**THE BASIC PRINCIPAL-AGENT PROBLEM: WHAT IS THE OPTIMAL PIECE RATE?**

Structure: 1. Principal (or principal and agent together) set the "rules of the game" or contract.
2. The agent maximizes her own utility, taking the contract/rules as given.

Solution Method: 1. First figure out what the agent is going to do under a set of rules (find the agent's "reaction function").
2. Solve for the optimal contract/rules given the agent's expected response.

Example: -one principal, one agent
-no uncertainty
-one output (*Q*), observed
-principal can't observe effort (*E*)

-$Q=E$
-$U=Y-C(E)$ where $C'>0$ and $C''<0$ (*Y*-income, *C(E)*-Cost of effort function)
-For example $C(E)=E^2/2$

-Compensation Schedule: $Y=a+bQ$
Contract is (*a*,*b*)
Agent's alternative Utility is $U^A=0.25$

First solve the **Agent's Problem** for an arbitrary contract (a,b):

Given (a,b), $\text{Max}_{E,Q,Y}\ U = Y-C(E)$,

Subject to: $Y=a+bQ$,
and to $Q=E$

Substituting the two constraints into the maximand, this is equivalent to:

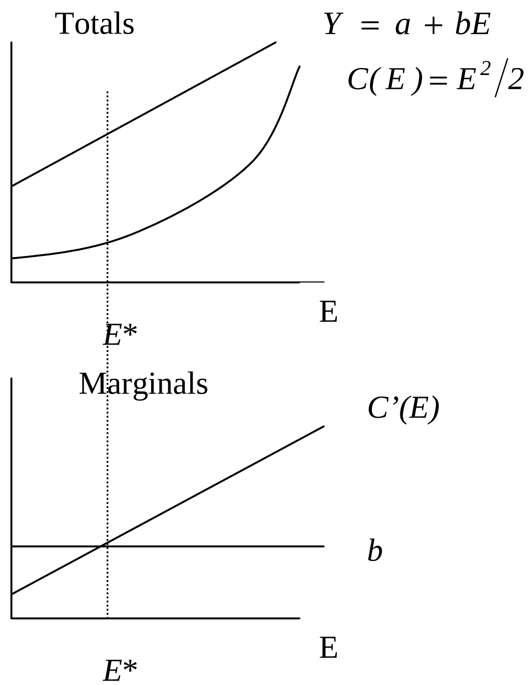
$\text{Max}_E\ U = a+bE-E^2/2$

*The first-order condition for a maximum is:*

$\frac{\partial U}{\partial E} = b - E = 0.$

*Solving this for the effort level the agent will choose yields:* $E^* = b.$

**Agent's Problem with Graphs:**




Principal-Agent Problem:

*E** maximizes the difference between *Y* and *C(E)*. At this effort level, *C'(E) = b*.

**BOTTOM LINE for the agent's problem:** Effort increases with *b*
Effort is independent of *a*

Now do the **Principal's problem**: figure out how high a commission rate (*b*) to pay the agent, and how much to pay the agent to show up (*a*).

Before doing the full-blown principal's problem (#4 below), we'll do three simpler, "warm-up" problems first, to shed light on what's going on.

Problems solved, below, in order:

1. Holding *a* fixed, which commission rate maximizes profits ($\Pi$)?
2. Holding *a* fixed, which commission rate does the worker most prefer (max *U*)?.
3. Which commission rate yields the *socially efficient* contract (*maxU+*$\Pi$)?.
4. **The principal's problem:** Holding the agent's *U* fixed at some predetermined level, choosing both *a* and *b*, and anticipating the agents response to changes in both *a* and *b*, which levels of *a* and *b* maximize $\Pi$?

1) Problem 1: Suppose $a = 0$. What level of b maximizes profits?

$\max_b \Pi = E^* - (a + bE^*)$ (from Agent's problem recall $E^* = b$)

$= b - (a + b^2)$

$= b - b^2 - a$

$\frac{\partial \pi}{\partial b} = 1 - 2b$

$b = 0.5$

*Therefore a 50% commission rate maximizes profits in this situation.*

*When this commission rate is set:*

- *the agent's effort level will be* $E^* = b = 0.5$
- *output will be (using the production function)* $Q=E=0.5$
- *profits will be* $\Pi=0.5-a-b^*E = 0.5 – 0 - 0.5^*0.5 = 0.25$

2) Problem 2: Suppose *a* is fixed (say, again, at 0 though it doesn't matter). Which commission rate does the worker prefer?

*Note that* $U = Y-C(E) = a + bQ - C(E)$

*When the agent's effort level is given by* $E^* = b$ this simplifies to:

$$U = a+b^2/2$$

*Now consider the effect of raising b on utility:*

$$\frac{\partial U}{\partial b} = b > 0.$$

The agent's utility always increases with *b*, no matter how high the current level of *b* already is. The FOC therefore has no internal solution: the worker prefers the highest *b* possible.

3) Problem 3: max the joint "surplus" of the worker and firm together:.

$$\text{Max}_{a,b}\ U+\Pi = E^* - (a+bE^*) + (a+bE^*) - C(E^*)$$
$$= E^*-C(E^*)$$
$$= b - b^2/2.$$

The FOC for a maximum is: $\frac{\partial U}{\partial b} = 1 - b = 0.$

Rearranging and solving for the optimal *b*: $b^*=1$

Therefore, for fixed *a*, the socially-efficient, Pareto-optimal, or surplus-maximizing commission rate is 100%.

4) Problem 4 (**The Principal's Problem**):

Max $\Pi$, s.t $U=U^a$ (attain the highest $\Pi$ consistent with a predetermined level of the agent's utility)

<u>STEP 1</u>: Find the level of *a* necessary to reach a given level $U^a$

Recall $U=Y - C(E) = a + bE - E^2/2$

$$= a + b^2/2$$

Hence $U^a = a + b^2/2$ and $a = U^a - b^2/2$

<u>STEP 2</u>: Max$_b$ $\Pi$ s.t. $a = U^a-b^2/2$

$$\Pi = E^* - (a - bE^*)$$
$$= b - (a+b^2) \text{ since } E^*=b$$
$$=b - b^2 - U^a + b^2/2$$
$$=b - b^2/2 - U^a$$

FOC for a maximum are: $\frac{\partial \pi}{\partial b} = 1 - b = 0$.
*Therefore b*=1*

To maximize profits, the principal should set a commission rate of 100%; i.e. increase the agent's pay by one dollar for every dollar the agent contributes to revenue.

To sum up, let's list the agent's effort, output, utility, and the firm's profits, etc. at two different commission rates (50% and 100%), with *a* set in both cases to guarantee the worker a utility level ($U^a$) of 0.25:

| ***b*** | ***E*** | ***bQ*** | ***a*** | ***U*** | ***Π*** |
|---|---|---|---|---|---|
| .5 | .5 | .25 | 0.125 | .25 | .125 |
| 1 | 1 | 1 | -0.25 | .25 | .5 |

**Lesson**: "Put rewards where the decisions are made". When the agent controls a decision that affects the utilities of a larger group (in this case herself plus the principal), it is best for all those involved to have the agent bear all the costs, as well as all the returns of her actions.

**Generality**:

The above result (that a 100% commission rate is optimal) does not depend on any of the following assumptions we made in the very simple model above:

-a linear production function
-a quadratic disutility of effort function
-linearity of the reward schedule
-no uncertainty in production (as long as agent is risk neutral).

On the other hand the result *does* change if:
- there is uncertainty in production *and* the agent is risk-averse
- the principal *also* makes some decision affecting both parties after the rules are in place (consider the case of hiring a lawyer)
-the agent produces more than one type of output, and some kinds of output are not fully observable.

We'll explore some of these extensions in future lectures. For now, let's just explore the first:

**The Principal-Agent Problem with Uncertainty and Risk Aversion**

First, let's add some uncertainty to the basic PA problem. In particular, let output not be given by $Q=dE+\varepsilon$, where $\varepsilon$ is a random variable. In this case a 100% piece rate ($b=1$) will expose the worker to high risk, which the worker will not like. If firms are risk neutral and workers risk averse, the firm may do better if, in addition to paying workers to do a job, it implicitly sells insurance to workers as well. This insurance could take the form of a lower piece rate.

<u>Case 1: Firm cannot observe state of nature</u>

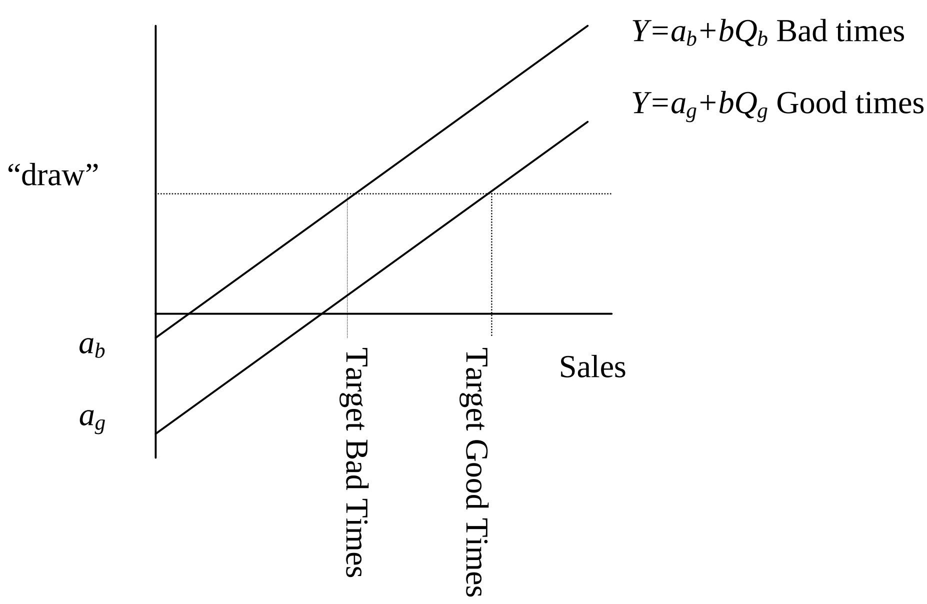
$Y = a+bQ$
$\varepsilon$ is either $k/2$ or $-k/2$
In good state: $Y^G=a+b(dE+k/2)$
In bad state: $Y^B=a+b(dE-k/2)$
(Note that the income difference is $bk$, a function of $b$)
$U = a + bdE - r(bk)^2$ ($r$- measure of worker's degree of risk aversion)

You will work through this example in this week's problem set. Here's what you'll find:

i) The optimal commission rate, $b^*$, is now less than 1.
ii) The optimal commission rate declines as $k$ and $r$ rise.
iii) For $k$ and $r$ high enough, the optimal $a^*$ becomes positive.

<u>Case 2: Firm does observe the state of nature.</u>

Now we can do better than above. The optimal commission rate is again simply 1, or 100%. The main new element is that the worker gets paid extra to show up in bad times ($a_b>a_g$). Because the firm knows when "times are bad" it simply insures workers by paying them a bigger lump sum in bad times. There is no longer a need to "blunt" incentives in order to provide insurance. In essence, the firm lowers a salesperson's sales target when it knows times are bad.



**Further Topics**:

- Safelite Glass Company
- Ratchet Effects
- Lincoln Electric
- Trivial Rewards and Intrinsic Motivation