

## Lesson 7: The Multi-Task Principal Agent Problem

We began the “incentives” section of this course by looking at the simplest incentive scheme: piece rates. Then we noted that explicit, “high powered” incentives such as these are quite uncommon in firms today. What explains this? One possibility is that most jobs are so complex that no good measures of job performance exist. Another possibility is that incentives do exist, but they are disguised in other forms: an implicit threat of dismissal, or a promotion scheme. Today we introduce a third possibility: measurement is possible, but what the principal wants the agent to do is multidimensional and not all dimensions are observed. In this case high-powered incentives on the observable activities can be dysfunctional. When the principal cares about different activities (observable and unobservable) of the agent, but compensates only based on the observable subset of these activities, the agent will exert greater effort in these activities while neglecting the unobservable ones. It might even be better to provide no incentives at all than to incentivize only one aspect of a job.

To begin with, **some examples** illustrate the point:

Ken O’Brien, a football quarterback in the mid-1980’s had a problem with throwing too many interceptions. His next contract tried to address this by penalizing him financially for every interception. The result: he stopped throwing the ball.

AT&T: at one time paid its programmers per line of code they wrote. They ended up with very inefficient programs. (incidentally the same has been said of Charles Dickens).

(source: Prendergast JEL 1999)

Dun & Bradstreet: at one point its salespeople received no commission unless a customer bought a bigger subscription than the previous year. As a result some of its salespeople deceived customers into increasing their order by misreporting their historical usage. This led to millions of dollars in lawsuits.

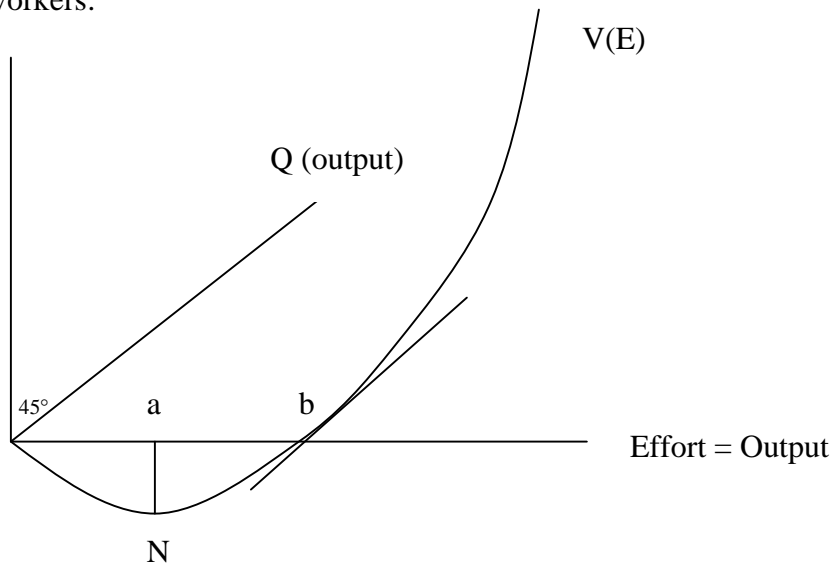
Sears auto shops, before 1992: its mechanics got a percentage of profits on repairs authorized by customers. As a result mechanics misled customers into authorizing unnecessary repairs. In 1992 the state of California threatened to shut down Sear’s auto repair business and Sears abandoned the commission scheme.

(source: Baker, Gibbons and Murphy 1994)

### One more introductory note: Working “hard” versus working “smart”.

Today’s model will depart from our assumption so far that all effort is costly and that, in the absence of financial incentives, workers will supply no effort at all. This really isn’t realistic but satisfied our purposes by illustrating a point. Also it doesn’t really affect the results: it’s only effort and incentives *on the margin* that matter, and at the margin where it’s efficient to operate, effort is costly to workers.

To see this point, go back to the optimal piece rate problem and suppose the first  $N$  units of effort on the job actually yielded **positive** utility (and therefore negative disutility) to workers:



Now, if the worker is not rewarded at all for performance, he/she will still work. Income = a horizontal line, disutility of effort is  $V(E)$ , and he/she will maximize the difference by choosing effort level  $N > 0$ .

But this is inefficient: it completely ignores the benefit of extra effort to the firm in the form of higher output: effort is productive in addition to being (up to a point) enjoyable to the worker. The efficient effort level is at point  $b$ , which, as always, requires a piece rate of 100%. So our analysis before was not affected by the assumption that every unit of effort causes disutility.

Today however we will drop this assumption explicitly and assume that even if not incentivized at all, people will still do *something* when they’re on the job. We will use this new assumption to address the idea that the **real problem** for most jobs is not getting people to work “hard” at a single task. The real problem in most jobs is getting employees to do what’s really useful to the firm as a whole from among a long list of things they might do. This is the **multitask principal-agent problem**.

## 1. A simple model: High School Teachers.

- Agent has two activities: teaching (T) and mentoring relationships with students (R).
- Of these, only teaching performance is observed, while R is unobserved.
- The agent has a total of 10 hours a day, and must divide them between the two activities. (there is no do-nothing “down time”). Thus  $10 = R + T$ .
- Let the agent’s utility be:  $U = Y - V(T) - V(R)$ , with  $V' > 0$  and  $V'' > 0$  where Y is the teacher’s income. The university solves the following, by-now familiar problem: *Given the teacher’s (agent’s) utility, the high school (principal!) wants to maximize profits.*

a) Find the agent’s indifference curves:

The indifference curve for utility level  $\bar{U}$  is defined by the equation:

$$\bar{U} = Y - T^2 - R^2 \Rightarrow Y = \bar{U} + T^2 + R^2$$

$$\text{Therefore, along an indifference curve, } Y = \bar{U} + T^2 + (10 - T)^2$$

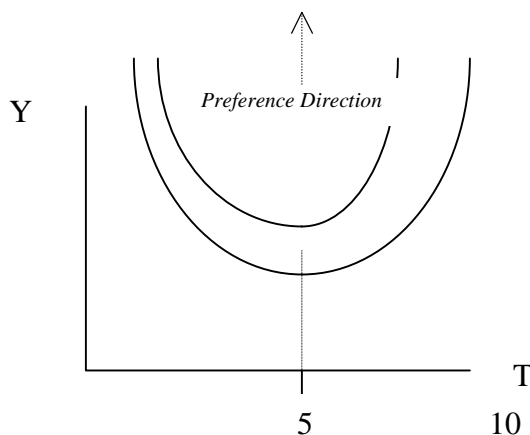
It turns out that these curves attain a minimum at  $T = R = 5$ . To see this, take the derivative wrt T and set it equal to zero:

$$\frac{\partial Y}{\partial T} = 2T - 2(10 - T) = 0$$

$$\frac{\partial Y}{\partial T} = 4T - 20 = 0$$

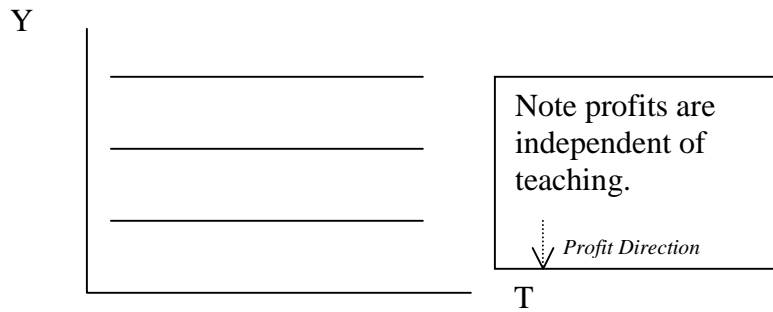
$$T^* = 5$$

Graphically, the teacher’s indifference curves therefore look like this

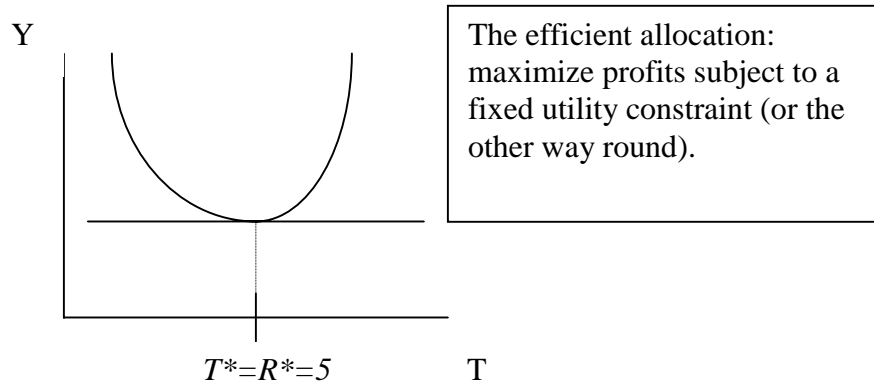


b) Now do the principal's iso-profit. Suppose output is  $Q = T + R$   
 "Profit" then is  $\pi = T + R - Y = T + (10 - T) - Y = 10 - Y$

As long as teachers therefore fill any non-teaching time with "R" time, profits in this example depend only on Y, not on T. (this is probably not realistic but we will relax this assumption below) Isoprofit curves are horizontal lines in Y,T space:

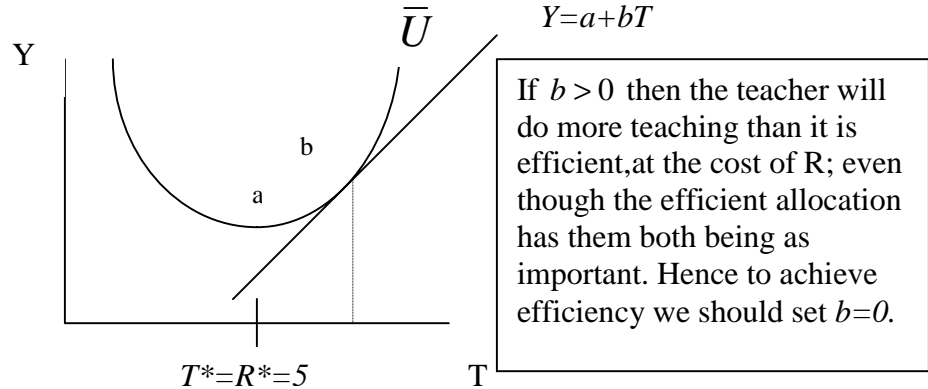


c) Now find the efficient allocation



To get agents to choose this efficient allocation, the principal needs to confront the agent with a compensation schedule that coincides with the horizontal isoprofit curve in the above diagram. In other words, the agent's compensation should be constant, independent of his/her teaching performance, i.e. there should be *no incentives at all*.

d) What happens if incentives *are* used? Suppose the principal, who can only observe T (not R), sets the following compensation scheme based on teaching only:  $Y=a+bT$  where  $b > 0$ .



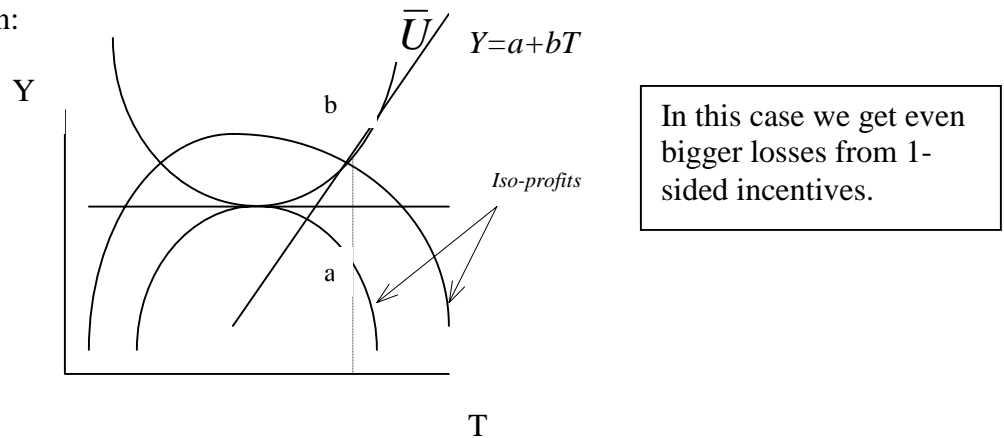
Note we could raise profits without any sacrifice in utility (i.e. make a Pareto Improvement) by moving from point *b* in the above diagram to point *a*. In doing so, we eliminate teaching incentives and lower total pay. The agent is happier doing an even mix of the two activities, output is the same, but the principal can get away with offering a lower wage.

Variations on this theme.

a) Complementarity of activities:

Suppose  $\pi = TR - Y$  where  $F(T, R) = TR$  is the production function and

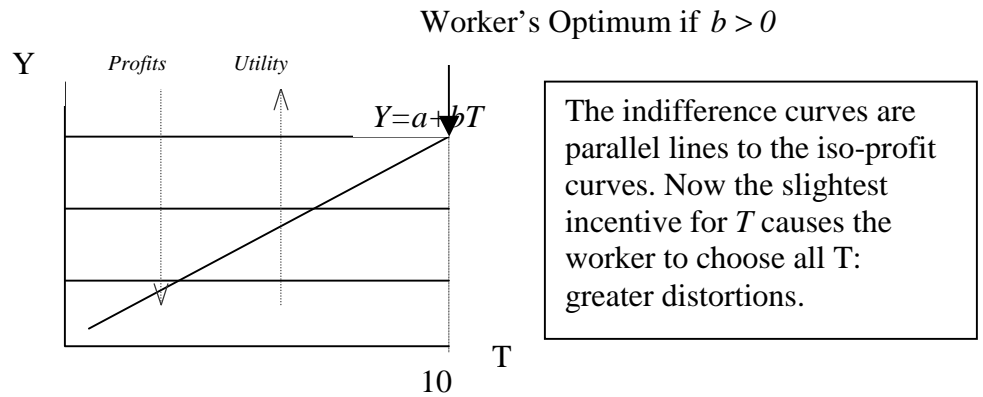
$\frac{\partial^2 F}{\partial R \partial T} > 0$ , then:



Now the firm is no longer indifferent to the teaching-relationship mix. For a given total wage bill, it is better off when there is some of both than when there is an uneven mix. The firm's profit gain from removing the teaching incentive and keeping utility the same is the vertical distance from point *b* to point *a*, which is greater than in the previous situation.

b) Substitutability of tasks.

Suppose  $U = Y - V(T + R)$ , i.e. the worker is indifferent between teaching and research.



## 2. Lessons

- When only a subset of activities is observed, providing some incentives can be worse than providing no incentives at all.
- The above is especially true if the activities are substitutable for the agent, but complementary to the principal.
- If it is impossible or very difficult to observe whether an activity is done “too much”, it may be better to totally prohibit that activity, even though a positive amount is efficient under full information.
- Discretion and high-powered incentives go together. Rules/Bureaucracy/No-Discretion go with low power incentives.
- Ideal Job Design: Group easily observable tasks into the same jobs, and pay these jobs with high-powered incentives. Group hard-to-observe activities into the same jobs and use low-powered incentives for these jobs.