Vickery-Clark-Groves Mechanism

Source:
http://auctiontheorycourse.wordpress.com/slides-and-notes/
Previously...

- We studied single-item auctions

- Bidders have values $v_i$ for an item

- A winning bidder gets a utility of $u_i = v_i - p_i$
  - A losing bidder pays nothing and get $u_i = 0$
Previously...

• **Seller possible goals:**
  – Maximize **social welfare** (efficiency)
    • 2nd-price *(Vickrey)* auction
  – Maximize **revenue**
    • 2\textsuperscript{nd}-price auction with a reserve price *(Myerson)*
      – For example, reserve-price=50 for the uniform distribution on [0,100]
      – Reserve price is independent of the number of players.
      – Optimality assumes a technical assumption on the distributions.
  • Revenue equivalence
Previously ...

- We saw that in single-item auctions we can achieve efficiency with dominant strategies.

- Can this be achieved in other models?
Today

• This class: Moving from a specific example (single-item auctions) to a more general mechanism design setting.

• **Main goal:** in the presence of incomplete information, design the right incentives such that the efficient outcome will be chosen.
Example 1: Roommates buy TV

• Consider two roommates who would like to buy a TV for their apartment.

• TV costs $100

• They should decide:
  – Do they want to buy a TV together?
  – If so, how should they share the costs?
Example 2: Selling multiple items

• Each bidder has a value of $v_i$ for an item.

• But now we have 5 items!
  – Each bidder wants only one item.

• An efficient outcome: sell the items to the 5 bidders with the highest values.

$70 \quad $30 \quad $27 \quad $25 \quad $12 \quad $5 \quad $2$
Vickrey-Clarke-Groves (VCG) mechanisms

• **Goal:** implement the *efficient outcome* in dominant strategies.

• **A general method** to do this: **VCG**
  – 2\textsuperscript{nd}-price auction is a special case

• **Solution (intuitively):** players should pay the “damage” they impose on society.
VCG basic idea (cont.)

In more details:

• You can maximize efficiency by:
  – Choosing the **efficient outcome** (given the bids)
  – Each player pays his “**social cost**” (how much his existence hurts the others).

\[ p_i = \text{Optimal welfare (for the other players) if player } i \text{ was not participating.} - \text{Welfare of the other players from the chosen outcome} \]
VCG idea in single item auctions

- \( P_i = \) Optimal welfare (for the other players) if player \( i \) was not participating.

- Welfare of the other players from the chosen outcome

  = 2\(^{nd}\)-highest value.

  When \( i \) is not playing, the welfare will be the second highest.

  = 0.

  When \( i \) wins, the total value of the other is 0.

→ By VCG payments, winners pay the 2\(^{nd}\)-highest bid, and loser pays nothing!
VCG in 5-item auctions

• \( p_i = \)

Optimal welfare (for the other players) if player \( i \) was not participating.

Welfare of the other players from the chosen outcome

\[ = 30 + 27 + 25 + 12 + 5 \]

The five winners when \( i \) is not playing.

\[ = 30 + 27 + 25 + 12. \]

The other four winners.

What is my VCG payment?

$70 $30 $27 $25 $12 $5 $2
VCG in $k$-item auctions

• VCG rules for $k$-item auctions:
  – Highest $k$ bids win.
  – Everyone pay the $(k+1)^{st}$ bid.

And truthfulness is a dominant strategy here too.
Truthfulness of VCG - intuition

• The trick is actually quite simple:
  — By lying, players may be able to change the outcome.
  — But their utility depends on the total efficiency.

→ Therefore, players want the efficient outcome to be chosen. Lying may ruin this.
The VCG family

- From the proof, we can see that the VCG mechanism is actually a family of mechanisms.

- The VCG mechanism:
  - Outcome \( w^* \) is chosen.
  - Each bidder pays:

\[
\sum_{j \neq i} v_j(t_j, w_{-i}^*) - \sum_{j \neq i} v_j(t_j, w^*)
\]

This could be any function of the other bids.
The VCG family

• From the proof, we can see that the VCG mechanism is actually a family of mechanisms.

The VCG mechanism:
– Outcome \( w^* \) is chosen.
– Each bidder pays:

\[
 h(t_{-i}) - \sum_{j \neq i} v_j(t_j, w^*)
\]

Choosing \( h(t_{-i}) = \sum_{j \neq i} v_j(t_j, w_{-i}^*) \)
ensures **individual rationality** (when values are positive)
the utility of each player is never negative and **no positive transfers**, i.e., players are not paid to participate.
Example 1: Roommates buy TV

- TV cost $100

- Bidders are willing to pay $v_1$ and $v_2$
  - Private information.

- VCG ensures:
  - Efficient outcome (buy if $v_1+v_2>100$)
  - Truthful revelation.

**In our model:**
- Welfare when buying: $v_1+v_2$
- Welfare when not buying: 100 (saved the construction cost)
Example 1: Roommates buy TV

• Let’s compute VCG payments.

• Consider values \( v_1 = 70, \ v_2 = 80 \).
  – With player 1: value for the others is 80.
  – Without player 1: welfare is 100.
  \( \rightarrow \quad p_1 = 100 - 80 = 20 \)
  – Similarly: \( p_2 = 100 - 70 = 30 \)
  – Total payment received: \( 20 + 30 < 100 \)

• Cost is not covered!

In general, \( p_1 = 100 - v_2, \ p_2 = 100 - v_1 \)

\[ p_1 + p_2 = 100 - v_1 + 100 - v_2 = 100 - (v_1 + v_2 - 100) < 100 \]

• Whenever we build, cost is not covered.
Example 1: Roommates buy TV

Conclusion: in some cases, the VCG mechanism is not budget-balanced. (spends more than it collects from the players.)

This is a real problem!

There isn’t much we can do: It can be shown that there is no mechanism that is both efficient and budget balanced.
Context: Public goods

• The roommate problem is known as the “public good” problem.

• Consider a government that wants to build a bridge.
  – When to build? If the total welfare is greater than the cost.
  – How the cost is shared?
  – Efficiency vs. Budget Balance (cannot achieve both).
Summary: VCG

• Efficiency is desired in various settings.
• We saw: one can always achieve this with (dominant-strategy) equilibrium.
  – “implementation”

• This is the only general goal that is known to be “implementable”.

• **Pros**: No distributional assumptions, strong equilibrium concept, individually rational.
• **Cons**: not budget balanced, prone to other manipulations.