Labor Demand Theory

Plan:

A. Demand for labor in SHORT run:
1. Production
2. Revenues
3. Costs and **k** -max
4. Shutdown decisions

B. Demand for labor in LONG run:
1. Production
2. C-min and Optimal Input mix
3. **k** -max and Optimal Output level
4. Overall Labor demand: Scale and Substitution effects

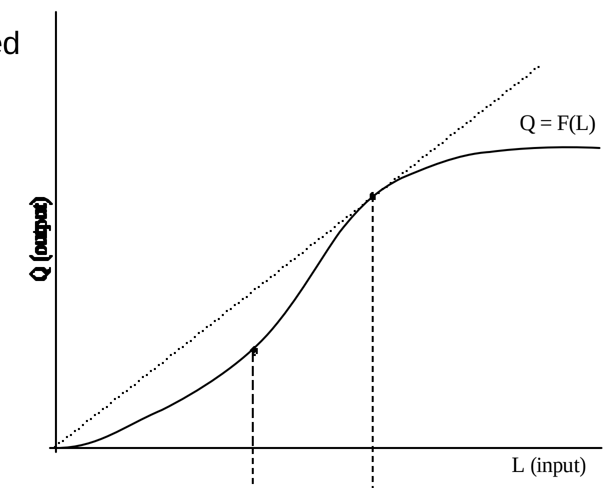
A. Short-run demand for labor

! consider:

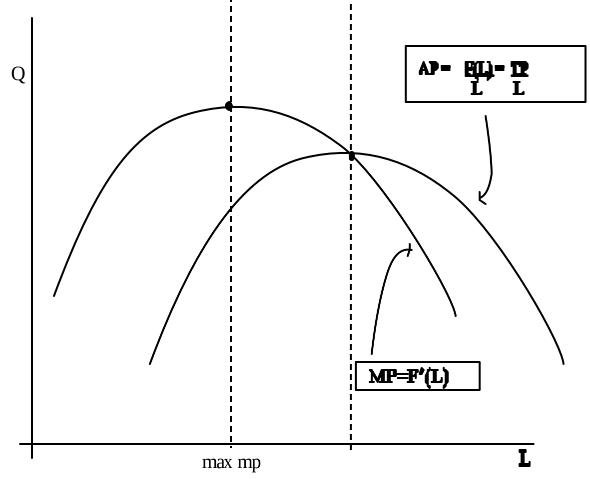
	- competitive firm	(price taking in output, input markets)
	- one output	(Q, price p)
	- one variable input	(L, price w)

1. Production

-production opps in SR summarized by TP curve (i.e. the production function):



-related curves:



2. Revenues To get an expression/graph for revenues as a function of L, just multiply the quantity of output produced (Q) by its price, p:

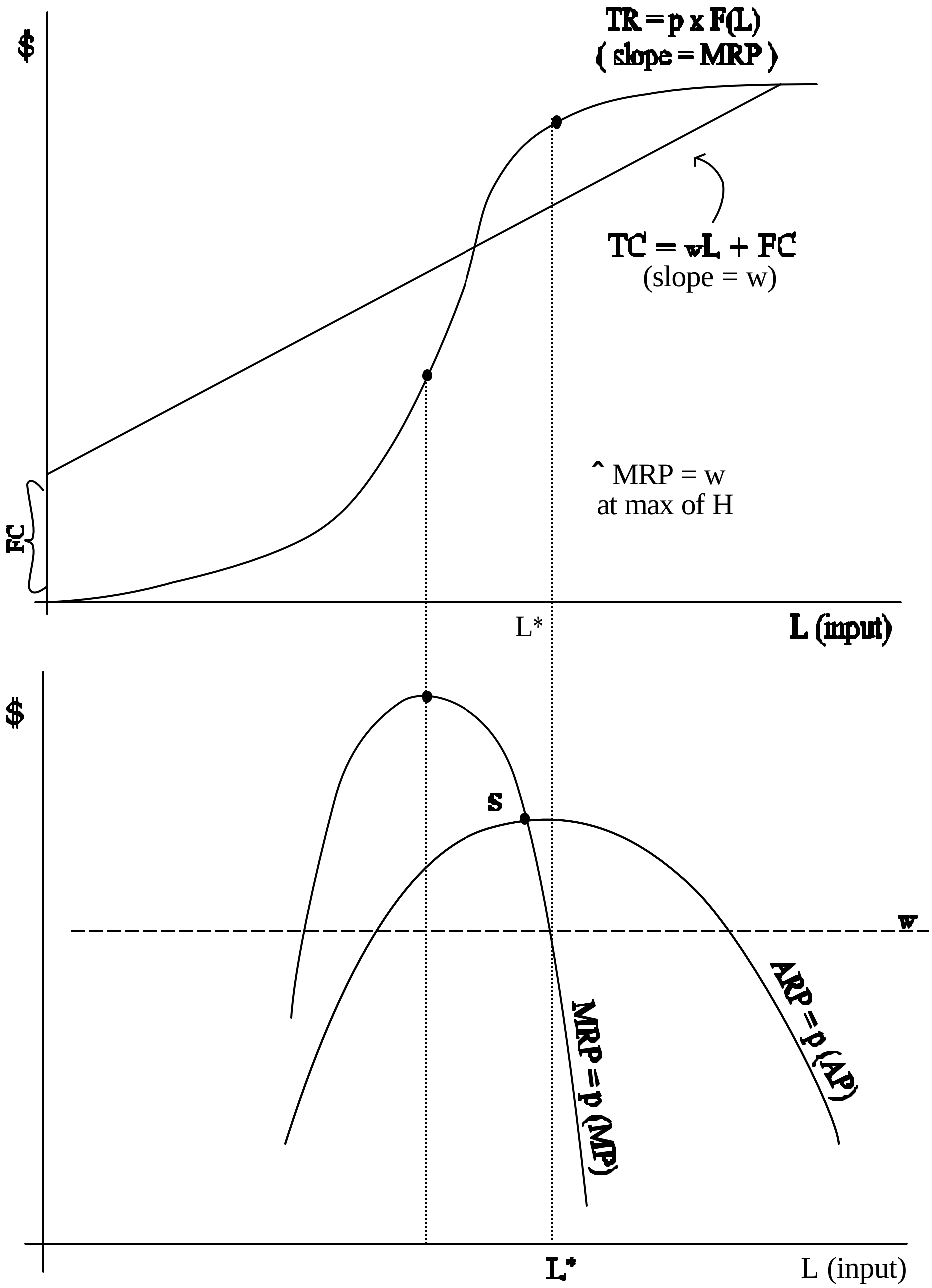
$$TR = p\,Q = p\,F(L)$$

for a price taking firm (p constant),

$$MRP = \frac{dTR}{dL} = p\,F'(L) = p\,(MP)$$

(also called VMP)

Diagrammatically,



3. Costs and profit-maximization

TC = wL + FC ! put in top diagram (slope =w)

k is max'ed at L*.

At this point, slope of TR = slope of TC

ˆ MRP = w ! **k** -max rule

This result can also be reached mathematically,

max **k** = p F(L) - wL - FC

FOC $\frac{d\,\mathbf{k}}{dL}$ = p F'(L) - w = 0

ˆ MRP = w

ˆ in bottom picture, w must be as shown

In words, “hire labor until the extra revenue produced by the last unit of labor hired just equals the wage paid for that unit of labor”.

4. Shutdown decisions

In SR, by definition we cannot escape FC even if shut down.

ˆ operate if:

Loss when operate < Loss when shut down

TC - TR < FC

wL + FC - TR < FC

wL < TR

$\frac{wL}{L} < \frac{p\,F(L)}{L}$

w < ARP

ˆ point s is shutdown ! operate iff wage less than this

So, high wages can lead to shutdown, the same way low p's can. Wages increases can induce firms to continue operating at a loss without shutting down.

! what if FC were avoided when shutdown?

operate if: TC - TR < 0

$$wL + FC - TR < 0$$

$$wL < pF(L) - FC$$

$$w < \frac{pF(L)}{L} - \frac{FC}{L}$$

$$w < ARP - AFC$$

now shutdown at a lower wage than before...

Summary of short-run labor demand:

! In SR, a firm's demand for labor is MRP below ARP
- downward-sloping (always - no ambiguity)
- adjustment to wage changes involves an output effect only, since K is fixed.

B. Long-run demand for labor

1. Production First, define a production function by F(L,K), where:

$$\frac{\partial F}{\partial K} = F_k = \text{Marg. product of } K > 0$$

$$\frac{\partial F}{\partial L} = F_L = \text{Marg. product of } L > 0$$

$$\frac{\partial}{\partial K}\left(\frac{\partial F}{\partial K}\right) = \frac{\partial^2 F}{\partial K^2} = F_{KK} = \text{effect of } \Delta K \text{ on } MP_K < 0$$

$$\frac{\partial}{\partial L}\left(\frac{\partial F}{\partial L}\right) = \frac{\partial^2 F}{\partial L^2} = F_{LL} = \text{effect of } \Delta L \text{ on } MP_L < 0$$

$$\frac{\partial}{\partial L}\left(\frac{\partial F}{\partial K}\right) = \frac{\partial^2 F}{\partial L \partial K} = F_{LK} = F_{KL} = \text{effect of } \Delta K \text{ on } MP_L \text{ and effect of } \Delta L \text{ on } MP_K$$

This can have any sign.

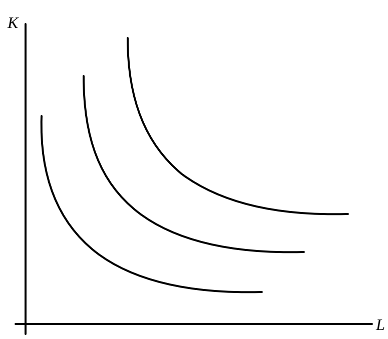
- the production function is sometimes summarized or depicted by an isoquant map:

$$\text{definition of isoquant } \overline{Q} = F(L,K)$$

$$0 = F_L + F_K\left(\frac{dK}{dL}\right)$$

$$\frac{dK}{dL} = -\frac{F_L}{F_K} = -\frac{MP_L}{MP_K}$$

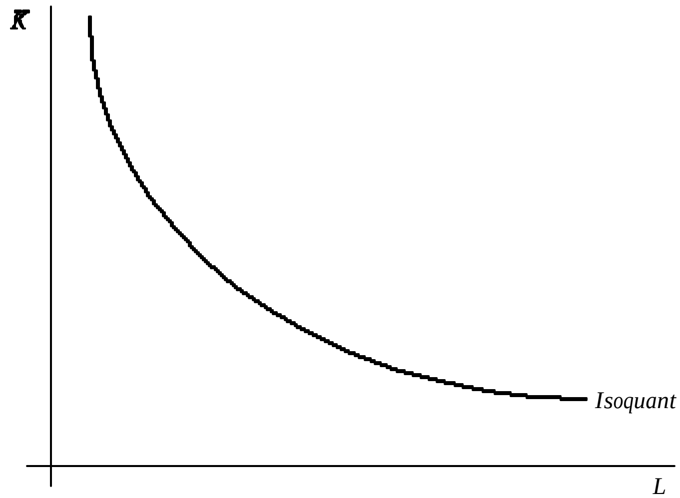
"Typical" isoquants look like this: (higher output levels are farther from the origin)



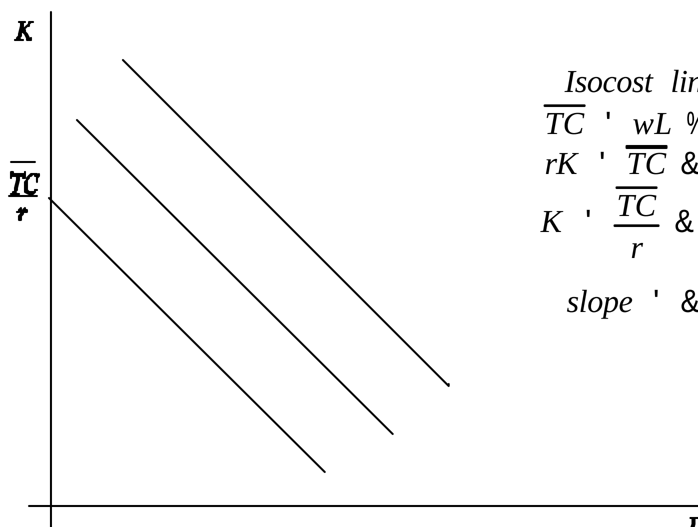
2. Optimal Input Mix

! Given Q, how does the input mix used to produce it respond to factor prices?

a) Production:



b) Costs:



Isocost lines:

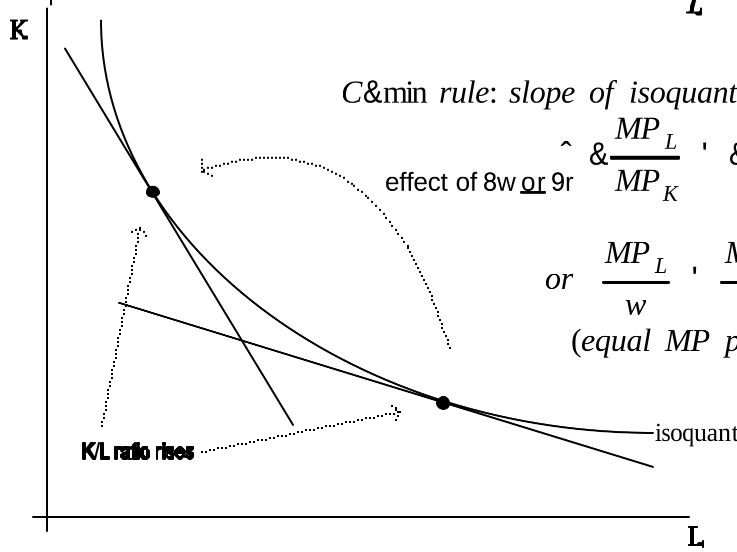
$\overline{TC}$ ' wL % rK

rK ' $\overline{TC}$ & wL

K ' $\frac{\overline{TC}}{r}$ & $\frac{w}{r}L$

slope ' & $\frac{w}{r}$

c) Cost-Min and Comp. Statics:



C&min rule: *slope of isoquant* ' *slope of isocost*

^ & $\frac{MP_L}{MP_K}$ ' & $\frac{w}{r}$

or $\frac{MP_L}{w}$ ' $\frac{MP_K}{r}$

(*equal MP per* $)

3. Profit-Maximization and the Optimal Output Level

Here, we will just state results that can be proven mathematically:

First, define a "normal" input, analogously to a normal good for consumers, as an input you use more of if you decide to produce more output, holding *prices* of *all* inputs fixed. An "inferior" input is used less as output expands, holding all input prices fixed (and assuming cost-minimization by the firm).

Intuitively, inferior inputs are inputs that are better suited to small scales of operation than to large scales (e.g. a roto-tiller versus a tractor).

Then:

! for a normal input, an increase in its price leads a profit-maximizing firm to produce *less* output.

! for an inferior input, an increase in its price leads a profit-maximizing firm to produce *more* output.

4. Overall Optimal Labor Demand and Comp. Statics

(i) Overall π-max rule:

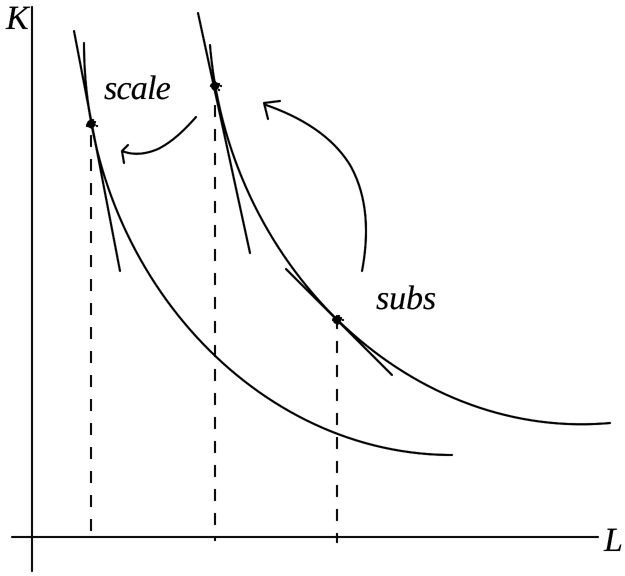
Choose L and K to:

$$\begin{aligned} Max\ \ Profits: &= p\ \ TR - TC \\ &= p\ F(L,K) - wL - rK \end{aligned}$$

$$FOC: \quad \frac{\partial Profits}{\partial L} = p\ F_L(L,\ K) - w = 0 \qquad \frac{\partial Profits}{\partial K} = p\ F_K(L,\ K) - r = 0 \qquad \Rightarrow \quad MRP_L = w, \quad MRP_K = r$$

! to get, solve above system of equations for K and L
- result: $\partial L / \partial W$ always negative
  $\partial L / \partial K$ ambiguous

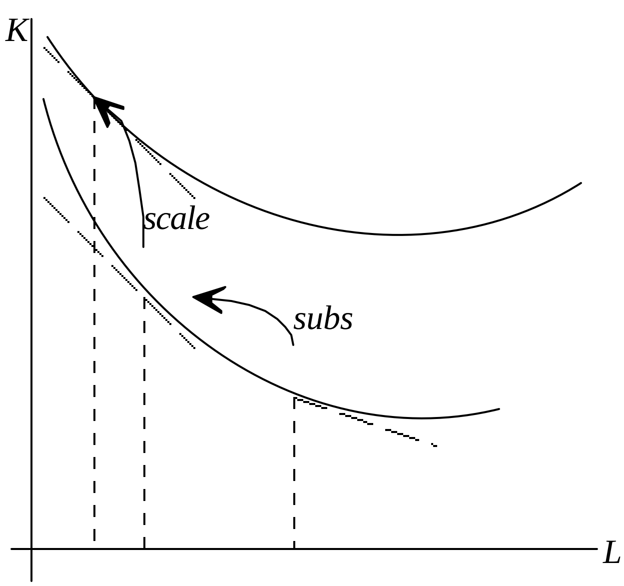
(ii) Decomposition of effects of factor p changes



! Scale and Substitution effects of a wage increase on the demand for labor (over all L^D elast.)

! if L, K are both normal inputs, then both effects are negative:

Thus, the scale and substitution effects reinforce each other.



! if L is inferior, both effects are *still* negative, because output expands when w rises:

In sum, no matter whether labor is normal *or* inferior, substitution and scale effects *always* reinforce each other, leading a wage increase to reduce labor demanded. There is no such thing as a "backward-bending labor demand curve.