

The N-worker Case

Now suppose there are n_1 type-1 workers and n_2 type-2 workers, $n_1+n_2 = N$ in all. Each worker's effort is still given by (1), where the co-worker wage is the mean wage of the worker's co-workers, i.e.:

$$(A1) \quad w_c = \frac{(n_1-1)w_1 + n_2w_2}{N-1} \text{ for type-1 workers, and}$$

$$(A2) \quad w_c = \frac{n_1w_1 + (n_2-1)w_2}{N-1} \text{ for type-2 workers.}$$

Total profits are then of course $n_1R(E_1) + n_2\theta R(E_2)$, and (3) and (4) become, respectively:

$$(A3) \quad aR'(E_1) + b(n_2/(N-1))[R'(E_1) - \theta R'(E_2)] = 1$$

$$(A4) \quad \theta aR'(E_2) + b(n_1/(N-1))[\theta R'(E_2) - R'(E_1)] = 1$$

Result A1. In the N-worker case, as in the two-worker case, profit-maximizing effort levels for each worker (E_1 and E_2) are identical to the 'egoistic' levels identified in Result 1, *regardless* of the value of b , the size of the firm (N), or the relative numbers of high- and low-productivity workers. As in the two-worker example, however, wages are not identical to the egoistic case: profit-maximizing firms compress wages relative to the egoistic equilibrium; i.e. $w_1 > w_1^e$ and $w_2 < w_2^e$. As b rises, w_1 rises and w_2 falls, but workers' wage rankings are never reversed, i.e. $w_1 < w_2$ regardless of b .

Proof. Using (A3) and (A4), the same argument as in Result 2 establishes that $E_1 = E_1^e$ and $E_2 = E_2^e$. Solving for wages as a function of these effort levels, (5) and (6) now become:

$$(A5) \quad w_1 = \frac{\alpha \tilde{b} E_2 + (a + (1-\alpha)\tilde{b})E_1}{a^2 + a\tilde{b}}$$

$$(A6) \quad w_2 = \frac{(1-\alpha)\tilde{b} E_1 + (a + \alpha\tilde{b})E_2}{a^2 + a\tilde{b}}$$

where $\alpha \equiv n_2/N$ (the share of high-productivity workers) and $\tilde{b} \equiv b \frac{N}{(N-1)}$. Parallel arguments to those in Result 2 establish the remaining claims. ■

Result A2. Holding firm size (N) fixed, the profit-maximizing wages of *both* worker types rise as the share of high-productivity workers in the firm (α) rises. The wage gap between the two worker types is invariant to α

Proof. This follows directly from (A5) and (A6), given that $E_1 < E_2$ and both effort levels are invariant to α . ■

Result A2 implies that, because of rent-sharing within firms, workers of *both* types would prefer that the firm would hire additional high-productivity workers rather than low-productivity workers.

To see the intuition behind Result A2 note from (A5) that, when type-1 (low-productivity) workers are predominant (as $\alpha \rightarrow 0$), the type-1 wage approaches the egoistic level (E_1/a). This is because, when type-1's dominate the firm, their co-worker wage essentially equals their own wage. The type-2 wage in this situation is strictly below its egoistic level. In contrast, as $\alpha \rightarrow 1$, the type-2 wage approaches the egoistic level (E_2/a), as type-2's are now, essentially, their own comparison group. Type-1 wages are strictly above their egoistic level (the fixed wage gap between the worker types being set to achieve efficient effort levels). Thus, as high-ability workers come to dominate the firm, their higher wages figure more prominently in the effort decisions of low-ability workers, and low-ability workers must be paid more in order to elicit efficient effort levels from them.

Result A3. Holding the share of type-2 workers (α) fixed, the wage gap between high- and low-productivity workers increases with firm size, (N).

Proof. Subtracting (A5) from (A6) and simplifying yields $w_2 - w_1 = (E_2 - E_1)/(a + \tilde{b})$. The result then follows from the fact that \tilde{b} is decreasing in N . ■

To see the intuition behind Result A3 consider firms of different sizes composed of equal numbers of type 1 and type 2 workers. In a two-worker firm, all of a worker's co-workers are of the 'other' type; in a very large firm only half of them are. In large firms, a larger change in the other type's wage is therefore needed to achieve a given change in a worker's average co-worker wage, w_c . Larger wage differentials are therefore required to achieve the efficient effort differential.