Introduction to Auction Theory

Econ 191AU

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Introduction: A Historical Perspective

- Herodotus reports that auction were used in Babylon as early as 500 B.C.

- 193 A.D. the Pretorian Guard sold the Roman Empire by means of an auction

- Wide array of commodities sold by means of auction
  - Tobacco, fish, fresh flowers
  - Bond issues by public utilities and long-term U.S. Treasury securities
  - Facilitate transfer of assets from public to private hands
    - Industrial enterprises in Eastern Europe and former Soviet Union
    - Rights to harvest natural resources such as oil and timber
    - Rights to use the electromagnetic spectrum for communications

Common Auction Formats

- Open ascending price or English auction
  - Auctioneer begins by calling out low price and raises it in small increments provided there are two or more active bidders
  - Auction ends when there is only one remaining bidder
- The open descending or Dutch auction
  - Descending counterpart to English auction
  - Not commonly employed, but it is of conceptual interest

Common Auction Formats

- First-Price, Sealed Bid Auction
  - Bidders submit bids in sealed envelopes
  - At a pre-determined time, auctioneer opens all envelopes and ranks bids
  - Highest bidder obtains object and pays his bid amount

- Second-Price, Sealed Bid Auction
  - Bidder submit bids in sealed envelopes
  - At a pre-determined time, auctioneer opens all envelopes and ranks bids
  - Highest bidder obtains object and pays second highest bid amount

Common Auction Formats

- All-Pay auction
  - Bidders submit bids (open or closed)
  - At a pre-determined time, auctioneer opens all envelopes and ranks bids
  - Every bidder pays what they bid regardless of whether or not they have the highest bid
  - Highest bidder obtains object
  - Examples: Elections, almost any kind of contest or sports event, research and development, wars, lobbying, queues

Valuations

- If seller knew values, she could offer object to bidder with highest values at or just below his willingness to pay

- Auctions are used because seller is unsure about values bidders attach to object being sold
  - Seller does not know the values of potential bidders
  - Bidders may know their own values, but not the values of potential opponents (or the seller)

Valuations

- Private values paradigm
  - Bidder knows value of object to himself but not others at time of bidding
  - Knowledge of other bidders' values would not affect how much the object is worth to given bidder
  - Assumption most plausible when values are derived from consumption or use alone

- Common values
  - Value is the same for all bidders
  - Each bidder has their own imprecise *estimate* of the value of the object
  - Appropriate when value of object is derived from a market price that is unknown at time of bidding
  - Tract of land with unknown quantity of oil underground

- Interdependent values
  - Each bidder receives a <u>private</u> signal about her value
  - Each bidder's value is based on a combination of the private signals.

Equivalence of Auction Formats

- Dutch, auction is strategically equivalent to first-price auction
  - In first-price auction, strategy maps type (private information) into bid
  - Dutch auction offers no useful information to bidders, only info available is that some bidder has agreed to buy at a given price
  - Bidding a certain amount in first-price auction is equivalent to offering to buy at that amount in Dutch auction

- In IPV setting, English auction is equivalent to second-price auction
  - In English auction, cannot be optimal to stay in after price exceeds value or exit before value is reached
  - Optimal to bid value in second-price auction
  - Equivalence breaks down with interdependent values (information revealed when bidders drop out may affect individual's estimate of value) or if jump bidding is allowed

Revenue versus Efficiency

- Two criteria for evaluating performance of an auction
  - Revenue or expected selling price
  - Allocative efficiency: Does the object get sold to the bidder who values it the most or who will make the best use of it for society?
    - Important goal in the case of privatization.
    - U.S. Congress directed the FCC to choose an auction design for allocating spectrum licenses that (to quote Al Gore) puts "licenses into the hands of those who value them the most" (Milgrom 2004, p.5)
  - Tradeoff between revenue and efficiency
- Other issues in choosing a mechanism
  - Susceptibility to collusion and simplicity/transparency of auction rules

Reserve Prices

- Until now we have implicitly assumed that the seller is willing to part with the object at any positive price.
- If the object has some intrinsic value to the seller, he would be foolish to sell it for less than this value.
- This possibility can be avoided by specifying a *reserve price* below which the object will not be sold.

Reserve Prices

- In some real-world auctions it is announced that there is a reserve price, but the actual reserve is kept as a secret
  - If the reserve is not met it is simply announced that the object has not been sold.
  - If the auction is a one-off event then this is **inconsequential**, as it does not convey any extra information to buyers and won't change their bidding behavior.
  - The seller may however want to keep the reserve a secret if he plans to negotiate with the highest bidder if the reserve is not met, or to try to re-auction again later.

Entry Fees

- Sometimes it costs money to participate in an auction.
- Entry fee is paid by participants whether they win the auction or not.
- Entry fees can result in negative payments for some bidders

Agenda

- We model auctions as a strategic game:
  - Players: The bidders and the seller(s)
  - Strategies: Bidders decide how much to bid; the seller decides the auction format and rules (the auction *design*)
  - Payoffs: Depend on the design of the auction and the bids, but typically the surplus from the trade is split in some way between the winning bidder and the seller.

Agenda

- Objectives of the game theoretic study of auctions are:
  - To model and predict auction outcomes
    - how do bidders decide how much to bid
  - To understand how changes in auction rules affect bidding behavior
  - To identify the circumstances in which auctions are an *efficient* mechanism for allocating scarce resources.
  - To understand why theoretical predictions may fail to predict auction outcomes

Experiment 1

- 2-bidder, second-price auction with no reserve price and no entry fee.
- 10 rounds without feedback.

Experiment 2

- FIVE-bidder, second-price auction with no reserve price and no entry fee.
- 10 rounds without feedback.

Experiment 3

- 2-bidder, second-price auction with a $50 reserve price and no entry fee.
- 10 rounds without feedback.

Experiment 4

- 2-bidder, second-price auction with no reserve price and a $25 entry fee.
- 10 rounds without feedback.

All-Pay Auctions

- In an all-pay auction, every bidder pays what they bid regardless of whether or not they win.
- Examples:
  - Elections
  - Almost any kind of contest or sports event
  - Research and Development
  - Wars
  - Lobbying
- Since bids are wasted if you don't win, bidders have a strong incentive to bid aggressively if they bid at all.

All-Pay Auctions

- A simple all-pay auction:
  - The object for sale is worth 1 to all of *n* identical bidders and all of them know this valuation exactly.
  - Clearly no one will bid more than 1.
  - Each bidder's bid is an amount *x* in the interval [0,1].
  - The winner is the highest bidder and gets a payoff of 1 – *x*
  - The payoff of all other bidders is –*x*
- There is no pure strategy equilibrium of this game.
- A mixed strategy of a player is a *probability distribution* over the possible bids between 0 and 1.
- We represent a mixed strategy by a probability distribution function P, where *P*(*x*) which gives the probability that a player bids *less* than some amount *x* between 0 and 1.

All-Pay Auctions

- We look for a symmetric equilibrium.
  - Suppose bidders 2,…,n used the mixed strategy P and consider bidder 1.
  - Suppose bidder 1 bids x.
  - Her expected payoff is given by

$$P(x)^{n-1}(1-x) + (1- P(x)^{n-1})(-x)$$

$$= P(x)^{n-1} - x$$

  - In order for bidder 1 to be willing to randomize we require $P(x)^{n-1} - x = c$.

All-Pay Auctions

- Moreover, since P is a probability distribution function we require P(0)=0
- Hence, c=0
- Hence, $P(x) = x^{1/(n-1)}$

All-Pay Auctions

- Equilibrium mixed strategies in this example: