

# Illiquidity in the Interbank Payment System Following Wide-Scale Disruptions

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## Abstract

We show how the interbank payment system can become illiquid following wide-scale disruptions. Two forces are at play in such disruptions—operational problems and changes in participants’ behavior. We model the interbank payment system as an  $n$ -player game and utilize the concept of a potential function to describe the process by which one of multiple equilibria emerges after a wide-scale disruption. If the disruption is large enough, hits a key geographic area, or hits a “too-big-to-fail” participant, then the smooth processing of payments can break down, and central bank intervention might be required to reestablish the socially efficient equilibrium. We also explore how the network topology of the underlying payment flow among banks affects the resiliency of the interbank payment system. The paper provides a theoretical framework to analyze the effects of events such as the September 11 attacks.

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# 1 Introduction

The events of September 11, 2001 and to a lesser extent the North American blackout of August 14, 2003 highlighted the fact that parts of the financial system are vulnerable to wide-scale disruptions. Moreover, the events underscored the fact that the financial system consists of a complex network of interrelated markets, infrastructures, and participants, and the inability of any of these entities to operate normally can have wide-ranging effects even beyond its immediate counter-parties. Not surprisingly, the financial industry and regulators are devoting considerable resources to business continuity measures and planning in order to strengthen the resiliency of the U.S. financial system. The primary objective is to minimize the immediate systemic effects on the financial system of large scale shocks.

At the apex of the U.S. financial system are a number of critical financial markets that provide the means for both domestic and international financial institutions to allocate capital and manage their exposures to liquidity, market, credit and other types of risks. These markets include Federal funds, foreign exchange, commercial paper, government and agency securities, corporate debt, equity securities and derivatives. Critical to the smooth functioning of these markets are a set of wholesale payments systems and financial infrastructures that facilitate clearing and settlement.<sup>1</sup> Operational difficulties involving these entities or their participants can create difficulties for other systems, infrastructures and participants. Such spillovers might cause liquidity shortages or credit problems and hence potentially impair the functioning and stability of the entire financial system.

Wide-scale disruptions may not only present operational challenges for participants of the core payment and securities settlement system, but may also induce participants to change behavior in terms of how they conduct their business. The actions of participants have both the potential to mitigate, but also to augment the adverse effects.<sup>2</sup> Hence, understanding how participants react when faced with operational adversity by their counter-parties is crucial for both operators and regulators in designing counter measures, devising policy, and providing emergency assistance if called for.

Despite their immense importance for the financial system, very little research is available on financial infrastructures and how they and their stakeholders respond to disruptions. Basically, two strands of literature exist. One strand investigates, by way of simulations, the potential contagion effects of a participant failing to meet obligations when due (see e.g. Humphrey (1996), Angelini (1996) and Devriese and Mitchell (2006)). The other strand consists of event studies of actual disruptions and their effects to the clearing, settlement and payment system. Bernanke (1990) looks at clearing and settlement during the stock market crash of 1987 whereas Fleming and Garbade (2002), McAndrews and Potter (2002) and Lacker (2004) all look at the impacts of the attacks of September 11, 2001.

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<sup>1</sup>Examples include the Depository Trust & Clearing Cooperation (DTCC), the Clearing House Inter-bank Payment System (CHIPS), Continuous Linked Settlement (CLS), and the Federal Reserve's Federal Funds and Securities Services.

<sup>2</sup>For a description of the extraordinary cooperative efforts undertaken by participants to overcome the problems for the payment and securities settlement system caused by the events of September 11, 2001 see the 2001 annual report of the Federal Reserve Bank of New York.

The backbone of the payment and securities settlement system in the U.S. is the Federal Reserve's Fedwire Funds Service (Fedwire). Fedwire is a Real Time Gross Settlement (RTGS) system in which payments are settled individually and with instant finality in real time. Over 9,500 participants use Fedwire to send or receive time critical and/or large value payments on behalf of corporate and individual clients, settle positions with other financial institutions stemming from other payment systems, clearing arrangements or securities settlement, submit federal tax payments and buy and sell Federal Reserve funds. The average daily number of payments in the first half of 2006 was 525,000 and the average value transferred was around \$2.1 trillion per day. Fedwire continued to operate during the events of September 11, 2001 and August 14, 2003, but in both cases the Federal Reserve had to intervene by extending the operating hours and by providing emergency liquidity.

On September 11, 2001, the massive damage to property and communications systems in lower Manhattan made it more difficult, and in some cases impossible, for many banks to execute payments to one another. The failure of some banks to make payments also disrupted the payments coordination by which banks use incoming payments to fund their own transfers to other banks. Once a number of banks began to be short of incoming payments, some became more reluctant to send out payments themselves. In effect, banks were collectively growing short of liquidity. The Federal Reserve recognized this trend toward illiquidity and provided liquidity through the discount window and open market operations in unprecedented amounts in the following week. Federal Reserve opening account balances peaked at more than \$120 billion compared to approximately \$15 billion prior. Moreover, the Federal Reserve waived the overdraft fees it normally charges. On September 14, daylight overdrafts peaked at \$150 billion, more than 60 percent higher than usual. (see Ferguson, 2003). Following the blackout in the Midwest and Northeast on August 14, 2003 commercial banks borrowed \$785 million from the discount window compared to the normal range of \$100 - \$200 million.

Several papers (e.g. McAndrews and Rajan, 2000, Bech and Garratt, 2003, and McAndrews and Potter, 2002) advocate interpreting the payment decisions of banks participating in Fedwire as a game. McAndrews and Rajan argue that the timing of payments resembles a coordination game. This idea emerges in the formal analysis of the intraday liquidity management game of Bech and Garratt where it is shown that the game played by banks is a type of coordination game known as the stag hunt game. In this game there are two Nash equilibria, one which involves early settlement of payments and the other late settlement. Evidence that banks select between these equilibria is found by McAndrews and Potter who document the breakdown and reemergence of coordinated payments following the events of September 11, 2001.

In what follows we shed light on why coordination on early settlement occurs in normal times and how operational difficulties for participants in Fedwire are likely to effect equilibrium selection. These are the central issues in study of systemic risk in payments systems, which is a major concern of central banks around the world.<sup>3</sup> Our argument involves con-

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<sup>3</sup>Hendricks, Kambhu and Mosser (2007, p. 83) state: "...the key characteristic of systemic risk is the movement from one stable (positive) equilibrium to another stable (negative) equilibrium for the economy and financial system. According to this view, research on systemic risk should focus on the potential causes

structuring a potential function for the game played by banks. The potential function is used to summarize welfare changes that result from movements by individual banks. Changes by individual banks that improve individual welfare necessarily increase potential. Hence, we look to adjustment processes that converge to potential maximizing strategy profiles as indicators of how the system will operate. We are able to characterize circumstances under which the system will converge to an early (versus late) payment equilibrium. Moreover, we describe the policy options of the central bank and discuss when moral suasion is likely to be effective. At first, we assume that all banks are identical. Subsequently, we introduce heterogeneity across banks and are able to add new insight into the question, when a bank is too big to fail. In addition, we investigate how different degrees of interconnectedness between banks, i.e., network topologies affect resiliency vis-a-vis wide-scale disruptions.

## 2 The $n$ -player intraday liquidity management game

Envision an economy with  $n$  identical banks using a RTGS system operated by the central bank to settle interbank claims. Banks seek to minimize their settlement costs. The business day consists of two periods: morning and afternoon. Banks start the business day with a zero balance on their settlement accounts at the central bank. At dawn each bank receives  $(n - 1)$  requests from customers to pay a customer of each of the other  $(n - 1)$  banks \$1 as soon as possible. For simplicity, assume that banks either process all requests in the morning period or postpone them all until the afternoon period. In order to process a payment request a bank must have sufficient funds in their account at the central bank. Banks without sufficient funds can borrow funds from the central bank. Unlike most other central banks, the Federal Reserve does not require intraday overdrafts to be backed by collateral for banks in good standing. Instead banks are charged a fee for such overdrafts. Here, it is assumed that banks are charged a fee  $F > 0$  per dollar if their settlement account is overdrawn at the end of a period.<sup>4</sup> The fee can be thought of as an insurance premium reflecting the credit risk assumed by the Federal Reserve.

Banks can avoid fees by delaying payments. However, delaying entails both private and social costs. First, delay of settlement might displease customers and counterparties as they are left with greater uncertainty or even explicit costs. They might demand compensation or take their business elsewhere in the future. Second, delaying payments creates an exposure to operational incidents that can prevent the settlement of payments before the close of business. Third, the process of delaying might itself be costly as additional resources have to be devoted to managing intraday positions. Fourth, as argued by Kahn et al. (2003) delay increases the length of time participants may be faced with credit risk exposures vis-a-vis one another. We assume that the cost of delaying faced by banks is  $D > 0$  per dollar. Hence, banks have to trade-off the potential cost of mobilizing liquidity against the cost of delay.

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and propagation mechanisms for the phase transition to a new but much less desirable equilibrium as well as the reinforcing feedbacks that tend to keep the economy and financial system trapped in that equilibrium.”

<sup>4</sup>See Coleman, (2002) for a discussion of the evolution of the Federal Reserve’s intraday credit policies.

Formally, the player set is  $N = \{1, 2, \dots, n\}$ . The set of strategies for each bank  $i$  is  $S_i = \{m, a\}$  where  $m$  denotes “all requests processed in the morning” and  $a$  denotes “all requests postponed until the afternoon.” Let  $N_{-i} = N \setminus \{i\}$ . The settlement cost of bank  $i$  depends on the strategies played by all banks and is given by  $c(s_i, s_{-i})$  where  $s_i \in S_i$  is the strategy played by bank  $i$  and where  $s_{-i} \in S_{-i} = \times_{j \in N_{-i}} S_j$  is the  $(n - 1)$  dimensional profile of opponents strategies. The price that banks charge for processing is assumed to be fixed and for simplicity it is set to zero. The payoff function for bank  $i$  is thus equal to the negative of the settlement cost function, that is

$$\pi_i(s_i, \mathbf{s}_{-i}) = -c(s_i, \mathbf{s}_{-i}) \quad (1)$$

The settlement cost a bank incurs by playing morning is the overdraft at noon times the overdraft fee per dollar. The overdraft at noon is given by the number of other banks playing afternoon, i.e., the number of banks from which the bank does not receive an offsetting payment (dollar) in the morning. On the other hand, if a bank plays afternoon then the settlement cost is the value of delayed payments times the delay cost per dollar. Formally, the payoff function of bank  $i$ , for each strategy choice in  $S_i$ , is given by

$$\pi_i(m, \mathbf{s}_{-i}) = -(n - 1 - |\mathbf{s}_{-i}|_m)F \quad (2)$$

$$\pi_i(a, \mathbf{s}_{-i}) = -(n - 1)D \quad (3)$$

where  $|\mathbf{s}_{-i}|_m$  is the number of banks playing the morning strategy in the strategy profile  $s_{-i}$ .

**Example 1** *In the 2-player case the game is given by*

		<i>Bank 2</i>	
		<i>m</i>	<i>a</i>
<i>Bank 1</i>	<i>m</i>	0,0	-F, -D
	<i>a</i>	-D, -F	-D, -D

*If  $D > F$  then  $(m, m)$  is the unique Nash equilibrium. If  $D < F$ , then the game is a stag hunt game. Both  $(m, m)$  and  $(a, a)$  are equilibria, and  $(m, m)$  is the efficient equilibrium.*

Due to the structure of the payoff function we can transform the game into an “aggregation game” (see Rausser et al., 2000) where it suffices to keep track of the number of opponents playing a particular strategy. When bank  $i$  plays the morning strategy its payoff depends only on the number of opponents that play the strategy  $m$ . When bank  $i$  plays the strategy  $a$ , its payoff is independent of the strategies played by the opponents. Hence, the payoff functions  $\pi_i$  can be replaced with the functions  $\tilde{\pi}_i(\cdot) : S_i \times Z_+ \rightarrow R$  that are specified as follows:  $\tilde{\pi}_i(m, |\mathbf{s}_{-i}|_m) \equiv \pi_i(m, s_{-i})$  and  $\tilde{\pi}_i(a, |\mathbf{s}_{-i}|_m) \equiv -(n - 1)D$ . The equilibria of the game are given by the following

**Proposition 1** *If intraday liquidity is relatively cheap, i.e.,  $F < D$  then the unique, pure-strategy Nash equilibrium is  $(m, \dots, m)$ . If intraday liquidity is relatively expensive, i.e.,  $F \geq D$  then  $(m, \dots, m)$  and  $(a, \dots, a)$  are the only pure-strategy Nash equilibria.*

**Proof.** From Eqs. (2) and (3) we have  $\tilde{\pi}_i(m, n-1) = 0 \geq \tilde{\pi}_i(a, n-1) = -(n-1)D < 0$  for every  $i \in N$ . Hence,  $(m, \dots, m)$  is a Nash equilibrium regardless of the relative magnitudes of  $F$  and  $D$ . Suppose  $F \geq D$ . Then, we have  $\tilde{\pi}_i(a, 0) = -(n-1)D \geq \tilde{\pi}_i(m, 0) = -(n-1)F$  for every  $i \in N$ . Hence,  $(a, \dots, a)$  is a Nash equilibrium if  $F \geq D$ . We now argue that there are no other Nash equilibria for any positive values of  $F$  and  $D$ . Suppose  $0 < |\mathbf{s}_{-i}|_m < n$  then at least one bank  $i \in N$  plays the strategy  $m$  and at least one bank  $j \in N$  plays the strategy  $a$ . For this to occur it has to be true that  $\tilde{\pi}_i(m, |\mathbf{s}_{-\{i,j\}}|_m) \geq \tilde{\pi}_i(a, |\mathbf{s}_{-\{i,j\}}|_m)$  and  $\tilde{\pi}_j(a, |\mathbf{s}_{-\{i,j\}}|_m + 1) \geq \tilde{\pi}_j(m, |\mathbf{s}_{-\{i,j\}}|_m + 1)$ . From Eq. (3) we have  $\tilde{\pi}_j(a, |\mathbf{s}_{-\{i,j\}}|_m) = \tilde{\pi}_j(a, |\mathbf{s}_{-\{i,j\}}|_m + 1)$ , and hence from the preceding strict inequalities,  $\tilde{\pi}_i(m, |\mathbf{s}_{-\{i,j\}}|_m) > \tilde{\pi}_j(m, |\mathbf{s}_{-\{i,j\}}|_m + 1)$ , which by Eq. (2) is a contradiction. ■

Whenever  $F \geq D$  the morning equilibrium  $(m, \dots, m)$  is the efficient equilibrium in terms of minimizing both individual and aggregate settlement costs. In the analysis that follows we assume that banks initially coordinate on the efficient, morning equilibrium and we examine the impact of a disruption that forces some banks to switch to the afternoon strategy. We are interested in the strategic adjustments made by banks in response to the disruption. In particular, we are interested in how the disruption influences equilibrium. We restrict ourselves to equilibria in pure strategies.

### 3 Wide-scale disruptions

The Interagency Paper on Sound Practices to Strengthen the Resilience of the U.S. Financial System issued by the Board of Governors of the Federal Reserve System, the Securities and Exchange Commission and the Office of the Comptroller of the Currency defines a wide-scale disruption as an event that causes a severe disruption or destruction of transportation, telecommunication, power, or other critical infrastructure components across a metropolitan or other geographical area and the adjacent communities that are economically integrated with it; or that results in wide-scale evacuation or inaccessibility of the population within normal commuting range of the disruption's origin.

In the context of our model, we take a wide-scale disruption to mean an event that prevents a subset of banks from making payments as normal. Specifically, some banks are temporarily forced to play to the afternoon strategy, which takes the system out of equilibrium. The size of the disruption can be measured by the share of banks that are disrupted. After the disruption we assume that the disrupted banks again become fully operational and that they are, like the non-disrupted banks, free to choose either the morning or afternoon strategy.

The question we ask is: Will the disruption trigger a move to the inefficient afternoon equilibrium or back to the efficient, morning equilibrium? The answer, of course, depends on the size of the disruption, its duration, and what the incentives are for banks to adjust their processing decisions following the disruption. In a game with many players this could be very difficult to assess. However, here it turns out that the game played by banks is a potential game a la Monderer and Shapley (1996), and as such, the dynamics of the system are conveniently and transparently understood by examining the potential function. A potential function is a mapping from strategy profiles (in this case vectors of processing decisions for the banks) to the reals that summarize all the information needed to compute the Nash equilibria of a game. The Nash equilibria we characterize in Proposition 1 appear as local (or global) maxima of the potential function.<sup>5</sup>

### 3.1 Potential function

We now turn to a formal characterization of the potential function we utilize to analyze the  $n$ -player, deterministic intraday liquidity management game.

**Definition 1 (Potential Function)** *A function  $P$  satisfying for every  $s_i, s'_i \in S_i, s_{-i} \in S_{-i}$  and  $i \in N$ ,*

$$\pi_i(s_i, \mathbf{s}_{-i}) - \pi_i(s'_i, \mathbf{s}_{-i}) = P(s_i, \mathbf{s}_{-i}) - P(s'_i, \mathbf{s}_{-i})$$

*is a potential function for the  $n$ -player, deterministic intraday liquidity management game.*

The function  $P$  is unique up to an additive constant. In order to pin down a particular function we choose the normalization  $P(m, \dots, m) = 0$ . The following example illustrates the construction of the potential function for the 2-player intraday liquidity management game.

**Example 2** *In the 2-player case, changes in bank  $i$ 's payoff that result from unilateral deviations are given by*

$$\begin{aligned}\pi_i(m, m) - \pi_i(a, m) &= D \\ \pi_i(m, a) - \pi_i(a, a) &= D - F\end{aligned}$$

*Hence, by Definition 1 the potential function has to satisfy*

$$\begin{aligned}P(m, m) - P(a, m) &= D & P(m, a) - P(a, a) &= D - F \\ P(m, m) - P(m, a) &= D & P(a, m) - P(a, a) &= D - F\end{aligned}$$

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<sup>5</sup>At a local maximum no unilateral deviation increases the potential of the game. At a global maximum there is no alternative strategy profile that has higher potential.

This implies that  $P(m, a) = P(a, m)$ . Letting  $P(m, m) = 0$  we have

$P(s)$	$m$	$a$
$m$	$0$	$-D$
$a$	$-D$	$F - 2D$

If  $F < 2D$  then  $(m, m)$  globally maximizes  $P$ , otherwise  $(a, a)$  is the global potential maximizer. If  $D < F < 2D$  then  $(a, a)$  is a local maximum. If  $F > 2D$  then  $(m, m)$  is a local maximum.

The function  $P$  maps strategy profiles into real numbers and hence for large numbers of players it is difficult to visualize. However, in this application the value of the potential function depends only on the total number of players playing the morning strategy. Hence, we can summarize  $P$  with a new function  $\tilde{P} : Z_+ \rightarrow R$ , defined in the following way. For any  $x \in \{0, 1, \dots, n\}$ ,  $\tilde{P}(x)$  is equal to the value of the function  $P(\cdot)$  evaluated at any strategy profile in the set  $\{s \in S : |s|_m = x\}$ .

From Eqs. (2) and (3) we have that

$$\tilde{P}(x) - \tilde{P}(x - 1) = (x - n)F + (n - 1)D \quad (4)$$

for all  $x \in \{1, \dots, n\}$ . Solving Eq. (4) by induction and assuming that  $\tilde{P}(n) = 0$  yields (see appendix A)

$$\tilde{P}(x) = \frac{1}{2}(n - 1 - x)(n - x)F - (n - x)(n - 1)D \quad (5)$$

which is a second order polynomial in continuous  $x$ .

The global and local maxima of  $\tilde{P}(x)$  are important for the analysis that follows and are characterized in the following

**Proposition 2** *If  $F < D$  then  $x = n$  globally maximizes  $\tilde{P}$  and there are no other local maxima. If  $D \leq F < 2D$  then  $x = n$  globally maximizes  $\tilde{P}$  and  $x = 0$  is a local maximum. If  $F = 2D$  then both  $x = 0$  and  $x = n$  globally maximize  $\tilde{P}$ . If  $F > 2D$  then  $x = 0$  globally maximizes  $\tilde{P}$  and  $x = n$  is a local maximum.*

## 4 Adjustment following disruptions

Monderer and Shapley (1996) specify a simple adjustment process that converges to a Nash equilibrium of a potential game in a finite number of steps. In their process it is assumed that, whenever the strategy profile is not a Nash equilibrium, one player deviates to a

strategy that makes him better off. Unilateral deviations that increase the payoff of the deviator raise the value of the potential while unilateral deviations that increase the payoff of the deviator lower it. Hence, once a Nash equilibrium is reached (there are no more self-improving, unilateral deviations) the process terminates and the potential function will be at a maximum in the sense that its value cannot be increased by varying *any single player's strategy*. Endpoints of the simple adjustment process are local maxima of the potential function, i.e., Nash equilibria.

We consider a generalization of the simple adjustment process that allows for simultaneous deviations. Our simultaneous adjustment process (SAP) is defined as follows: Each step in the adjustment process occurs during one day, which we refer to as a period. Assume the disruption occurs in period 0 and lasts until period  $\tau$ . At period 0 the player set is partitioned into two groups. Let those who are disrupted and hence forced to play the afternoon strategy be denoted by  $N^d$  and let those who are not disrupted, and hence free to choose their processing strategy, be denoted by  $N^{nd}$ . The adjustment process starts in period 1. At this time the initial strategy profile is given by  $s^0 = (s_{N^{nd}}^0, s_{N^d}^0)$  where  $s_{N^{nd}}^0 = \{m, \dots, m\}$  has  $|N^{nd}|$  components and  $s_{N^d}^0 = \{a, \dots, a\}$  has  $|N^d|$  components.

The period  $t \geq 1$  adjustment involves two steps.

Step 1. Nature randomly selects a set  $D^t$  of eligible deviators. If the disruption is not over (i.e., if  $t < \tau$ ), the set of eligible deviators is selected according to the probability function  $f^{nd}$ , which is defined on the elements of  $2^{N^{nd}}$ , the power set of  $N^{nd}$ . If the disruption is over, the set of eligible deviators is selected according to the probability function  $f$ , which is defined on the elements of  $2^N$ , the power set of  $N$ .

Step 2. Banks in the set  $D^t$  choose a strategy  $s_i^t$  to solve

$$\max_{s_i^t \in \{m, a\}} u_i(s_i^t, s_{-i}^{t-1}).$$

Some comments are in order. First, notice that the set of eligible deviators in a period depends on whether the disruption is over or not. Moreover, the random process used to select deviators allows a bank that was already selected as the eligible deviator in period  $t'$  to be selected again in period  $t'' > t'$ . Thus, the process can switch directions. Banks make adjustments that are optimal based on the previous period's strategy profile. This captures the notion that at any point of time during or after a disruption some (arbitrarily determined) set of banks will act, more or less simultaneously, without knowing who else is making adjustments to their processing strategy or what adjustments they are making. The implicit assumption is that banks behave myopically. That is, they do not anticipate future adjustments by other banks and they are not allowed to coordinate their adjustments. This assumption seems justified in the U.S. system with over 9,000 participants.

Under the simultaneous adjustment process, the transition probability from a given strategy profile to any other one depends on the given strategy profile, but not on the strategy profiles reached in previous periods. This means the adjustment process is a Markov process whose state space is the set of strategy profiles. In addition, the transition probabilities take

on one set of values before the disruption ends and another set after it ends (because the power set from which each period's set of eligible deviators is drawn changes). This means the SAP is represented by two Markov chains, one that applies before the disruption ends and one that applies after it ends. The initial state of the first chain is determined by the size of the disruption (i.e., it will be an exogenously determined state). The initial state of the second chain is given by the state of the first chain in the period before the disruption ends.

We now use the information revealed by the potential function to explain the learning dynamics under the SAP. In the case where  $F > D$ , we will use the notion of potential minimizers. Let  $\mu$  denote the integer value that minimizes  $\tilde{P}$  over the set of integers from 0 to  $n$ .<sup>6</sup> The SAP has the following properties, which are described in

**Proposition 3** (a) Suppose  $F < D$ . The SAP leads to the equilibrium  $(m, \dots, m)$  from any starting point. (b) Suppose  $F = D$ . The SAP leads to the equilibrium  $(m, \dots, m)$  from any starting point except  $(a, \dots, a)$ . At  $(a, \dots, a)$ , all players are indifferent between playing  $a$  or making a unilateral deviation to  $m$ . (c) Suppose  $F > D$ . The SAP leads to the equilibrium  $(m, \dots, m)$  from starting points on the right of  $\mu$  and to the equilibrium  $(a, \dots, a)$  from starting points to the left of  $\mu$ .

**Proof.** (a) If  $F < D$  then, by Eq. (4),  $\tilde{P}(x) - \tilde{P}(x-1) = D(n-1) - F(n-x) > 0$  for all  $1 \leq x \leq n$ . Hence all unilateral deviations from  $a$  to  $m$  are profitable, but deviations from  $m$  to  $a$  are not. Therefore, in periods before the disruption ends all eligible deviators stick with the morning strategy. Moreover, once the disruption ends, all of the previously disrupted banks who become eligible deviators switch to the morning strategy. Hence, the morning strategy profile is accessible from any strategy profile, but no other profile is accessible from the morning profile. By Theorem 4.7 of Seneta (1981), the Markov chain and hence the adjustment process converges from any given strategy profile to the efficient morning equilibrium.

(b) If  $F = D$  then, by Eq. (4),  $\tilde{P}(x) - \tilde{P}(x-1) = (x-1)F > 0$  for all  $2 < x \leq n$ . The proof is the same as part (a). If  $x = 1$  then  $\tilde{P}(x) - \tilde{P}(x-1) = 0$ . No one can deviate until the disruption is over, and it is not profitable for anyone to deviate once it does end.

(c) If  $F > D$  then, by Eq. (4),  $\tilde{P}(x) - \tilde{P}(x-1) = D(n-1) - F(n-x)$  is strictly greater than 0 for all  $n \leq x < \mu$ . Hence all unilateral deviations from  $a$  to  $m$  are profitable, but deviations from  $m$  to  $a$  are not. Likewise, if  $F > D$ , then  $\tilde{P}(x) - \tilde{P}(x-1) = D(n-1) - F(n-x)$  is strictly less than 0 for all  $\mu > x \geq 0$ . Hence, all unilateral deviations from  $m$  to  $a$  are profitable, but deviations from  $a$  to  $m$  are not. By the same reasoning as part (a), the process converges to the morning equilibrium from the initial state  $s^0 = (s_{N^{nd}}^0, s_{N^d}^0)$  if  $|N^{nd}| \in \{\mu + 1, \dots, n\}$  and it converges to the afternoon equilibrium if  $|N^{nd}| \in 0, \dots, \mu - 1$ . ■

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<sup>6</sup>Because  $\tilde{P}$  is strictly convex (over continuous  $x$ ) the minimum defined over discrete points may occur at either one or two points. However, instances where the minimum occurs at two points are of measure zero in the parameter space and hence are ignored.

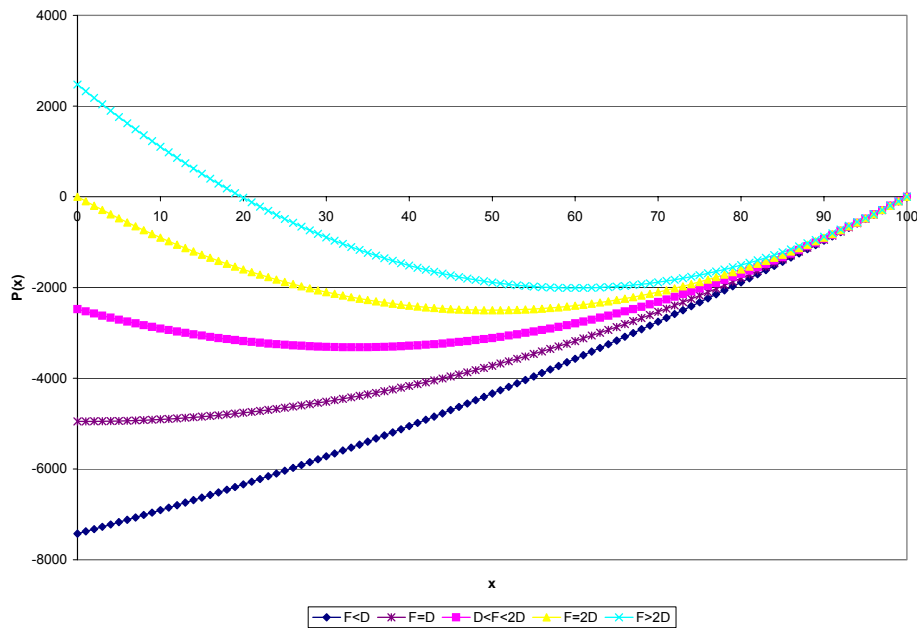


Figure 1: Potential function.

The different cases outlined in Proposition 1 are illustrated in Fig. 1. The figure displays the function  $\tilde{P}$  for  $n = 100$  and different values of  $D$  and  $F$ . The case of  $F < D$  is illustrated by the lowest curve. As indicated by property (a) of Proposition 1 all deviations to  $m$  are profitable after a disruption of any size in this case. The implication is that the system would converge to the (efficient) morning equilibrium regardless of the size of the shock. In other words, the system is resilient to a disruption of any size in terms of maintaining coordination on early processing. The same is true for the case where  $F = D$ , save the situation where the disruption hits all  $n$  banks, cf. property (b) of Proposition 1.

Three instances of the case  $F > D$  (cf. property (c) of Proposition 1) are illustrated by the top three curves in Fig. 1. In each instance disruptions that are sufficiently large move the system to a point to the left of the minimum of the potential, causing the banks to converge to the (inefficient) afternoon equilibrium. The figure illustrates an important fact which we will address more formally in the next section: The higher the cost of liquidity from the central bank,  $F$ , is relative to the cost of delay,  $D$ , the lower is the number of banks that minimizes the potential, and thus the larger is the “basin of attraction” for the afternoon equilibrium. For example, if  $F = 2D$  then the disruption needs to hit half or more of the banks in order for the system to converge to the inefficient equilibrium. However, if  $F > 2D$  then a disruption that hits less than half of the banks will cause the system to converge to the inefficient equilibrium.

The different cases of potential functions shown in Figure 1 make it easy to understand the Federal Reserve’s response to the terrorist attacks on September 11, 2001 (See Lacker,

2004, for a detailed overview). Shortly after the attacks, the Federal Reserve issued several statements to the effect that the Fedwire was open and operating and that the discount window was available to meet liquidity needs. Moreover, the Federal Reserve began to make liquidity widely available and for the period from September 11 through September 21, it waived the daylight overdraft fees for all account holders and eliminated the penalty normally charged in excess of the effective Federal Funds rate on overnight overdrafts. Both policy responses are consistent with this model. The statements can be seen as an attempt to encourage the banks to keep coordinating on the efficient equilibrium. Moreover, the infusion of liquidity and elimination of fees served to lower the cost of liquidity  $F$  to virtually zero so that deviations to  $m$  became profitable. Once the system returned to the morning equilibrium (or had made sufficient progress) the original  $F$  was be restored.

## 5 Banking structure and network topology

So far, we have considered an economy with a homogenous banking structure consisting of  $n$  identical banks. Obviously, this is - in many ways - an over-simplification. For example, in Fedwire the top 100 participants account for 95% of the value and 60% of the volume. Moreover, it is often argued that the financial sector's resiliency to different types of shocks depends critically on the interconnectedness of the interbank market cf. Allen and Gale (2000). In the following, we provide an extension of our model that allows us to investigate how both heterogeneity across size and different degrees of completeness of interbank relationships impact the resiliency of the interbank payment systems to (wide-scale) disruptions.

### 5.1 Mergers and payment topology

Envision a new economy with one large bank and a number of smaller identical banks. Assume that the large bank is created by merging  $k > 1$  out of the  $n$  identical banks in our previous economy.<sup>7</sup> For notational convenience, assume that banks 1 through  $k < n$  are the ones that are merged. Denote the merged bank by  $k$ . The new player set is  $\tilde{N} = \{k, k + 1, \dots, n\}$ . A key distinction is that transfers between customers of the merged bank are handled internally within that bank. Consequently, following a merger, the total volume of payments transferred via the RTGS system decreases. The larger the merger the larger the decrease. Specifically, the total volume (and value) of payments transferred falls to  $n(n - 1) - k(k - 1)$ , compared to  $n(n - 1)$  before the merger.

In addition, we assume that a merger can lead to a shift in the payment flow. For example, the merger might lead to an increase in the volume of transactions involving the merged bank. This would occur, for example, if there are economies of scale or scope in the processing of payments. We do not formally model payment flows as an endogenous

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<sup>7</sup>It has been suggested to us by practitioners that some banks have close relationships and do not behave strategically with respect to one another. An alternative interpretation is that  $k$  is the size of the coordinated sector.

variable. Rather, we consider different exogenous payment flow effects by including the parameter  $\alpha \geq 0$ , which represents the fraction of each dollar that small banks continue to transact with each other following a merger. The remaining  $1 - \alpha$  of each dollar small banks transacted with each other before the merger now goes to the merged bank. For example, if the merger is size  $k$ , each small bank sends  $\alpha$  dollars to each of the other small banks and sends  $k + (1 - \alpha)(n - k - 1)$  dollars to the merged bank. We keep the total payment flow balanced, and hence the large bank sends the same amount to each of the small banks.

If  $\alpha = 1$ , then there is no shift in payment volume between the large and the smaller banks, but the analysis is still effected by the merger as the decision of the merged bank now carries special weight. Values  $\alpha < 1$  correspond to the scenario discussed above where the small banks are losing market share vis-a-vis the merged bank. If  $\alpha > 1$ , then the small banks are gaining market share vis-a-vis the merged bank. In order to keep payments from the small banks to the merged bank positive we require

$$\alpha \leq \frac{n - 1}{n - k - 1} = \bar{\alpha}. \quad (6)$$

By varying  $\alpha$ , we create different degrees of interconnectedness or network topologies. If  $0 < \alpha < \bar{\alpha}$ , we have a complete graph, where all banks exchange payments with each other. In the two special cases where  $\alpha = 0$  or  $\alpha = \bar{\alpha}$ , we get topologies where not all banks are exchanging payments with each other. If  $\alpha = 0$ , then we have a star graph where the small banks do not interact with each other, but instead, only interact with the “money center” bank. If  $\alpha = \bar{\alpha}$ , then we have a disconnected graph where the small banks are interacting with each other, but not with the merged bank, which only process internal transactions. Examples of the payment flows with  $0 \leq \alpha \leq \bar{\alpha}$  in an economy with five banks before and after a merger of size  $k = 2$  are shown in Fig. 2.

## 5.2 Potential function for the heterogenous economy

The payoff function of the merged bank  $k$  is given by

$$\pi_k(m, \mathbf{s}_{-k}) = -(k + (1 - \alpha)(n - k - 1))(n - k - |\mathbf{s}_{-k}|_m)F \quad (7)$$

$$\pi_k(a, \mathbf{s}_{-k}) = -(k + (1 - \alpha)(n - k - 1))(n - k)D \quad (8)$$

The payoff functions of the  $i = k + 1, \dots, n$  small banks are given by

$$\pi_i(m, s_k, \mathbf{s}_{-\{i,k\}}) = -(n - 1 - (k + (1 - \alpha)(n - k - 1))1_{s_k=m} - \alpha |\mathbf{s}_{-\{i,k\}}|_m)F \quad (9)$$

$$\pi_i(a, s_k, \mathbf{s}_{-\{i,k\}}) = -(n - 1)D \quad (10)$$

where  $1_{s_k=m}$  is an indicator function taking value one when  $s_k = m$  and zero otherwise.

As before, the payoff to a bank, that plays afternoon, is independent of the strategies

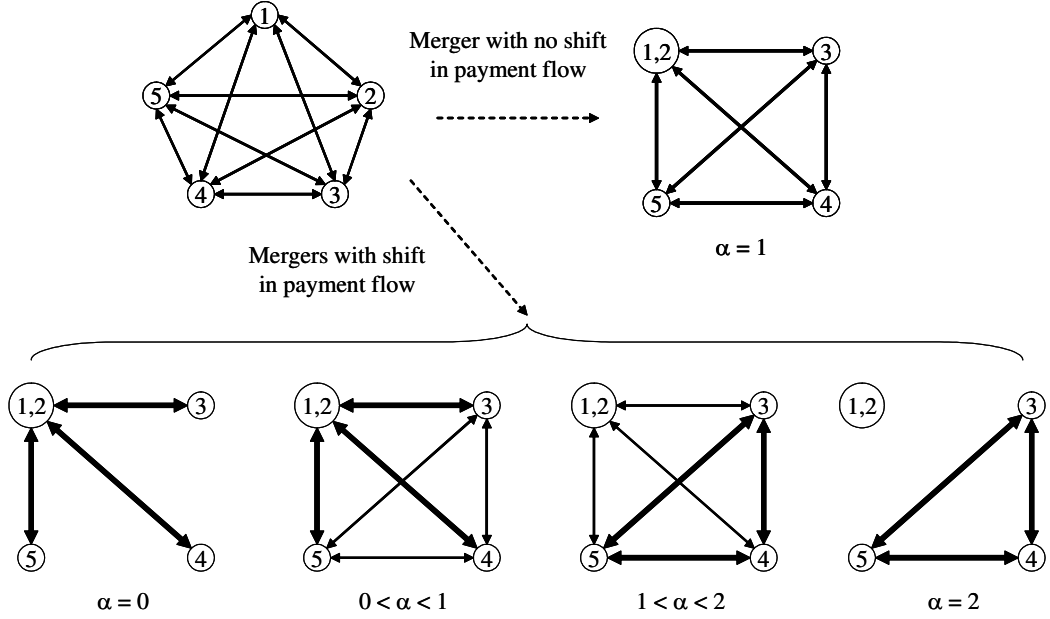


Figure 2: Payment flows before and after merger ( $n = 5$ ,  $k = 2$ ,  $\bar{\alpha} = 2$ ).

played by the opponents and the structure of the payoff function allows us to transform the game into an aggregation game. For the small banks it suffices to keep track of the bank's own strategy, the strategy of the large bank and the number of other small banks playing the morning strategy. Hence, the payoff functions  $\pi_i$ ,  $i \neq k$ , can be replaced with the functions  $\hat{\pi}_i(\cdot) : S_i \times S_k \times Z_+ \rightarrow R$  that are specified as  $\hat{\pi}_i(s_i, s_k, |\mathbf{s}_{-\{i,k\}}|_m) \equiv \pi_i(s_i, s_k, s_{-\{i,k\}})$ . For the merged bank, it suffices to keep track of its own strategy and the number of small banks playing the morning strategy. Thus, we define  $\hat{\pi}_k(\cdot) : S_k \times Z_+ \rightarrow R$  as  $\hat{\pi}_k(s_k, |\mathbf{s}_{-k}|_m) \equiv \pi_k(s_k, s_{-k})$ . For each strategy choice of the big bank, the value of the potential function depends only on the total number of small banks playing the morning strategy. Hence, we can proceed using a potential function for the game,  $\hat{P}(\cdot) : S_k \times Z_+ \rightarrow R$ , defined to satisfy

$$\hat{P}(s_k, |(s_{k+1} \dots s_i \dots s_n)|_m) - \hat{P}(s_k, |(s_{k+1} \dots s'_i \dots s_n)|_m) = \hat{\pi}_i(s_i, s_k, |\mathbf{s}_{-\{i,k\}}|_m) - \hat{\pi}_i(s'_i, s_k, |\mathbf{s}_{-\{i,k\}}|_m) \quad (11)$$

and

$$\hat{P}(s_k, |\mathbf{s}_{-k}|_m) - \hat{P}(s'_k, |\mathbf{s}_{-k}|_m) = \hat{\pi}_k(s_k, |\mathbf{s}_{-k}|_m) - \hat{\pi}_k(s'_k, |\mathbf{s}_{-k}|_m) \quad (12)$$

for all  $i \in \{k + 1, \dots, n\}$  and  $s_i, s_k \in \{m, a\}$ . Using Eqs. (7) - (10), we have that

$$\hat{P}(m, x) - \hat{P}(m, x - 1) = -\alpha(n - k - x)F + (n - 1)D \quad (13)$$

$$\hat{P}(a, x) - \hat{P}(a, x - 1) = -(n - 1 - \alpha(x - 1))F + (n - 1)D \quad (14)$$

$$\hat{P}(m, x) - \hat{P}(a, x) = -(\alpha k + (1 - \alpha)(n - 1))((n - k - x)F - (n - k)D) \quad (15)$$

for  $x = 1, \dots, n - k$ . Solving Eqs. (13) - (15) by assuming that  $\hat{P}(m, n - k) = 0$  yields

$$\hat{P}(m, x) = \frac{\alpha}{2}(n - k - 1 - x)(n - k - x)F - (n - k - x)(n - 1)D \quad (16)$$

and

$$\begin{aligned} \hat{P}(a, x) &= [n - 1 - \frac{\alpha}{2}(n - k + x - 1)](n - k - x)F \\ &\quad - \{(n - k - x)(n - 1) + [\alpha k + (1 - \alpha)(n - 1)](n - k)\}D \end{aligned} \quad (17)$$

for  $x = 0, \dots, n - k$  (see Appendix B).

**Remark 1** Both  $\hat{P}(m, x)$  and  $\hat{P}(a, x)$  are second order polynomials in continuous  $x$ , and hence each function has a unique minimum point when evaluated over a continuous domain.

As before, the local or global maxima of the potential function correspond to the Nash equilibria of the game and we summarize them in the following proposition.

**Proposition 4** *If  $F < D$  then  $s_k = m$  and  $x = n - k$  globally maximize  $\hat{P}$  and there are no other local maxima. If  $D \leq F < 2D$  then  $s_k = m$  and  $x = n - k$  globally maximizes  $\hat{P}$  and  $s_k = a$  and  $x = 0$  is a local maximum. If  $F = 2D$  then  $s_k = m$  and  $x = n - k$  as well as  $s_k = a$  and  $x = 0$  globally maximize  $\hat{P}$ . If  $F > 2D$  then  $s_k = a$  and  $x = 0$  globally maximizes  $\hat{P}$  and  $s_k = m$  and  $x = n - k$  is a local maximum.*

If liquidity is relatively cheap ( $F < D$ ) then our game has a unique equilibrium and coordination is not needed to sustain the efficient equilibrium. Moreover, the adjustment process always leads to the efficient equilibrium. Hence, early processing of payments will not be affected by any wide-scale disruption. In contrast, if liquidity is relatively expensive ( $F \geq D$ ) then our game has two equilibria, one where all banks play the morning strategy and one where they play the afternoon strategy. We shall focus on the latter case below.

### 5.3 Convergence following disruptions

Once more, we assume that the system initially is in the efficient morning equilibrium, but that it is hit by a disruption which forces a set of banks - that may or may not include the merged bank - to play the afternoon strategy. The adjustment process following a disruption is governed by the potential functions  $\hat{P}(s_k, x)$ ,  $s_k \in \{m, a\}$  in Eqs. (16) and (17). If  $\hat{P}(s_k, x - 1) - \hat{P}(s_k, x) > 0$  for  $s_k \in \{m, a\}$  then deviations by small banks from the morning to the afternoon strategy increase the potential and are profitable for the small banks. Conversely, if  $\hat{P}(s_k, x - 1) - \hat{P}(s_k, x) < 0$  then deviations from the morning strategy decrease the potential and are not profitable for the small banks. For the merged bank the interesting quantity is the difference between the two potentials, i.e.,  $\hat{P}(m, x) - \hat{P}(a, x)$ , which gives the change in potential associated with its deviation. If this change in potential is positive then it is profitable for a merged bank playing the afternoon strategy to switch to the morning strategy and vice versa if it is negative. Consequently, the intersection point of  $\hat{P}(m, x)$  and  $\hat{P}(a, x)$ , defined in terms of continuous  $x$ , is important in terms of the dynamics of the adjustment process. We denote this intersection by  $x^c$ .

The following lemma summarizes key relationships between the potential functions for each strategy of the merged bank, defined with respect to continuous  $x$ .

**Lemma 1**  $\hat{P}(a, 0) > \hat{P}(m, 0)$ . Moreover, for  $F \geq D$ , there exists a unique point of intersection  $x^c$  of  $\hat{P}(m, x)$  and  $\hat{P}(a, x)$  in the interval  $[0, n - k]$ , which does not depend on  $\alpha$ .

**Proof.** Quick calculations show that  $\hat{P}(a, 0) - \hat{P}(m, 0) = (n - k)(n - 1 - \alpha(n - k - 1))(F - D) \geq 0$  for all  $\alpha \leq \bar{\alpha}$  provided  $F \geq D$ . Moreover,  $\partial \hat{P}(m, x) / \partial x \Big|_{x=0} > \partial \hat{P}(a, x) / \partial x \Big|_{x=0}$  and  $\partial^2 \hat{P}(m, x) / \partial x^2 = \partial^2 \hat{P}(a, x) / \partial x^2 = \alpha F > 0$  for all  $x \in [0, n - k]$ . This means  $\hat{P}(m, x)$  starts below  $\hat{P}(a, x)$  and can intersect it at most once. In fact, explicit calculation of the intersection shows that it occurs at  $x^c = \frac{(n-k)(F-D)}{F}$ , which does not depend on  $\alpha$ . This intersection always occurs in the interval  $[0, n - k]$  for  $F \geq D$ , since  $(F - D)/F = 1 - D/F < 1$ . ■

Let  $\mu_m$  denote the integer value that minimizes  $\hat{P}(m, x)$  over the set of integers from 0 to  $n - k$ .<sup>8</sup> Potential functions are shown for two qualitatively distinct cases allowed by Lemma 1 in Figs. 3 and 4.

#### 5.3.1 Disruptions to small banks

In this case we are looking at a movement to the left (from the far right) along the blue curve in Figs. 3 and 4. We distinguish three qualitatively distinct cases: self-reversing, self-perpetuating and indeterminate. Self-reversing refers to the fact that the system will converge back to the efficient, morning equilibrium on its own following a disruption. There are always small enough disruptions that are self-reversing. Self-perpetuating means the

<sup>8</sup>We assume  $\mu_m$  is unique. See footnote 6.

system will converge to the inefficient, afternoon equilibrium in the absence of central bank intervention. This case exists if and only if  $F > \frac{\bar{\alpha}}{\alpha}D$ , which implies  $\hat{P}(m, 0) > \hat{P}(m, 1)$ . The scalar  $\frac{\bar{\alpha}}{\alpha}$  is decreasing in  $\alpha$  and increasing in  $k$ . Hence, if funds are concentrated in the merged bank (i.e.,  $\alpha$  is low) or the merged sector ( $k$ ) is large the self-perpetuating disruptions are only a problem in “high” fee regimes. In the indeterminate case the dynamic adjustment process will converge to one of the two equilibria, but neither is reached with probability 1. The existence of indeterminate dynamics depends on the relative positions of  $\mu_m$  and  $x^c$ . The indeterminate case does not exist (except at the single point  $x = \mu_m$ ) if  $\mu_m > x^c$  (See Figs. 3 and 4).

**Self-reversing** Any disruption to small banks that keeps the system at a point to the right of the intersection of  $\hat{P}(m, x)$  and  $\hat{P}(a, x)$  where  $\hat{P}(m, x - 1) - \hat{P}(m, x) < 0$  will be self-reversing (See Figs. 3 and 4.). During the disruption none of the unaffected banks will switch to the afternoon strategy, nor will the merged bank since  $\hat{P}(a, x) - \hat{P}(m, x) < 0$  for  $x > x^c$ . Once the disruption ends the disrupted banks will switch back to the morning strategy as soon as they are selected to deviate by the adjustment process. This is true because  $\hat{P}(m, x - 1) - \hat{P}(m, x) < 0$  implies  $\hat{P}(m, x + 1) - \hat{P}(m, x) > 0$ . In these cases the length of the disruption does not affect the outcome of the adjustment process.

Since self-reversing disruptions do not require intervention by the Federal Reserve to restore efficient processing it is interesting to ask how the cost parameters and the concentration of payments in the system affect the range of self-reversing disruptions. We have the following result.

**Proposition 5** *Suppose  $D$ ,  $n$ , and  $k$  are fixed. The largest size of a disruption to the small banks that is self-reversing decreases or remains unchanged as  $F$  or  $\alpha$  increases.*

**Proof.** Since  $\mu_m$  minimizes  $\hat{P}(m, x)$  over the set of integers from 0 to  $n - k$ ,  $\min\{x \in \{1, \dots, n - k\} | \hat{P}(m, x - 1) - \hat{P}(m, x) < 0\} = \mu_m + 1$ .

Case 1 :  $x^c < \mu_m$  (see Fig. 3). The largest size of a disruption to the small banks that is necessarily self-reversing is given by  $d^{SR} = n - k - \mu_m - 1$ . By (13),  $\hat{P}(m, x - 1) - \hat{P}(m, x) = \alpha(n - k - x)F - (n - 1)D$ , hence increases in  $\alpha$  or  $F$  cause  $\mu_m$  to increase or stay the same. Hence,  $d^{SR}$  decreases or stays the same as  $\alpha$  or  $F$  increase.

Case 2:  $x^c > \mu_m$  (see Fig. 4). The largest size of a disruption to the small banks that is necessarily self-reversing is given by  $d^{SR} = \text{int}(n - k - x^c)$ , where  $\text{int}(r)$  denotes the integer part or floor of the real number  $r$ . By lemma 1,  $x^c$  does not depend on  $\alpha$ . From the proof of lemma 1,  $x^c = \frac{(n-k)(F-D)}{F}$ , and hence  $\frac{\partial x^c}{\partial F} = \frac{(n-k)D}{F^2} > 0$ . Hence, increases in  $\alpha$  or  $F$  cause  $x^c$  to increase or stay the same. Hence,  $d^{SR}$  decreases or stays the same as  $\alpha$  or  $F$  increase. ■

The above proposition says that banking systems where more funds are concentrated in the merged banks (i.e., with smaller  $\alpha$ ) are more resilient to shocks in the sense that larger shocks are self-reversing. This is intuitive because, the coordination problem that leads to inefficient outcomes is reduced when more transfers involve the merged bank. In the extreme

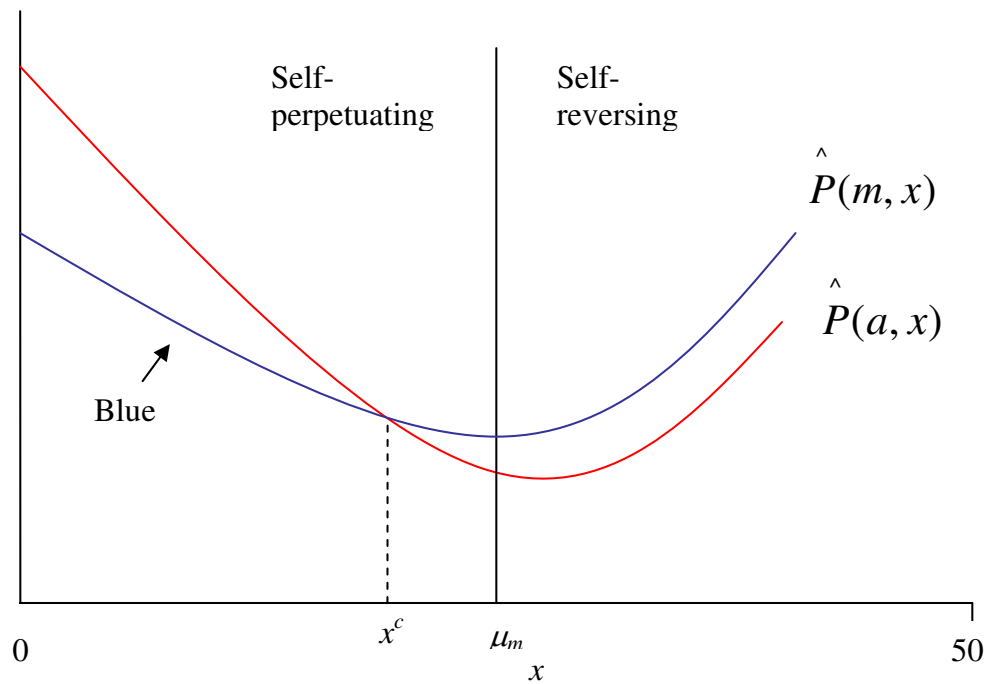


Figure 3:  $\mu_m > x^c$ .

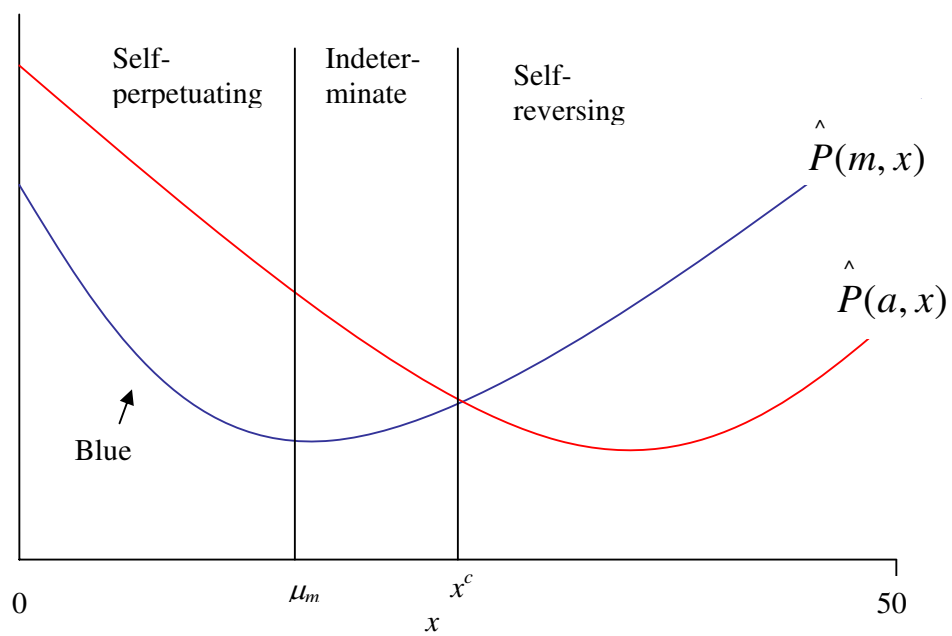


Figure 4:  $\mu_m < x^c$ .

case where  $\alpha = 0$ , for instance, all transfers are bilateral exchanges between each of the small banks and the merged bank. There are no issues of coordination between each pair of small banks.

**Self-perpetuating** Any disruption to small banks that puts the system at a point where  $\hat{P}(m, x - 1) - \hat{P}(m, x) > 0$  will be self-perpetuating (See Figs. 3 and 4.). Our next result explains how different concentrations of funds within the banking system affects the minimum size of disruption that is self-perpetuating.

**Proposition 6** *Choose  $F, D, n, k$  and  $\alpha$  with  $F > \frac{\bar{\alpha}}{\alpha}D$ . Then, the smallest sized disruption to the small banks that is self-perpetuating decreases or remains unchanged as  $\alpha$  or  $F$  increases.*

**Proof.** Since  $\mu_m$  minimizes  $\hat{P}(m, x)$  over the set of integers from 0 to  $n - k$ ,  $\max\{x \in \{1, \dots, n - k\} : \hat{P}(m, x - 1) - \hat{P}(m, x) > 0\} = \mu_m$ . The condition  $F > \frac{\bar{\alpha}}{\alpha}D$  implies that  $\hat{P}(m, 0) - \hat{P}(m, 1) > 0$  and hence it ensures that  $\mu_m \geq 1$ , which is necessary and sufficient for the self-perpetuating case to exist.

There are two cases to consider. In the first case,  $\mu_m > x^c \geq 0$  (See Fig. 3). Then, the smallest sized disruption to the small banks that is self-perpetuating is  $d^{SP} = n - k - \mu_m + 1$ . This is true because for integer points  $x$  satisfying  $x^c < x < \mu_m - 1$ ,  $\hat{P}(m, x - 1) - \hat{P}(m, x) > 0$ ,  $\hat{P}(m, x + 1) - \hat{P}(m, x) < 0$  and  $\hat{P}(m, x) - \hat{P}(a, x) > 0$ . Moreover, for points  $x$  satisfying  $x < x^c$ ,  $\hat{P}(a, x - 1) - \hat{P}(a, x) > 0$  and  $\hat{P}(a, x) - \hat{P}(m, x) > 0$ . Hence, all selected deviators switch to the afternoon strategy and the adjustment process converges toward the afternoon equilibrium while the disruption is on and does not reverse direction once the disruption ends. In contrast, disruptions that correspond to points to the right of  $\mu_m$  are self-reversing for the reasons given above. Since, by (13),  $\hat{P}(m, x - 1) - \hat{P}(m, x) = \alpha(n - k - x)F - (n - 1)D$ , increases in  $\alpha$  or  $F$  cause  $\mu_m$  to increase or stay the same. Hence,  $d^{SP}$  decreases or stays the same as  $\alpha$  or  $F$  increase.

In the second case,  $\mu_m < x^c$  (See Fig. 4). Disruptions that affect  $n - k - \mu_m + 1$  or more small banks are self-perpetuating since  $\hat{P}(m, x - 1) - \hat{P}(m, x) > 0$ ,  $\hat{P}(m, x + 1) - \hat{P}(m, x) < 0$  and  $\hat{P}(a, x) - \hat{P}(m, x) > 0$  for  $x \leq \mu_m - 1$ . Disruptions that affect fewer than  $n - k - \mu_m + 1$  small banks are not (necessarily) self-perpetuating since  $\hat{P}(m, x + 1) - \hat{P}(m, x) > 0$  for  $x \geq \mu_m$  and hence disrupted small banks that are selected by the adjustment process may switch back to the morning strategy one the disruption is over (provided the large bank has not switched to the afternoon strategy). Once again, the result holds because increases in  $\alpha$  or  $F$  cause  $\mu_m$  to increase or stay the same. ■

**Indeterminate** In cases where  $x^c > \mu_m$ , any disruption to small banks that places the system to the right of  $\mu_m$  and to the left of  $x^c$  (see Fig. 4) will have indeterminate dynamics. This is because, for  $x$  such that  $\mu_m < x < x^c$  we have that  $\hat{P}(m, x - 1) - \hat{P}(m, x) < 0$  and  $\hat{P}(m, x) - \hat{P}(a, x) < 0$ . The merged bank has an incentive to switch to  $a$ , and the outcome of the adjustment process depends on whether the disruption ends and enough small banks

deviate back to the morning strategy before the merged bank deviates to the afternoon strategy (in which case the system converges to the afternoon equilibrium). The likelihood that the process will converge to the inefficient equilibrium increases if the disruption is longer.

**Proposition 7** *Suppose  $F$ ,  $D$ ,  $n$ ,  $k$  and  $\alpha$  are such that  $\mu_m < x^c$ . The probability that the system converges to the inefficient, afternoon equilibrium following a disruption to the small banks of size  $n - k - |N^{nd}|$ , with  $x^c < |N^{nd}| < \mu_m$  increases as  $\tau$  increases.*

**Proof.** Let  $f^{nd}(k)$  denote the probability the adjustment process selects the merged bank  $k$  to deviate from the set  $N^{nd}$  in any round of the adjustment process before the disruption is over. The probability the merged bank will still be playing the morning strategy after  $\tau$  periods is

$$(1 - f^{nd}(k))^\tau.$$

After period  $\tau$  the previously disrupted small banks will begin to deviate back to the morning strategy. The probability that  $x^c - |N^{nd}|$  small banks in  $N^d$  are selected to deviate by the process before bank  $k$  is selected depends on the probability function  $f$  but does not depend on the duration  $\tau$ . Hence, the probability that the process will converge to the afternoon equilibrium is increasing in  $\tau$ . ■

### 5.3.2 Disruptions to the merged bank

A disruption to the merged banks knocks the system from the end (far-right) point of the blue curve down to the right end point of the red curve in Figs 3 and 4. The response of the small banks depends on the sign of  $\hat{P}(a, n - k) - \hat{P}(a, n - k - 1)$ . A positive sign means the small banks will continue to play the morning strategy and the system will return to the efficient equilibrium once the disruption to the merged bank is over. A negative sign means the small banks will adjust to the afternoon strategy. Where the system ends up in the latter case will depend on the length of the disruption to the merged bank and how fast the small banks adjust to the afternoon strategy.

Quick calculations show that the former case occurs if and only if  $F < \frac{\bar{\alpha}}{\bar{\alpha} - \alpha} D$ . The scalar  $\frac{\bar{\alpha}}{\bar{\alpha} - \alpha} D$  is increasing in  $\alpha$  and decreasing in  $k$ . Hence, if funds are concentrated in the merged bank (i.e.,  $\alpha$  is low) or the merged sector ( $k$ ) is large disruptions to the merged bank will only be self-reversing in “low” fee regimes.

We have the following, immediate results

**Proposition 8** *Disruptions to the merged bank are self-reversing if and only if  $F < \frac{\bar{\alpha}}{\bar{\alpha} - \alpha} D$ .*

**Proof.** From Eqs. (16) and (17),  $\hat{P}(a, n - k) - \hat{P}(a, n - k - 1) = D(n - 1) - F(n - 1 - \alpha(n - k - 1)) > 0$  if  $F < \frac{\bar{\alpha}}{\bar{\alpha} - \alpha} D$ . Hence no small banks will switch to the afternoon strategy. Since  $\hat{P}(m, n - k) > \hat{P}(a, n - k)$ , once the disruption is over the merged bank will switch back to the morning strategy. ■

**Corollary 1** *If a disruption to the merged bank is self-reversing at  $\alpha$ , then it is also self-reversing at any  $\alpha' < \alpha$ ;  $\alpha, \alpha' \in (0, \bar{\alpha})$ .*

In the latter case we can again relate the probability of converging to the inefficient equilibrium to the duration of the disruption.

**Proposition 9** *Suppose  $F > \frac{\bar{\alpha}}{\bar{\alpha} - \alpha}D$ . The probability that the system converges to the inefficient equilibrium following a disruption to the merged bank increases as  $\tau$  increases.*

**Proof.** From Eqs. (16) and (17),  $\hat{P}(a, x) - \hat{P}(a, x - 1) < 0$  for all  $x \leq n - k$  if  $F > \frac{\bar{\alpha}}{\bar{\alpha} - \alpha}D$ . Hence, along  $\hat{P}(a, x)$  deviations from  $m$  to  $a$  are profitable from any starting point. Whether or not the afternoon equilibrium is reached in this setting depends on how many small banks deviate to the afternoon strategy before the disruption to the merged bank ends. The system will converge to the inefficient equilibrium if the number of banks that deviate to the afternoon strategy before the disruption ends is greater than  $n - k - x^c = (n - k)\frac{D}{F}$ . Since all banks playing  $m$  that are selected by the SAP in each period deviate to  $a$  (and banks playing  $a$  do not deviate back to  $m$ ) the probability that  $(n - k)\frac{D}{F}$  small banks deviate from  $a$  to  $m$  in  $\tau$  rounds is increasing in  $\tau$ .<sup>9</sup> Once more than  $(n - k)\frac{D}{F}$  banks have deviated to  $a$  and the disruption ends the merged bank will, when selected, not deviate back to  $m$ . Hence the situation is the same as the homogeneous case covered by Proposition 3 (c) where the number of banks is  $n - k$  and the starting point is to the left of  $\mu$ . Hence, the SAP leads to the inefficient afternoon equilibrium.

The system may converge to the inefficient equilibrium even if the disruption ends with  $x^\tau > x^c$ , where  $x^\tau = |\mathbf{s}^\tau|_m$  is the number of banks playing the morning strategy when the disruption ends. Then convergence depends on whether the merged bank is selected to deviate before the number of small banks that deviate to  $a$  under the SAP reaches  $(n - k)\frac{D}{F}$ . However, here we can apply the fact that  $x^t < x^{t'}$  for  $t < t'$ . That is, the longer the disruption occurs, the more banks there will be that are playing  $a$ , and hence the fewer the number of additional deviations that are required for  $x$  to fall below  $x^c$  (at which point the system converges to the inefficient equilibrium for sure by the argument given in the preceding paragraph). Hence, the probability that enough remaining banks are selected by the distribution function  $f$  to switch from  $m$  to  $a$  before  $k$  is selected to switch from  $a$  to  $m$  increases as  $\tau$  increases. ■

## 6 Policy

The appropriate policy response of the central bank is different across the many cases and the challenge for the central bank is to quickly identify the nature of the disruption. Ceteris

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<sup>9</sup>In this proof we do not compute probabilities explicitly as this is not necessary for the stated result. However, explicit computations are possible using the distribution function  $f^{nd}$  defined on the power set  $2^{N^{nd}}$ .

paribus, a central bank prefers not to change its (intraday) liquidity regime because doing so has implications for the credit risk it assumes and its monetary policy stance.<sup>10</sup> In the self-reversing cases, the central bank obviously need take no action. In the self-perpetuating cases the central bank can drive the cost of liquidity towards zero in order the change the nature of the game or it can attempt to impact behavior through moral suasion. Moral suasion can work in this instance because the central bank facilitates a correlated equilibrium (as defined by Aumann (1974, 1987)), in which the correlating device recommends the morning profile  $(m, \dots, m)$  with probability one. It is in the best interests of any given bank to follow this recommendation given that each of the other banks will follow. In systems as large as the US it is not clear whether the central bank can affect the equilibrium outcome through moral suasion alone.

In the indeterminate case discussed above the outcome of the adjustment process was shown to depend crucially on the reaction (or lack thereof) of the large bank. Here there is a more hopeful role for the central bank to perform moral suasion. The inefficient equilibrium can be avoided if the central bank is able to convince the large bank to be patient and not switch to the afternoon strategy. Patience would be rewarded because after the disruption ends the system would converge back to the morning equilibrium.

Finally, in cases with  $F > \frac{\bar{\alpha}}{\bar{\alpha} - \alpha} D$  where the disruption affects the large bank the ability of the large bank to recover quickly is instrumental in terms of achieving coordination on the efficient equilibrium. Ex ante, stakeholders in the interbank payment system may push for stronger resiliency requirements for larger banks.<sup>11</sup> The pivotal role of the merged bank, in these instances, suggests that a bank might be considered “too big to fail” in part due to its role in terms of maintaining coordination on smooth processing of payments.<sup>12</sup>

## 7 Conclusion

The containment of systemic risk within the U.S. financial system in the event of a wide-scale disruption rests on the rapid recovery and resumption of the core clearing and settlement activities that support the financial markets. Fedwire is a one of the most critical components of clearing and settlement process and hence the resilience of the system itself and its participants is of paramount importance. As shown by McAndrews and Rajan (2000)

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<sup>10</sup>“On Monday morning September 17, the Federal Open Market Committee (FOMC) met by conference call at 7:30 a.m. eastern time and voted lower the target for the federal funds rate by 50 basis points to 3 percent. In its statement the Committee said: The Federal Reserve will continue to supply unusually large volumes of liquidity to the financial markets, as needed, until more normal market functioning is restored. As a consequence the FOMC recognizes that the actual federal funds rate may be below its target on occasion in these unusual circumstances.” (Lacker, 2004, p. 949).

<sup>11</sup>See, for example, “Best Practices to Assure Telecommunications Continuity for Financial Institutions and the Payment & Settlements Utilities” published by the Payments Risk Committee ([www.newyorkfed.org/prc/telecom.pdf](http://www.newyorkfed.org/prc/telecom.pdf)). The Payments Risk Committee is a private sector group of senior managers from U.S. banks that is sponsored by the Federal Reserve Bank of New York.

<sup>12</sup>See Stern and Feldman (2004, chapter 12) for a general discussion of too big to fail policy and the payment system.

and McAndrews and Potter (2002), coordination in the transmission of payments is a vital part of the day-to-day operations in Fedwire. The ability to maintain coordination, where banks tend to settle promptly and synchronize their payment activity, can potentially be instrumental in mitigating the impact of a wide-scale disruption to the financial system.

In this paper we have investigated how a wide-scale disruption is likely to impair the smooth functioning of the interbank payment system. Such a disruption will almost by definition - create operational difficulties for the system and its participants in some way or another. However, we address a problem that is perhaps less obvious: the operational difficulties of some participants may induce other participants to change behavior in terms of how they process payments. In particular, this may lead to a break down of coordination.

This paper argues that the ability of banks in Fedwire to maintain payment coordination following a wide-scale disruption depends critically on a number of different factors. These included the duration of the disruption ( $\tau$ ), the size of the merged or coordinated sector ( $k$ ), and the fraction of each dollar that small banks transact with each other after a merger ( $\alpha$ ). The main findings are summarized below.

1. High fees are bad for disruptions to either small banks or the merged bank (Props. 2, 3, 5, 6 and 8)
2. High  $\alpha$  is bad for disruptions to the small banks because it reduces the minimum size disruption that is self-perpetuating. (Prop. 6)
3. High  $\alpha$  is good for disruptions to the merged bank because it allows disruptions to the merged bank to be self-reversing under high fee regimes. (Prop. 8)
4. High  $\tau$  is bad for disruptions to either small banks or the merged bank because it can increase the probability of convergence to the inefficient equilibrium in intermediate cases (Props. 7 and 9).
5. High  $k$  is good for disruptions to small banks because it means disruptions to small banks are only self perpetuating in high fee regimes (Sec. 5.3.1).
6. High  $k$  is bad for disruptions to the merged bank because it means low fees are required for disruptions to the merged bank be to self-reversing (Sec. 5.3.2)

Absent from our discussion are insolvency concerns, which were the driver behind the settlement problems during the stock markets crash of 1987 and the settlement strains after the failure of Bankhaus Herstatt in 1974. As argued by Lacker (2004) it was fortuitous that the banking system was in a relatively healthy condition on September 11, 2001 but the combination of a wide scale operational disruption and banking sector weakness is not inconceivable and something future research should pursue. Another important factor, not addressed specifically in this paper, is the timing of the disruption. The clearing and settlement cycle over the course of the day consists of a range of critical times where different types of processes take place. For example, settlement of foreign exchange transactions through Continuous Linked Settlement (CLS) bank occurs in a narrow window very early in the

morning whereas the bulk of the activity from the money and U.S. Treasury markets occur in the afternoon. The effect of a disruption is going to depend on what time of the day it occurs and how long it persists.

## 8 Appendix

### A Potential function, homogenous case

From Eq. (4) we have

$$\begin{aligned}
\tilde{P}(n) - \tilde{P}(n-1) &= (n-1)D \\
\tilde{P}(n-1) - \tilde{P}(n-2) &= -F + (n-1)D \\
\tilde{P}(n-2) - \tilde{P}(n-3) &= -2F + (n-1)D \\
\tilde{P}(n-3) - \tilde{P}(n-4) &= -3F + (n-1)D \\
&\vdots \\
\tilde{P}(2) - \tilde{P}(1) &= -(n-2)F - (n-1)D \\
\tilde{P}(1) - \tilde{P}(0) &= -(n-1)F + (n-1)D
\end{aligned} \tag{18}$$

Let  $\tilde{P}(n) = 0$ . Then  $\tilde{P}(n-1) = -(n-1)D$  and so forth

$$\begin{aligned}
\tilde{P}(n-2) &= \tilde{P}(n-1) + F - (n-1)D = F - 2(n-1)D \\
\tilde{P}(n-3) &= \tilde{P}(n-2) + 2F - (n-1)D = 3F - 3(n-1)D \\
\tilde{P}(n-4) &= \tilde{P}(n-3) + 3F + (n-1)D = 6F - 4(n-1)D \\
\tilde{P}(n-5) &= \tilde{P}(n-4) + 4F + (n-1)D = 10F - 5(n-1)D \\
&\vdots
\end{aligned} \tag{19}$$

Hence, we have

$$\tilde{P}(x) = \frac{1}{2}(n-1-x)(n-x)F - (n-x)(n-1)D \tag{20}$$

## B Potential function, heterogeneous Case

Let  $\hat{P}(m, n - k) = 0$ . From Eqs. (13) - (15) we have that  $\hat{P}(m, n - k - 1) = -(n - 1)D$ . Moreover, we have

$$\begin{aligned}\hat{P}(m, n - k - 2) &= \hat{P}(m, n - k - 1) + \alpha(n - k - (n - k - 1))F - (n - 1)D \\ &= \alpha F - 2(n - 1)D, \\ \hat{P}(m, n - k - 3) &= \hat{P}(m, n - k - 2) + \alpha(n - k - (n - k - 2))F - (n - 1)D \\ &\quad 3\alpha F - 3(n - 1)D, \\ \hat{P}(m, n - k - 4) &= \hat{P}(m, n - k - 3) + \alpha(n - k - (n - k - 3))F - (n - 1)D \\ &= 6\alpha F - 4(n - 1)D\end{aligned}\tag{21}$$

and so forth. Hence, we have

$$\hat{P}(m, x) = \frac{\alpha}{2}(n - k - 1 - x)(n - k - x)F - (n - k - x)(n - 1)D\tag{22}$$

for  $x = 1, \dots, n - k$ . Furthermore, from Eq. (15) we know that

$$\hat{P}(a, x) = \hat{P}(m, x) + (\alpha k + (1 - \alpha)(n - 1))((n - k - x)F - (n - k)D)\tag{23}$$

Substituting in Eq (22) and simplifying yields

$$\begin{aligned}\hat{P}(a, x) &= [n - 1 - \frac{\alpha}{2}(n - k + x - 1)](n - k - x)F \\ &\quad - \{(n - k - x)(n - 1) + [\alpha k + (1 - \alpha)(n - 1)](n - k)\}D\end{aligned}\tag{24}$$

for  $x = 1, \dots, n - k$ . Both  $\hat{P}(m, x)$  and  $\hat{P}(a, x)$  are second order polynomials in  $x$ .

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