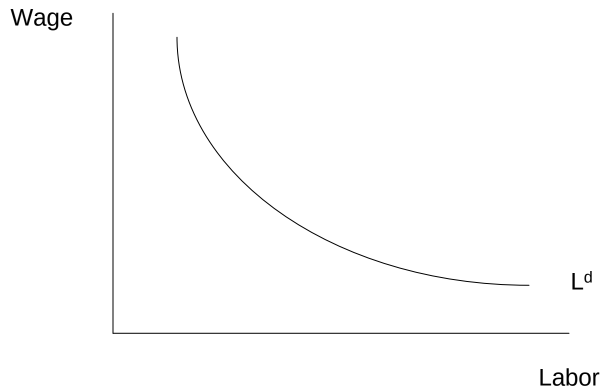
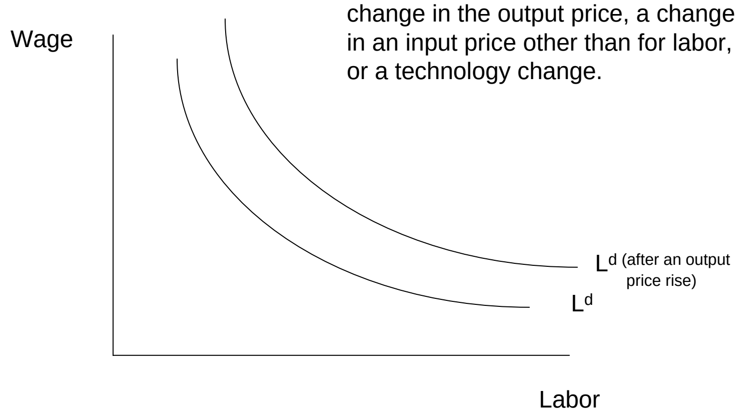
The Demand for Labor in a Competitive Market

As wages rise the demand for labor falls. We will prove this in a few days.



The Demand for Labor in a Competitive Market

Recall that a change in the wage is a movement along the labor demand curve, while a shift is caused by a change in the output price, a change in an input price other than for labor, or a technology change.



How do firms decide how much labor to hire?

- The fundamental assumption of labor demand theory is that FIRMS ARE PROFIT MAXIMIZERS
  - Firms are always trying to find ways to make larger profits
- The standard OUTPUT interpretation
  - Increase output if the revenue from doing so exceeds the cost
  - Marginal Revenue (MR) > Marginal Cost (MC)
- The INPUT interpretation
  - Use more of an input if the income from doing so exceeds the cost

How do firms decide how much labor to hire?

More formally,

- The marginal product of labor: $MP_L = \frac{\partial Q}{\partial L}$
- The marginal product of capital: $MP_K = \frac{\partial Q}{\partial K}$

A firm can expand or contract output by increasing or decreasing its use of labor or capital. However, the firm is really interested in the revenue generated by these decisions.

How do firms decide how much labor to hire?

- The marginal revenue product of labor is:
  $MRP_L = MP_L \times MR$ (in general)
  $MRP_L = MP_L \times p$ (in a competitive product market)

- The marginal revenue product of capital is:
  $MRP_K = MP_K \times MR$ (in general)
  $MRP_K = MP_K \times p$ (in a competitive product market)

How do firms decide how much labor to hire?

In general,

- Increase the amount of labor if: $MRP_L > MC_L$
- Decrease the amount of labor if: $MRP_L < MC_L$
- Keep the same amount of labor if: $MRP_L = MC_L$

In a competitive labor market: $MC_L = w$
In a competitive capital market: $MC_K = r$

In a competitive labor market:

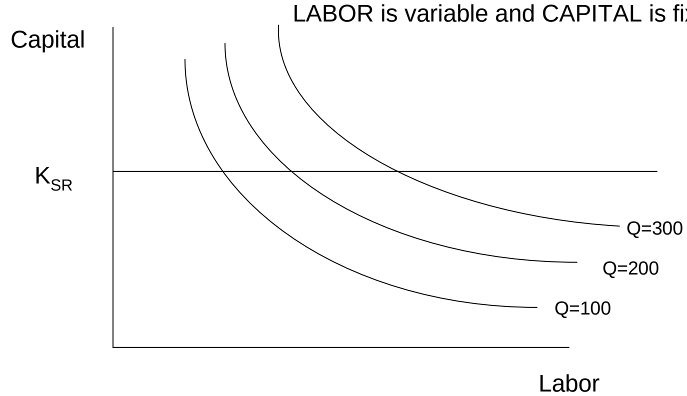
- Increase the amount of labor if: $MRP_L > w$
- Decrease the amount of labor if: $MRP_L < w$
- Keep the same amount of labor if: $MRP_L = w$

The short-run versus the long-run

- Short Run: Firms can only alter their variable inputs (labor)
- Long Run: Firms can alter all inputs

Labor Demand in the SHORT RUN

For simplicity, assume that in the short run LABOR is variable and CAPITAL is fixed



With capital fixed, it requires increasingly more labor to generate the same increase in output. This is known as the DECREASING MARGINAL PRODUCTIVITY assumption. There are diminishing returns to adding more labor when capital is held constant. Stated somewhat differently, as more labor is forced to use the same capital stock marginal productivity falls.

Labor Demand in the SHORT RUN

As we saw earlier, a firm will increase output (or add more labor) to the point where the benefit from adding one more unit of labor is just equal to the cost of doing so ($MRP_L = w$)

- When both the output (Q) and labor (L) markets are competitive:

$$MP_L \times p = w$$

$$MP_L = \frac{w}{p}$$

- In other words, the productivity of the last person hired must equal their “real wage”

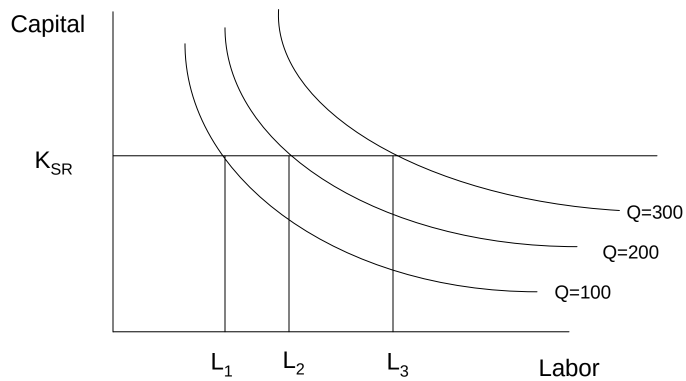
Labor Demand in the SHORT RUN

Example: w = $10 per hour, p = $3, and the last worker hired produces 3 units per hour. Should you keep this worker?

Answer: $MP_L = 3, \quad \frac{w}{p} = 3.33 \quad \Rightarrow \quad MP_L < \frac{w}{p}$

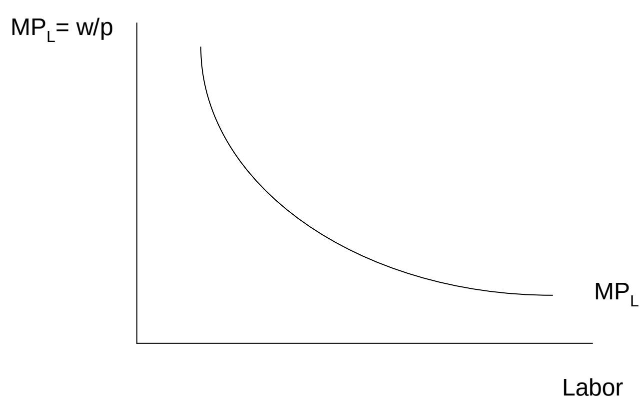
Since the last worker hired is worth less than he costs you should fire him.

Labor Demand in the SHORT RUN



Since we know that there are diminishing returns to adding more labor when capital is fixed, the MP_L must be downward sloping.

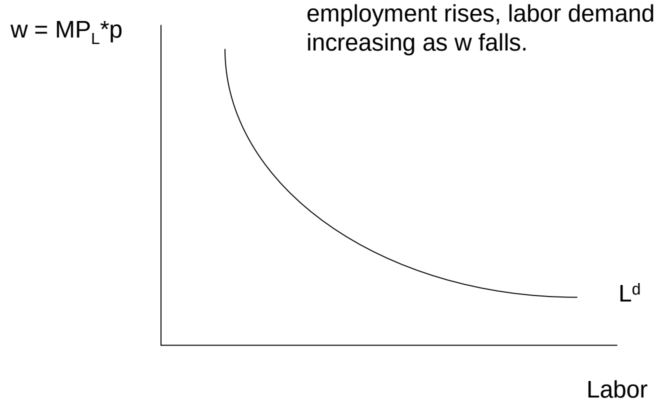
Labor Demand in the SHORT RUN



Labor Demand in the SHORT RUN

Since in equilibrium, $MP_L = w/p$,

then $w = p * MP_L$. And since MP_L is falling as employment rises, labor demand must be increasing as w falls.



Optimal amount of labor in the short run

- In the short run capital is fixed. The profit maximizing labor choice is thus given by:

$$MP_L = \frac{w}{p}$$

Optimal amount of labor in the short run

- Example: $Q = 2K^{1/4}L^{1/2}, \quad p = 10, \quad w = 2, \quad r = 1, \quad K_{SR} = 10{,}000$
- Solution:

$$MP_L = \frac{\partial Q}{\partial L} = \frac{w}{p} \quad \Rightarrow \quad 2(1/2)K_{SR}^{1/4}L^{-1/2} = \frac{w}{p} \quad \Rightarrow \quad \frac{K_{SR}^{1/4}}{L^{1/2}} = \frac{w}{p}$$

$$L^{1/2} = \frac{pK_{SR}^{1/4}}{w} \quad \Rightarrow \quad L = \left(\frac{pK_{SR}^{1/4}}{w}\right)^2 \quad \Rightarrow \quad L_{SR} = \frac{p^2K_{SR}^{1/2}}{w^2}$$

$$L_{SR} = \frac{10^2 10{,}000^{1/2}}{2^2} = 2500$$

Optimal amount of labor in the short run

- How much profit is earned?

$$\begin{aligned}\pi &= pQ - wL - rK_{SR} \\ &= p(2K_{SR}^{1/4}L^{1/2}) - wL - rK_{SR} \\ &= 10 \times 2 \times 10 \times 50 - 2 \times 2500 - 1 \times 10{,}000 \\ &= -5{,}000\end{aligned}$$

Optimal amount of labor in the short run

- What if p increases to 20?
- Labor demand: $L_{SR} = \frac{p^2 K_{SR}^{1/2}}{w^2} = \frac{20^2 10{,}000^{1/2}}{2^2} = 10{,}000$
- Profit: $\pi = 20 \times 2 \times 10 \times 100 - 2 \times 10{,}000 - 1 \times 10{,}000$
  $= 10{,}000$

Optimal amount of labor in the short run

- What if w decreases to 1?
- Labor demand: $L_{SR} = \frac{p^2 K_{SR}^{1/2}}{w^2} = \frac{10^2 10{,}000^{1/2}}{1^2} = 10{,}000$
- Profit: $\pi = 10 \times 2 \times 10 \times 100 - 1 \times 10{,}000 - 1 \times 10{,}000$
  $= 0$

Optimal amount of labor in the short run

- What is the elasticity of short-run labor demand?

$$\eta_{SR} = \frac{\partial L_{SR}}{\partial w} \times \frac{w}{L_{SR}} = -\frac{2p^2 K_{SR}^{1/2}}{w^3} \times \frac{w}{\frac{p^2 K_{SR}^{1/2}}{w^2}} = -2$$

Labor Demand in the LONG RUN

- Both LABOR and CAPITAL are flexible in the long run
- A firm adds more **labor and/or capital** to the point where the benefit from adding one more unit of input is just equal to the cost of doing so (MRP_L = w and MRP_K = r)
- When both the output (Q) and input (L and K) markets are competitive:

$$pMP_L = w \quad \Rightarrow \quad MP_L = \frac{w}{p}$$

$$pMP_K = r \quad \Rightarrow \quad MP_K = \frac{r}{p}$$

Labor Demand in the LONG RUN

- If we simply rearrange the equations:

$$p = \frac{w}{MP_L} \quad and \quad p = \frac{r}{MP_K} \quad \Rightarrow \quad \frac{MP_L}{MP_K} = \frac{w}{r}$$

- The benefit/cost ratio for using one more unit of labor or capital to produce more output must be equal at the profit maximizing point.
- Otherwise, the firm could use less of one input and more of the other input to produce the same output at a lower cost.

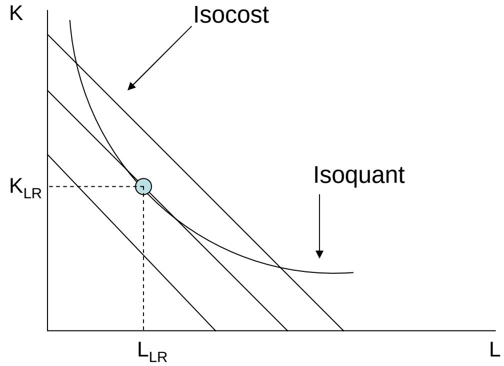
Alternative solution method

- Alternatively, you might want to think about the problem as the following two-stage problem:

(1) The firm chooses the profit maximizing level of output
$\rightarrow$ the firm increases output until MR = MC

(2) The firm then chooses the least cost way to produce the profit maximizing level of output

Graphically



Notice that the cost minimizing method of producing the profit maximizing level of output occurs where the slope of the isocost is the same as the slope of the isoquant.

In words

- The slope of the Isocost curve is -w/r
  - → wL + rK = Cost
  - → K = Cost/r - (w/r) L
- The slope of the Isoquant curve:

$$MTRS = -\frac{\Delta K}{\Delta L} = -\left(\frac{\Delta K}{\Delta Q}\right) \Big/ \left(\frac{\Delta L}{\Delta Q}\right) = -\left(\frac{\Delta Q}{\Delta L}\right) \Big/ \left(\frac{\Delta Q}{\Delta K}\right) = -\frac{MP_L}{MP_K}$$

- At Q* the slope of the Isoquant and Iscost curves are the same

$$MRTS = -\frac{w}{r} \quad \Rightarrow \quad -\frac{MP_L}{MP_K} = -\frac{w}{r}$$

- The benefit/cost ratio for using one more unit of labor or capital to produce more output must be equal at the profit maximizing point

Optimal amount of labor in the long run

- Example: $Q = 2K^{1/4}L^{1/2}, \quad p = 10, \quad w = 2, \quad r = 1$
- Solution:

$$(1)\ MP_L = \frac{w}{p}, \quad (2)\ MP_K = \frac{r}{p}, \quad (3)\ \frac{MP_L}{MP_K} = \frac{w}{r}$$

$$\text{Using (3)}: \quad \frac{MP_L}{MP_K} = \frac{w}{r} \quad \Rightarrow \quad \frac{\frac{K^{1/4}}{L^{1/2}}}{\frac{L^{1/2}}{2K^{3/4}}} = \frac{w}{r} \quad \Rightarrow \quad \frac{2K}{L} = \frac{w}{r} \quad \Rightarrow \quad K = \frac{wL}{2r}$$

$$\text{Subbing into (1)}: \quad \frac{K^{1/4}}{L^{1/2}} = \frac{w}{p} \quad \Rightarrow \quad L = \frac{p^2 K^{1/2}}{w^2} = \frac{p^2\left(\frac{wL}{2r}\right)^{1/2}}{w^2}$$

$$\text{Rearranging to isolate L}: \quad L_{LR} = \frac{p^4}{2rw^3}$$

Optimal amount of capital in the long run

$$\text{Subbing L into } K = \frac{wL}{2r}: \quad K_{LR} = \frac{w\left(\frac{p^4}{2rw^3}\right)}{2r} = \frac{p^4}{4r^2w^2}$$

$$\text{Numerically}: \quad L_{LR} = 625 \quad and \quad K_{LR} = 625$$

$$and \quad \pi = 10 \times 2 \times 5 \times 25 - 2 \times 625 - 1 \times 625 = 625$$

What if p increases to $20?

$$L_{LR} = \frac{p^4}{2rw^3} = 10{,}000 \quad and \quad K_{LR} = \frac{p^4}{4r^2w^2} = 10{,}000$$

$$and \quad \pi = 20 \times 2 \times 5 \times 25 - 2 \times 10{,}000 - 1 \times 10{,}000 = 10{,}000$$

What if w decreases to $1?

$$L_{LR} = \frac{p^4}{2rw^3} = 5{,}000 \quad and \quad K_{LR} = \frac{p^4}{4r^2w^2} = 2{,}500$$

$$and \quad \pi = 10 \times 2 \times 7.07 \times 70.7 - 1 \times 5{,}000 - 1 \times 2{,}500 = 2496.98$$

What is the elasticity of long run labor demand?

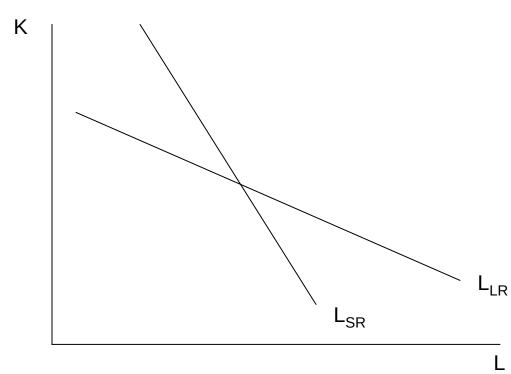
$$\eta_{LR} = \frac{\partial L_{LR}}{\partial w} \times \frac{w}{L_{LR}} = -\frac{3p^4}{2w^4 r} \times \frac{w}{\frac{p^4}{2w^3 r}} = -3$$

Compare this to the short run elasticity of :

$$\eta_{SR} = \frac{\partial L_{SR}}{\partial w} \times \frac{w}{L_{SR}} = -\frac{2p^2 K_{SR}^{1/2}}{w^3} \times \frac{w}{\frac{p^2 K_{SR}^{1/2}}{w^2}} = -2$$

- The long-run elasticity is always greater (in absolute value) because all inputs are flexible

Short and Long Run Elasticity



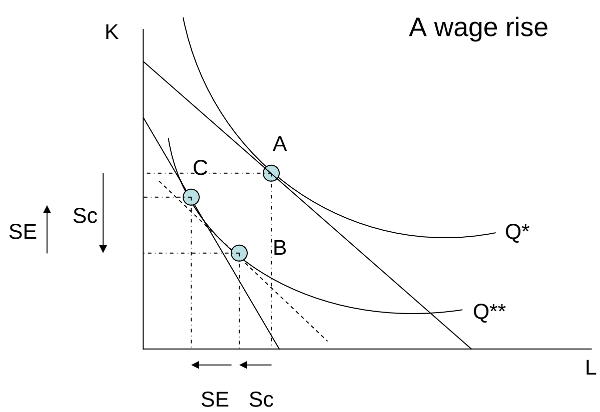
Note: the labor demand curves are not necessarily straight lines.

Elasticity

Labor demand elasticity is high under the following conditions:

- When the price elasticity of demand for the output produced is high
  - Implications:
    - labor demand is more elastic at the firm level than the industry level
    - labor demand is more elastic in the long-run
- When the other factors of production can easily be substituted for labor
- When labor costs are a large share of total cost

Scale and Substitution Effects



Since L and K are adjustable in the long run, it is important to understand how a change in the price of one input changes the demand for the other input

Scale and Substitution Effects

- The scale and substitution effects are reinforcing for the input whose price changes and off-setting for the input whose price does not change.

Downward sloping long run L^D

- Labor demand is downward sloping in the long-run because:
  - A wage rise leads to substitution away from labor
  - A wage rise lowers the profit maximizing scale of output

In other words, the Sc and SE ensure a downward sloping labor demand curve in the long-run.

Sc and SE

- We know that the Sc and SE go in the same direction for labor is w changes, but how do we determine whether the SE or Sc dominates for K?

- Example: $Q = 2K^{1/4}L^{1/2}, \quad p = 10, \quad w = 2, \quad r = 1$

$$\text{Long run L and K are}: \quad L_{LR} = \frac{p^4}{2rw^3} = 625 \quad and \quad K_{LR} = \frac{p^4}{4r^2w^2} = 625$$

$$\text{If w} \downarrow \text{to 1}: \quad L_{LR} = \frac{10^4}{2(1)(1)^3} = 5000 \quad and \quad K_{LR} = \frac{10^4}{4(1)^2(1)^2} = 2500$$

$$K \uparrow \quad \Rightarrow \quad |Sc| > |SE|$$

Let's work through and example

$$Q = 4L^{1/4}K^{1/4}, \quad p = 120, \quad w = 4, \quad r = 16, K_{SR} = 64, \text{and a franchise fee} = 25p$$

- Short-run:

$$MP_L = \frac{w}{p} \quad \Rightarrow \quad L^{-3/4}K_{SR}^{1/4} = \frac{w}{p} \quad \Rightarrow \quad L^{-3/4} = \frac{w}{pK_{SR}^{1/4}}$$

$$\Rightarrow \quad \left(L^{-3/4}\right)^{-4/3} = \left(\frac{w}{pK_{SR}^{1/4}}\right)^{-4/3}$$

$$\Rightarrow \quad L_{SR} = \frac{p^{4/3}K_{SR}^{1/3}}{w^{4/3}}$$

Numerical Answer

$$L_{SR} = \frac{p^{4/3}K_{SR}^{1/3}}{w^{4/3}} = \frac{120^{4/3}(64^{1/3})}{4^{4/3}} = \frac{591.89\times 4}{6.35} = 372.84$$

$$\begin{aligned}\pi &= pQ - wL - rK_{SR} - 25p = 4pL^{1/4}K_{SR}^{1/4} - wL - rK_{SR} - 25p \\ &= 4\times 120\times 372.84^{1/4}\times 64^{1/4} - 4\times 372.84 - 16\times 64 - 25\times 120 \\ &= 5965.72 - 1491.36 - 1024 - 3000 \\ &= 450.36\end{aligned}$$

Since profits are positive, the profit maximizing level of short-run labor demand is 372.84 units of labor.

Long-run:

$$(1)\ MP_L = \frac{w}{p}, \quad (2)\ MP_K = \frac{r}{p}, \quad (3)\ \frac{MP_L}{MP_K} = \frac{w}{r}$$

$$(1)\ L = \frac{p^{4/3}K^{1/3}}{w^{4/3}}$$

$$(3)\ \frac{MP_L}{MP_K} = \frac{w}{r} \quad \Rightarrow \quad \frac{L^{-3/4}K^{1/4}}{L^{1/4}K^{-3/4}} = \frac{w}{r} \quad \Rightarrow \quad \frac{K}{L} = \frac{w}{r} \quad \Rightarrow \quad K = \frac{wL}{r}$$

$$\text{Subbing into (1)}: \quad L = \frac{p^{4/3}\left(\frac{wL}{r}\right)^{1/3}}{w^{4/3}} \quad \Rightarrow \quad L = \frac{p^{4/3}L^{1/3}}{wr^{1/3}}$$

$$\text{Rearranging to isolate L}: \quad L_{LR} = \left(\frac{p^{4/3}}{wr^{1/3}}\right)^{3/2} = \frac{p^2}{w^{3/2}r^{1/2}}$$

$$\text{Solving for K}: \quad K_{LR} = \frac{w\left(\frac{p^2}{w^{3/2}r^{1/2}}\right)}{r} = \frac{p^2}{w^{1/2}r^{3/2}}$$

Numerical Answer

$$L_{LR} = \frac{p^2}{w^{3/2}r^{1/2}} = \frac{120^2}{4^{3/2}16^{1/2}} = \frac{14{,}400}{8\times 4} = 450$$

$$K_{LR} = \frac{p^2}{w^{1/2}r^{3/2}} = \frac{120^2}{4^{1/2}16^{3/2}} = \frac{14{,}400}{64\times 2} = 112.5$$

$$\begin{aligned}\pi &= pQ - wL - rK - 25p = 4pL^{1/4}K^{1/4} - wL - rK - 25p\\ &= 4\times 120\times 450^{1/4}\times 112.5^{1/4} - 4\times 450 - 16\times 112.5 - 25\times 120\\ &= 7200 - 1800 - 1800 - 3000\\ &= 600\end{aligned}$$

Since profits are positive, the profit maximizing level of long-run labor demand is 450 units of labor and the long-run capital demand is 112.5 units.

What if p increases to $130?

$$L_{SR} = \frac{p^{4/3}K_{SR}^{1/3}}{w^{4/3}} = \frac{130^{4/3}(64^{1/3})}{4^{4/3}} = \frac{658.55\times 4}{6.35} = 414.84$$

$$\begin{aligned}\pi &= pQ - wL - rK_{SR} - 25p = 4pL^{1/4}K_{SR}^{1/4} - wL - rK_{SR} - 25p\\ &= 4\times 130\times 414.84^{1/4}\times 64^{1/4} - 4\times 414.84 - 16\times 64 - 25\times 130\\ &= 6637.64 - 1659.36 - 1024 - 3250\\ &= 704.28\end{aligned}$$

Since profits are positive, the profit maximizing level of short-run labor demand is 414.84 units of labor.

What if p increases to $130?

$$L_{LR} = \frac{p^2}{w^{3/2}r^{1/2}} = \frac{130^2}{4^{3/2}16^{1/2}} = \frac{16{,}900}{8\times 4} = 528.13$$

$$K_{LR} = \frac{p^2}{w^{1/2}r^{3/2}} = \frac{130^2}{4^{1/2}16^{3/2}} = \frac{16{,}900}{64\times 2} = 132.03$$

$$\begin{aligned}\pi &= pQ - wL - rK - 25p = 4pL^{1/4}K^{1/4} - wL - rK - 25p \\ &= 4\times 130\times 528.13^{1/4}\times 132.03^{1/4} - 4\times 528.13 - 16\times 132.03 - 25\times 130 \\ &= 8448.12 - 2112.53 - 2112.48 - 3250 \\ &= 973.11\end{aligned}$$

Since profits are positive, the profit maximizing level of long-run labor demand is 528.13 units of labor and the long-run capital demand is 132.03 units.

What if w increases to $6?

$$L_{SR} = \frac{p^{4/3}K_{SR}^{1/3}}{w^{4/3}} = \frac{120^{4/3}(64^{1/3})}{6^{4/3}} = \frac{591.89\times 4}{10.9} = 217.21$$

$$\begin{aligned}\pi &= pQ - wL - rK_{SR} - 25p = 4pL^{1/4}K_{SR}^{1/4} - wL - rK_{SR} - 25p \\ &= 4\times 120\times 217.21^{1/4}\times 64^{1/4} - 6\times 217.21 - 16\times 64 - 25\times 120 \\ &= 5212 - 1303.26 - 1024 - 3000 \\ &= -115.26\end{aligned}$$

Since profits are negative, the profit maximizing level of short-run labor demand is 0 units of labor.

What if w increases to $6?

$$L_{LR} = \frac{p^2}{w^{3/2}r^{1/2}} = \frac{120^2}{6^{3/2}16^{1/2}} = \frac{14{,}400}{14.7 \times 4} = 244.9$$

$$K_{LR} = \frac{p^2}{w^{1/2}r^{3/2}} = \frac{120^2}{6^{1/2}16^{3/2}} = \frac{14{,}400}{64 \times 2.45} = 91.84$$

$$\begin{aligned}\pi &= pQ - wL - rK - 25p = 4pL^{1/4}K^{1/4} - wL - rK - 25p \\ &= 4 \times 120 \times 244.9^{1/4} \times 91.84^{1/4} - 6 \times 244.9 - 16 \times 91.84 - 25 \times 120 \\ &= 5878.20 - 1469.4 - 1469.44 - 3000 \\ &= -60.64\end{aligned}$$

Since profits are negative, the profit maximizing level of long-run labor demand is 0 units of labor and the long-run capital demand is 0 units.

What is the elasticity of labor demand?

- Short-run: $$\eta_{SR} = \frac{\partial L_{SR}}{\partial w} \times \frac{w}{L_{SR}} = -\frac{(4/3)p^{4/3}K_{SR}^{1/3}}{w^{7/3}} \times \frac{w}{\frac{p^{4/3}K_{SR}^{1/3}}{w^{4/3}}} = -4/3$$
- Long-run: $$\eta_{LR} = \frac{\partial L_{LR}}{\partial w} \times \frac{w}{L_{LR}} = -\frac{(3/2)p^2}{w^{5/2}r^{1/2}} \times \frac{w}{\frac{p^2}{w^{3/2}r^{1/2}}} = -3/2$$
- The long-run elasticity is always greater (in absolute value) because all inputs are flexible

Sc and SE

- We know that the Sc and SE go in the same direction for labor is w changes, but how do we determine whether the SE or Sc dominates for K?

Long run L and K are: $L_{LR} = \frac{p^2}{w^{3/2}r^{1/2}} = 450 \quad and \quad K_{LR} = \frac{p^2}{w^{1/2}r^{3/2}} = 112.5$

If w $\downarrow$ to 3: $L_{LR} = \frac{120^2}{3^{3/2}16^{1/2}} = 692.82 \quad and \quad K_{LR} = \frac{120^2}{3^{1/2}16^{3/2}} = 129.9$

$K \uparrow \quad \Rightarrow \quad |Sc| > |SE|$