Economics 150A - Labor Economics
Assignment #3 - Answers

Question 1

A manager is trying to decide how many workers to hire. His factory has the following production function: $Q=100L^{1/2}K^{1/4}$. The price of output is \$10 the wage rate is \$100, and the rental rate for capital is \$100 per unit.

(a) What is the short-run (i.e. you must pay the capital cost whether or not you produce anything) profit maximizing level of labor demand if k=100?

$$L_{SR} = \frac{50^2 p^2 K^{1/2}}{w^2} = 250 \quad and \quad \pi = 15{,}000$$

Since profit is less negative than the fixed capital cost you should produce in the short run using 250 units of labor.

a. What is the profit maximizing level of short-run labor demand if the price of output falls to \$5?

$$L_{SR} = \frac{50^2 p^2 K^{1/2}}{w^2} = 62.5 \quad and \quad \pi = -3{,}750$$

Since profit is less negative than the fixed capital cost (-10,000) you should produce in the short run using 62.5 units of labor.

b. What is the profit maximizing level of short-run labor demand if the wage falls to \$50?

$$L_{SR} = \frac{50^2 p^2 K^{1/2}}{w^2} = 1{,}000 \quad and \quad \pi = 40{,}000$$

Since profit is less negative than the fixed capital cost you should produce in the short run using 1,000 units of labor.

c. What is the profit maximizing level of short-run labor demand if short-run k=10000?

$$L_{SR} = \frac{50^2 p^2 K^{1/2}}{w^2} = 2{,}500 \quad and \quad \pi = -750{,}000$$

Since profit is less negative than the fixed capital cost (-1,000,000) you should produce in the short run using 2,500 units of labor.

(b) What is the long-run profit maximizing level of labor demand?

$$L_{LR} = \frac{50^4 p^4}{2w^3 r} = 312.5 \quad K_{LR} = \frac{50^4 p^4}{4w^2 r^2} = 156.25 \quad and \quad \pi = 15{,}625$$

Since profit is positive you should produce in the long run using 312.5 units of labor.

a. What is the profit maximizing level of long-run labor demand if the price of output falls to $5?

$$L_{LR} = \frac{50^4 p^4}{2w^3 r} = 19.53 \quad K_{LR} = \frac{50^4 p^4}{4w^2 r^2} = 9.77 \quad and \quad \pi = 976.56$$

Since profit is positive you should produce in the long run using 19.53 units of labor.

b. What is the profit maximizing level of long-run labor demand if the wage falls to $50?

$$L_{LR} = \frac{50^4 p^4}{2w^3 r} = 2500 \quad K_{LR} = \frac{50^4 p^4}{4w^2 r^2} = 625 \quad and \quad \pi = 62{,}500$$

Since profit is positive you should produce in the long run using 2500 units of labor.

c. What is the profit maximizing level of long-run labor demand if short-run k=10000?

The short-run capital is irrelevant for the long run. You should continue to produce using 312.5 units of labor.

Question 2

Laura (of Luke and Laura's Itty-Bitty Drink Umbrella Inc.) is trying to decide how many workers to hire. The Itty-Bitty Drink Umbrella production function is given by: $Q=6L^{1/3}K^{1/2}$. The price of output is p, the wage rate is w, and the rental rate for capital is r.

(a) In the short-run, while capital is fixed, what is the profit maximizing level of labor demand.

Answer

$$MAX\ \Pi = pQ - wL - rK = 6pL^{1/3}K^{1/2} - wL - rK$$

$$FOC: \frac{\partial \Pi}{\partial L} = 0 \quad \Rightarrow \quad (6/3)pL^{-2/3}K^{1/2} = w \quad \Rightarrow \quad L_{SR}^{D} = \left(\frac{w}{2pK^{1/2}}\right)^{-3/2} = \left(\frac{2pK^{1/2}}{w}\right)^{3/2} = \frac{\sqrt{8}p^{3/2}K^{3/4}}{w^{3/2}}$$

(b) In the long-run, when Laura can choose both labor and capital levels to maximize profits, what is the profit maximizing level of labor demand?

Answer

$MAX\ \Pi = pq - wL - rK = 6pL^{1/3}K^{1/2} - wL - rK$

FOCs

(1) $\frac{\partial \Pi}{\partial L} = 0 \quad \Rightarrow \quad (6/3)pL^{-2/3}K^{1/2} = w$

(2) $\frac{\partial \Pi}{\partial K} = 0 \quad \Rightarrow \quad (6/2)pL^{1/3}K^{-1/2} = r$

and,

(3) $\frac{MP_L}{MP_K} = \frac{w}{r} \quad \Rightarrow \quad \frac{2K}{3L} = \frac{w}{r} \quad \Rightarrow \quad K = \frac{3wL}{2r}$

substituting (3) into (1),

$(6/3)pL^{-2/3}\left(\frac{3wL}{2r}\right)^{1/2} = w$

$\frac{2^{1/2}3^{1/2}pw^{1/2}L^{-1/6}}{r^{1/2}} = w$

$L_{LR}^{D} = \left(\frac{w^{1/2}r^{1/2}}{2^{1/2}3^{1/2}p}\right)^{-6} = \left(\frac{2^{1/2}3^{1/2}p}{w^{1/2}r^{1/2}}\right)^{6} = \frac{216p^6}{w^3r^3}$

Substituting into K,

$K_{LR}^{D} = \frac{324p^6}{w^2r^4}$

(d) Is labor demand more elastic in the long-run or the short-run, and WHY? Show the math and explain why your answer in words.

Answer

In the Short Run: $\eta_{SR}^{D} = \frac{\partial L_{SR}^{D}}{\partial w} \times \frac{w}{L_{SR}^{D}} = \frac{\left(\sqrt{8}\right)(-3/2)p^{3/2}\overline{K}^{3/4}}{w^{5/2}} \times \frac{w}{\frac{\left(\sqrt{8}\right)p^{3/2}\overline{K}^{3/4}}{w^{3/2}}} = -\frac{3}{2}$

In the Long Run: $\eta_{LR}^{D} = \frac{\partial L_{LR}^{D}}{\partial w} \times \frac{w}{L_{LR}^{D}} = \frac{(216)(-3)p^6}{w^4r^3} \times \frac{w}{\frac{216p^6}{w^3r^3}} = -3$

The elasticity of labor demand is greater in the long-run because both labor and capital are flexible.