Symmetric, Nonlinear Effort Supply Case: Some Results

Let the effort supply function be:

$$E = F(w) + bG(w - w_c), \quad (1)$$

Where w is the worker's own wage, w is his co-worker's wage, $F(0) = 0$; $F' > 0$, $F'' < 0$; $G(0)=0$, $G'>0$, and $G(x) = -G(-x)$. (This final condition defines symmetry).

Profits can now be written:

$$\Pi = R\left[F(w_1) + bG(w_1 - w_2)\right] + \theta R\left[F(w_2) + bG(w_2 - w_1)\right] - w_1 - w_2 \quad (2)$$

where $R(0) = 0$; $R' > 0$, and $R'' < 0$.

First-order conditions for a maximum are:

$$R'(E_1) \cdot \left[F'(w_1) + bG'(w_1 - w_2)\right] - \theta R'(E_2) bG'(w_2 - w_1) = 1 \quad (3)$$

$$\theta R'(E_2) \cdot \left[F'(w_2) + bG'(w_2 - w_1)\right] - R'(E_1) bG'(w_1 - w_2) = 1 \quad (4)$$

Using $G'(w_1-w_2) = G'(w_2-w_1)$ (which follows from symmetry), (3) and (4) can be written:

$$R'(E_1)F'(w_1) + bG'(w_1 - w_2)\left[R'(E_1) - \theta R'(E_2)\right] = 1 \quad (5)$$

$$\theta R'(E_2)F'(w_2) + bG'(w_1 - w_2)\left[\theta R'(E_2) - R'(E_1)\right] = 1 \quad (6)$$

We can now show the following:

Result 1: When workers' effort supply behavior is difference-neutral (b=0), profit-maximization implies $w_2 > w_1$.

Proof: by contradiction. Suppose b=0. Then (5) and (6) imply:

$$R'(E_1)F'(w_1) = \theta R'(E_2)F'(w_2)\,. \quad (7)$$

Now suppose $w_1 > w_2$. From the effort supply function, this implies $E_1 > E_2$. By concavity of the R and F functions, it follows that $R'(E_1) < R'(E_2)$, and that $F'(w_1) < F'(w_2)$. Given $\theta > 1$, this violates (7). Therefore we cannot have $w_1 > w_2$ at a profit maximum. The case $w_1 = w_2$ can be eliminated by the same argument.

Result 2: Suppose $F(w)$ is linear (but $G(w)$ is not). Then E_1 and E_2 remain at their difference-neutral levels even when b>0. As a consequence, $w_1 > w_1^n$ and $w_2 < w_2^n$, where the *n* superscript denotes the difference-neutral wage.

Proof: Let $F(w) = aw$. (Recall that $F(0)$ =0). Now (7) becomes $R'(E_1) = \theta R'(E_2)$. Substituting this condition into the first-order conditions (5) and (6) yields $R'(E_1)a = 1$ and $\theta R'(E_2)a = 1$, which are identical to the first-order conditions under difference-neutrality. The wage-compression result then follows via a parallel argument to Result 2 in the paper.

Result 3: When *F* is concave, maximized profits *rise* with *b.*

Proof: Applying the envelope theorem to the objective function (2) yields:

$$d\Pi / db = \partial\Pi / \partial b = R'(E_1)G(w_1 - w_2) + \theta R'(E_2)G(w_2 - w_1) .$$

By symmetry, this can be written:

$$d\Pi / db = G(w_2 - w_1)\left[\theta R'(E_2) - R'(E_1)\right].$$

Given $w_2 > w_1$, $sign(d\Pi / db) = sign\left[\theta R'(E_2) - R'(E_1)\right]$. Now suppose that $\theta R'(E_2) - R'(E_1) < 0$. Then the FOC (5) implies that $R'(E_1)F'(w_1) < 1$. FOC (6) implies that $\theta R'(E_2)F'(w_2) > 1$. Combining these two conditions yields: $\theta R'(E_2) > R'(E_1)\left[F'(w_1) / F'(w_2)\right]$. Given concavity of *F*, the quantity in square brackets exceeds one, which in turn contradicts our original assumption that $\theta R'(E_2) - R'(E_1) < 0$. The case where $\theta R'(E_2) - R'(E_1) = 0$ is ruled out by a parallel argument. It follows that $\theta R'(E_2) - R'(E_1) > 0$ at an optimum, and maximized profits must increase with *b*.