THE HIRING DECISION: WHAT KIND OF WORKERS TO USE?

1. Optimal Worker mix when production is completely independent across workers.

- **a) Example**
- **b) Lessons**
- **c) Derivation using isocosts and isoquants**

a) Example: Days Inn

They can hire two kinds of workers: Youth (Y) and Seniors (S).
At the beginning they only hired youths at wage W_Y=$5.00. The question management faced was: Does it makes any sense to replace the young workers with seniors, even if the latter receive a higher wage W_S=$7.50.

	Seniors	Youth
Hourly Wage	$7.50	5.00
$ Reservations/Hour	$16,000	$10,000
Total $ Reservations/Hour	$1.6 million	$1.6 million

What we want to do is to find how many workers of each kind we need to hire in order to sell $1.6 million dollars of reservations. Note from the information in the table above that senior workers are more expensive, but also are more productive. Also note that young workers and senior workers are substitutes for each other.

- What happens if we only hire seniors?

Number of workers we should hire to achieve $1.6m in reservations:

$$\#L=(Total\ \$\ Reservations\ per\ hour/\$\ reservations\ per\ hour)$$
$$\#L_S=\$1.6m/\$16{,}000$$
$$\#L_S=100$$

Total wage bill (TW) if we only hire seniors:

$$TW=W_S*\#L_S$$
$$TW=\$7.50*100=\$750/hour$$

- What happens if we hire only youth?

$$\#L_Y=\$1.6m/\$10{,}000=160$$
$$TW=\$5.00*160=\$800/hour$$

- What happens if we hire a mix of youth and seniors ($\#L_S$=50, $\#L_Y$=80)[1]?

$$TW=\$7.50*50+\$5.00*80=775/hour$$

b) Lessons:

1) Cheap labor is not necessarily low cost.
2) The best hiring decision is not always to hire the best worker.

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[1] Note that we are in the same Isoquant curve since:

$$Total\ \$\ Reservation/hours=\$16{,}000*\#L_S+\$10{,}000*\#L_Y$$

3) When production is independent across workers of different types; the best hiring policy is an extreme policy: all of one or all of the other (i.e. a “corner solution”).
4) When production is independent across workers, you want to hire the worker with highest productivity to wages ratio (*PWR*). This is the cost minimizing decision.

$$PWR=\$\ Reservation\ per\ hour/Wage$$
$$PWR_S=\$213.33 \text{ vs. } PWR_Y=\$200$$

5) An alternative statement of (4) is to compare the wage ratio ($WR=W_S/W_Y$) with the productivity ratio ($PR=a_S/a_Y$). If $PR>WR$ hire seniors, otherwise hire youth.

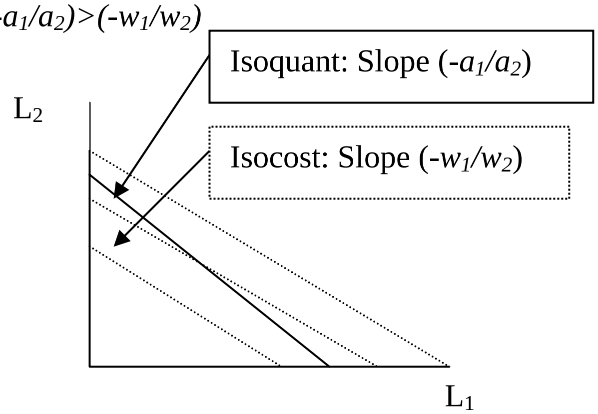
c) Derivation using isocosts and isoquants

Two types of labor: L_1 and L_2 receiving wages w_1 and w_2
Production Function: $Q=a_1L_1+a_2L_2$
Costs: $TC=w_1L_1+w_2L_2$

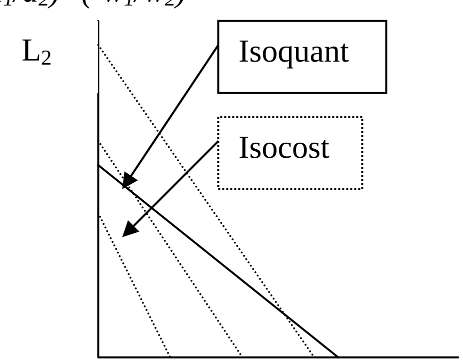
Problem *min* $w_1L_1+w_2L_2$ *subject to* $Q=a_1L_1+a_2L_2$

Recall slope of isoquant is $(-a_1/a_2)$ and the slope of the isocost is $(-w_1/w_2)$.

i) Case 1 $(-a_1/a_2)>(-w_1/w_2)$



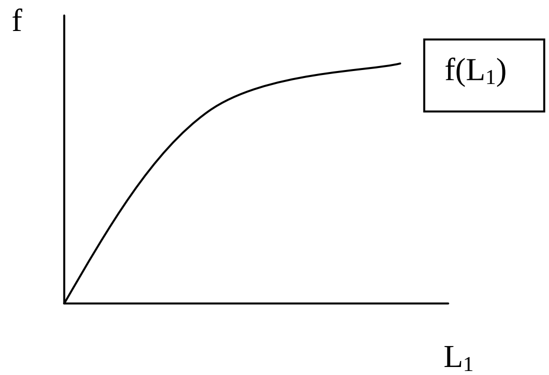
ii) Case 2 $(-a_1/a_2)<(-w_1/w_2)$



LESSON: Hire only L_1 in case 1, L_2 otherwise.

2. Optimal worker mix: Independent tasks with diminishing returns to each

Production function $Q=f_1(L_1)+f_2(L_2)$
Assume: $f_1'>0, f_1'' <0, f_2>0, f_{22}<0.$
$|Slope\ Isoquant|=|f_1'/f_2'|$

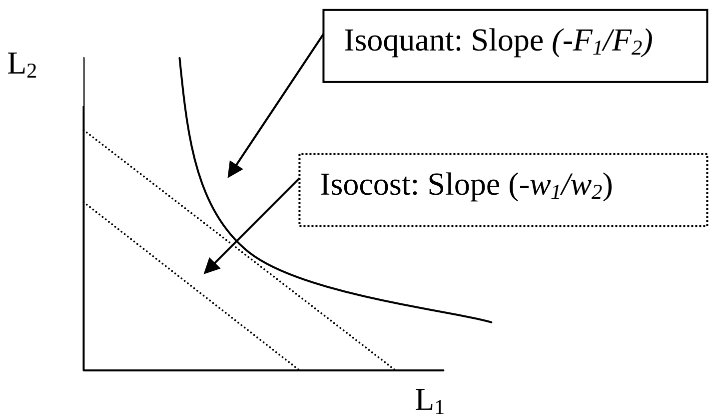


Cost minimizing input mix is at tangency point:

$$|w_1/w_2|=|f_1'/f_2'|$$

3. Optimal worker mix: Positive interactions between workers.

Production Function: $Q=F(L_1,L_2)=L_1^a L_2^b$. With this Prod. Function, |*Slope of Isoquant*| $= \left|\frac{aL_2}{bL_1}\right|$, *which is decreasing as we move from left to right (i.e. as* L_1 increases and L_2 falls).

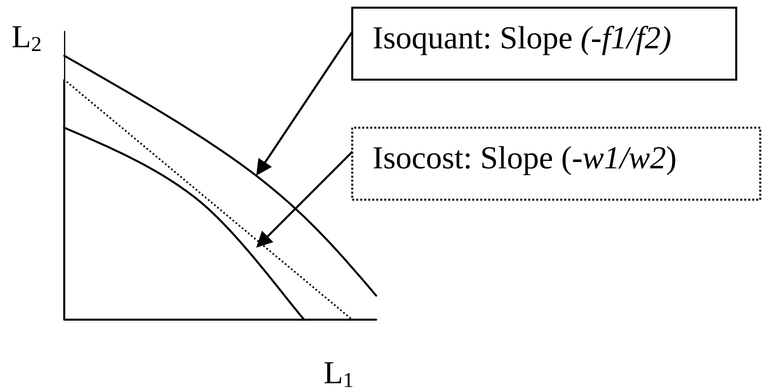


Cost minimizing mix at tangency point:

$$\frac{F_1}{F_2}=\frac{w_1}{w_2} \Rightarrow \frac{aL_2}{bL_1}=\frac{w_1}{w_2}$$

4. Optimal Worker Mix: Negative Interactions between workers.

Production Function: $Q=F(L_1,L_2)=(L_1^2+L_2^2)^{1/2}$
Now, $F_{12}<0$.
$|$*Slope of Isoquant*$|=|F_1/F_2|=|L_1/L_2|$, *which increases as we move to the right.*



Lesson: Hire only one kind of worker!

5. Hiring with Imperfect Information: Risky Workers

OVERALL QUESTION:

How does uncertainty over whether a worker's productivity (whether he/she "works out/ fits in/ has the "right stuff") affect profit-maximizing hiring decisions?

THE ANSWER: is more complicated than you might think, and depends on costs of dismissing workers who don't pan out.

Our Example:

You must choose between hiring two salespeople.

Both workers:

-will stay with you for (at most) ten years.
-must be paid $50,000 per year.

The "safe" worker:

produces *net* revenues of $100,000. per year, each year, for sure.

The "risky" worker:

-with probability 0.4, produces net sales of $400,000. every year;
-with probability 0.6 produces net sales of *minus* $100,000. every year

A worker's productivity is a permanent feature of the worker.

You don't know if the risky worker has "the right stuff" (i.e. produces 400K rather than minus 100K) till he/she has been with you for two years.

After the second year, you are free to dismiss the risky (or any other) worker if he/she hasn't worked out.

Whom should you hire?

Note: the *expected* annual output of the two workers in this example is exactly the same:

0.4($400,000) + 0.6(-100,000) = $160,000 – $60,000 = $100,000.

Plausible answers:

1. Since the two workers yield the same profits in expectation, but there is more uncertainty surrounding the risky worker, always hire the "safe" worker in this situation.

This is false because it ignores the option value of the risky worker.

2. It depends on how patient you are (or can afford to be), i.e. it depends on your discount rate. For example, the risky worker has a greater "payoff" in the future, because you can keep him/her only if he/she turns out to be "good". But if you discount future payoffs highly, it may not be worth making this risky investment that yields returns later in time.

This is false because, if the expected output of the two workers is the same (and the employer is risk neutral), the employer is *never* worse off in an expected sense with the "risky" worker. Before you know the worker's type you are, on average, just as well off; after you find out, you are better off (in expectation).

3. It depends on the probability that the worker will quit, for the same reasons as in (2) above. This is false for the same reasons that (2) is false.

The Right Answer: In the situation described (equal expected productivities, zero firing costs), you should always hire the risky worker. .

Detailed Lessons from the Spreadsheet Problem:

1. The relative value of the risky worker falls with the discount rate.

To see this: look at the numbers for the "base case" in the spreadsheet, for the **SUM of PV(Sales-Salary)**. For the "base" discount rate of 10 percent, this number is 774 for the risky worker versus 338 for the safe worker. Now raise the discount rate higher and higher, e.g to 15 or 20 percent. The risky worker's advantage shrinks.

2. However, as long as firing costs are zero and the two workers expected per-period productivities are the same, then no matter how high you raise the discount rate, it is always more profitable to hire the risky worker than the safe worker.

To see this: Experiment with higher and higher discount rates. If you pick an extreme amount of impatience for the employer, e.g. a discount rate of 5 (i.e. 500 percent per year), the totals get close. But they never reverse. Remember: in expectation, the risky worker is now worth the same as the safe worker in the early years, and strictly more in later years. Changing the discount rate therefore can't change the relative rankings of the two worker types.

3. The relative value of the risky worker falls as the expected turnover rate (of both worker types) rises.

To see this, go back to the base case and look at the columns labelled "EPV(Profits) after quits". These show profits after a quit rate of 20 percent. The relevant totals now are the **SUM of EPV(Profits) after quits**. Expected profits are now lower with both types of workers but the risky worker's advantage is lower. As you raise the turnover rate above 20 percent the risky worker's advantage shrinks.

4. However, as long as firing costs are zero and the two workers expected per-period productivities are the same, then no matter how high you raise the quit rate, it is always more profitable to hire the risky worker than the safe worker.

To see this, go back to the base case and raise the quit rate. Experiment with quit rates up to 1 (this is 100 percent of your workers leaving every year). The intuition is the same as in point 2.

5. Depending on parameter values you may may prefer to hire "risky" workers over "safe" workers with the *lower* expected productivity.

To see this keep everything the same as the base case but change the productivity of the risky worker if he/she is a "lemon" to –200. It still pays to hire the risky worker even though his/her expected sales are only 40, versus 100 for the safe worker.

6. The relative value of the risky worker falls with the level of firing costs.

Raise firing costs from the base case level of zero. The risky worker's advantage shrinks. With all parameters (including the quit rate) at their base case levels, you can eliminate the risky worker's advantage with a one-time firing cost of 270.

7. Holding expected productivity fixed, the relative value of the risky worker *increases* with how risky he/she is.

Set firing costs = 650 and p=.5, but everything else at its base-case value. Note that the expected annual sales of the risky worker is now 150, but that it is now best to hire the safe worker. Now, raise the risky worker's productivity in the "good" state from 400 to 500, and reduce it in the bad state from –100 to –200. Note that his/her expected productivity is still 150, but he/she is now even "riskier" than before. But now the risky worker is the better choice to hire! By making him/her riskier you have raised the worker's option value.

Finally, going beyond this particular example:

8. The relative value of the risky worker falls with the length of time needed to observe the worker's productivity (the probationary period).

To show this you would need to change row 3 of the spreadsheet to look like row 2: that would be the case where it takes three years to see the worker's type. The risky worker's advantage will fall.

9. The relative value of the risky worker falls if his/her ability becomes visible to other firms after it becomes known to you.

If other firms know your worker "has the right stuff" in the good state, you will have to pay him/her more than 50K per year in this eventuality. This reduces the risky worker's advantage.