &= \int_v^z [G(z) - G(y)]dy > 0
 \end{aligned}$$

Consider $z < v$

$$\begin{aligned}
 (*) &= \int_z^v G(y)dy - G(z)(v - z) \\
 &= \int_z^v [G(y) - G(z)]dy > 0
 \end{aligned}$$

Hence, deviating from $b^I(v)$ is always costly.

In the case of F uniform, the equilibrium bid function is

$$\begin{aligned}
 b^I(v_i) &= \int_0^{v_i} y \frac{(n-1)y^{n-2}}{v_i^{n-1}} dy \\
 &= \frac{(n-1)}{v_i^{n-1}} \int_0^{v_i} y^{n-1} dy \\
 &= \frac{(n-1)}{v_i^{n-1}} \left[\frac{y^n}{n} \right]_0^{v_i} \\
 &= \frac{(n-1)}{v_i^{n-1}} \frac{v_i^n}{n} \\
 &= \frac{(n-1)}{n} v_i.
 \end{aligned}$$

It is relatively easy to show that this is a symmetric equilibrium bidding strategy. We can apply the Nash equilibrium logic directly. Suppose bidders $i = 2, \dots, n$ are following the strategy $b^I(v_i) = \frac{(n-1)}{n}v_i$ and consider an arbitrary bid b for bidder 1. Bidder 1 wins with bid b with probability $G(b)$, which is the probability that the other $n-1$ bidders place bids less than b . All the other bidders are assumed to bid $\frac{n-1}{n}v_i$. Hence the probability that any one of their bids is less than b is $Pr[\frac{n-1}{n}v_i < b] = Pr[v_i < \frac{n}{n-1}b] = \frac{n}{n-1}b$, in the case of uniform F. The probability that all of the other $n-1$ bids is less than b is, assuming independent value draws, $(\frac{n}{n-1}b)^{n-1}$. Thus, the expected payoff of bidder 1 if she bids an amount b is

$$\begin{aligned}
 G(b)(v_1 - b) &= \left(\frac{n}{n-1}b\right)^{n-1}(v_1 - b) \\
 &= \left(\frac{n}{n-1}\right)^{n-1}b^{n-1}v_1 - b^n.
 \end{aligned}$$

Now let's maximize this with respect to the choice of b . The first-order condition for a maximum is

$$\left(\frac{n}{n-1}\right)^{n-1}[(n-1)b^{n-2}v_1 - nb^{n-1}] = 0$$

or

$$(n-1)v_1 - nb = 0$$

The solution is

$$b^* = \frac{(n-1)}{n}v_1.$$

Hence, if bidder 1 believes that everyone else will follow the symmetric bidding strategy she best responds by following it as well. Since this is true for any bidder, we have verified that $b^I(v_i) = \frac{(n-1)}{n}v_i$, for $i = 1, \dots, n$, is a symmetric Nash equilibrium in the case of uniform F .

Report on Experiments 5 and 6: FPA with no entry fee and no reserve for 2- and 5-bidder auctions.

More generally, assume that the first-price auction has an entry fee, c , and a reserve, r , where $0 \leq c \leq 1 - r \leq 1$. An equilibrium of FPA(r, c) consists of each bidder following the strategy

$$b^I(v_i) = \begin{cases} \int_{v_0}^{v_i} y \frac{g(y)}{G(v_i)} dy + r \frac{G(v_0)}{G(v_i)} & \text{if } v \geq v_0 \\ \text{No} & \text{if } v < v_0 \end{cases},$$

where v_0 is defined implicitly by the equation $G(v_0)(v_0 - r) = c$. We will not go over a formal proof of this claim (See Matthews, 1995, Section 8.1). The value v_0 is same as in the second-price auction. This is the unique symmetric equilibrium.

Example 4.1 *In the case of F uniform and $n = 2$,*

$$b^I(v_i) = \begin{cases} \frac{1}{2} \frac{v_i^2 - v_0^2}{v_i} + r \frac{v_0}{v_i} & \text{if } v_i \geq v_0 \\ \text{No} & \text{if } v_i < v_0 \end{cases},$$

where v_0 is defined implicitly by the equation $v_0(v_0 - r) = c$.

Report on Experiments 7 and 8: 2-bidder FPA with no entry fee and reserve $r = \$50$, 2-bidder FPA with entry fee $c = \$25$ and no reserve. Useful Information: For $r = 0$ and $c = 25$, we have $v_0 = 50$ and hence

$$b^I(v_i) = \begin{cases} \frac{1}{2}v_i - \frac{1}{8v_i} & \text{if } v_i \geq .5 \\ \text{No} & \text{if } v_i < .5 \end{cases}.$$

For $r = 50$ and $c = 0$, we have $v_0 = 50$ and hence

$$b^I(v_i) = \begin{cases} \frac{1}{2}v_i + \frac{1}{8v_i} & \text{if } v_i \geq .5 \\ \text{No} & \text{if } v_i < .5 \end{cases}.$$

4.2.1 Risk Aversion

In this section we examine the impact of risk aversion on bidding behavior in first-price auctions. For simplicity we will assume there is no entry fee or reserve. We did not discuss risk aversion in the context of second-price auctions because risk aversion does not affect bidding behavior in that auction format when there is no entry fee and no reserve. This is because in a second-price auction a person's bid only affects the probability that they win the auction; it does not affect their payoff. Hence, the decision of how much to bid for any given value does not depend on the person's preferences over payoffs.² In the first-price auction things are more complicated. Below we will describe the equilibrium bid functions for the case of n bidders under the assumption that the bidders have constant relative risk aversion (CRRA) utility functions.³ Again we will only consider the case of no entry fee and no reserve. We will discuss how the theoretical bid functions compare to the experimental classroom data.

We wish to introduce the possibility that bidders exhibit differing degrees of risk aversion. We do so by assuming that each bidder's utility over money payoffs depends on a risk parameter α_i . In particular, suppose the utility over money payoffs for each bidder i is given by the function $u(z) = z^{\alpha_i}$, where $0 < \alpha_i < 1$.⁴ This means that bidder i evaluates a gamble over two different wealth levels in the following way. Her utility of a gamble that gives her a wealth of \$0 with probability .5 and a wealth of \$100 with probability .5 is $.5u(0) + .5u(100) = .5 * 100^{\alpha_i}$. If, for example, $\alpha_i = \frac{1}{2}$, then $.5u(0) + .5u(100) = .5 * 100^{\frac{1}{2}} = .5 * 10 = 5$. In contrast, her utility to receiving the expected value of this gamble, $.5 * 0 + .5 * 100 = 50$, is $u(50) = 50^{\frac{1}{2}}$, which is approximately equal to 7.07. Hence bidder i , with risk parameter $\alpha_i = \frac{1}{2}$, will prefer to be paid \$50, the expected value of the gamble, for sure to the gamble itself. This means she is risk averse.

In fact, there are a variety of ways to demonstrate that higher degrees of risk aversion are consistent with lower values of α_i (i.e., closer to 0). For instance, we could contemplate increasing the probability of the \$100 prize in order to get her to accept the gamble rather than a certainty cash payment of \$50. The minimum probability p that would work is determined by solving

$$p100^{\alpha_i} = 50^{\alpha_i}.$$

If $\alpha_i = \frac{1}{2}$, the solution is $p = .707$. Hence a bidder with risk parameter $\alpha_i = \frac{1}{2}$, would accept a gamble that paid \$100 with probability p to a certainty cash payment of \$50 so long as $p > .707$. For general α_i , the value of p that solves the above expression is $p = \left(\frac{1}{2}\right)^{\alpha_i}$. The lower α_i is, the higher p must be. That is, the more risk averse the person is, the more we must compensate her (by

²In the case of a positive entry fee, the minimum value v_0 at which a bidder would be willing to enter the auction would be higher.

³See Cox, Smith and Walker, 1982, as a primary source for this material.

⁴This implies constant relative risk aversion equal to $1 + \alpha_i$.

giving her favorable terms) to accept a gamble. This will be true for any two money amounts.

Suppose each bidder has a value that is drawn from the uniform distribution on $[0, 1]$. I.e., $F(v) = v$ for all bidders. We will now verify that the symmetric equilibrium bid function is

$$b^I(v_i) = \frac{n-1}{n-1+\alpha_i}v_i,$$

Once again we apply the Nash equilibrium logic directly. Suppose bidder $i = 2, \dots, n$ follows the strategy $b^I(v_i) = \frac{1}{\alpha_i+1}v_i$ and consider an arbitrary bid b for bidder 1. Bidder 1 wins with bid b with probability $G(b)$, which is equal to

$$\begin{aligned} \prod_{i=2}^n Pr\left[\frac{n-1}{n-1+\alpha_i}v_i < b\right] &= \prod_{i=2}^n Pr\left[v_i < \frac{n-1+\alpha_i}{n-1}b\right] \\ &= \prod_{i=2}^n \frac{n-1+\alpha_i}{n-1}b \\ &= \left(\prod_{i=2}^n \frac{n-1+\alpha_i}{n-1}\right)b^{n-1}, \end{aligned}$$

in the case of uniform F . Thus, the expected payoff of bidder 1 if she bids an amount b is

$$G(b)(v_1 - b)^{\alpha_1} = \left(\prod_{i=2}^n \frac{n-1+\alpha_i}{n-1}\right)b^{n-1}(v_1 - b)^{\alpha_1}.$$

Now let's maximize this with respect to the choice of b . The first-order condition for a maximum is

$$\left(\prod_{i=2}^n \frac{n-1+\alpha_i}{n-1}\right) \left((n-1)b^{n-2}(v_1 - b)^{\alpha_1} - b^{n-1}\alpha_1(v_1 - b)^{\alpha_1-1}\right) = 0.$$

Dividing both sides by $\left(\prod_{i=2}^n \frac{n-1+\alpha_i}{n-1}\right)b^{n-2}(v_1 - b)^{\alpha_1-1}$ yields

$$(n-1)(v_1 - b) - b\alpha_1 = 0$$

or

$$b^* = \frac{n-1}{n-1+\alpha_1}v_1.$$

Hence, if bidder 1 believes that all the other bidders will follow the symmetric bidding strategy she best responds by following it as well. Hence, we have verified that $b^I(v_i) = \frac{v_i}{\alpha_i+1}$, for $i = 1, \dots, n$, is a symmetric Nash equilibrium in the case of uniform F .

Note that the equilibrium bid functions are still linear and have an intercept of 0. However, now the slope is somewhere between the risk-neutral prediction of $\frac{n-1}{n}$ that corresponds to $\alpha_1 = 1$ and a slope of 1, the slope that corresponds to

$\alpha_1 = 0$. Note also that each bidder's strategy depends on her own risk-aversion parameter, α_i , but not on those of the other bidders. Hence, we do not require that individuals know or make any conjectures about the risk aversion of their opponents!

The result that equilibrium bids do not depend on opponent's degree of risk aversion may seem surprising, but it is actually apparent from the risk-neutral case. Specifically, the best response of a bidder in a first price auction involving two bidders is to bid one-half of her value regardless of the fraction between $1/2$ and 1 that her opponent bids. In the case of a risk-averse opponent, if we set $\alpha_1 = 1$ and vary α_2 from 0 to 1 we trace out the same scenarios.

Report on individual data from Experiments 5 and 6 using calculations from Assignment 2.

4.3 English and Dutch Auctions

4.3.1 Strategic Equivalence of First-Price and Dutch Auctions

4.3.2 Jump Bidding in English Auctions

