Tournament Rewards and Risk Taking

Hans K. Hvide, *Norwegian School of Economics and Business*

This article considers a Lazear-Rosen tournament model where agents can influence both the spread of their output distribution (risk taking) and its mean. The unique equilibrium induces excessive risk taking and a low level of effort. By modifying the tournament to give the highest prize to the agent with the "most moderate" output, a low level of risk taking and high level of effort can be sustained as an equilibrium. The first result can be useful to understand the Relative Performance Evaluation Puzzle of executive compensation, and the second result can be useful to understand puzzling workplace norms promoting mediocrity.

I. Introduction

To the extent that real world rewards are based on measures of performance, they often depend on relative performance. For example, promotion is awarded to the most productive member of a level in an organization, the CEO of the least profitable firm in an industry gets fired, and the mutual fund with the highest return in one year gets a larger investor inflow the next year.

The main theoretical rationale for rewarding relative performance stems

Thanks to Eddie Dekel, Thore Johnsen, Rudolf Kerschbamer, Eirik G. Kristiansen, Jim March, Trond Olsen, Harald Troye, Yoram Weiss, Knut Aase, and to seminar participants at Cornell University, Econometric Society World Meeting 2000, Norwegian School of Economics and Business, Norwegian School of Management, Tel-Aviv University, Tromsø University, Catholic University of Louvain, and Wissenschaft Zenter Berlin for valuable comments and discussions. The main bulk of this article was written while the author was on a post-doc stay at the Berglas School of Economics in Tel-Aviv. I warmly thank the school for its hospitality. Thanks also to the Maritime Program at NHH for financial support.

[*Journal of Labor Economics*, 2002, vol. 20, no. 4]
© 2002 by The University of Chicago. All rights reserved.
0734-306X/2002/2004-0006$10.00

from the Informativeness Principle (Holmstrom 1979), which, informally, states that an optimal compensation contract conditions rewards on any variable that is (incrementally) informative about work intensity (effort). Recently, a corollary of the Informativeness Principle known as the Relative Performance Evaluation (RPE) Hypothesis has been extensively tested in the empirical literature on CEO compensation (see Murphy [1999] or Prendergast [1998] for overviews). The idea behind the RPE Hypothesis is that, if firms in the same industry face some common random shock, like changes in industry demand, an optimal compensation contract for a CEO makes his payment conditional on the relative performance of the firm (in addition to its absolute performance); the higher the profit of the other firms, the lower the reward of the CEO. In the empirical literature, researchers tend to be puzzled by the lack of evidence for RPE in the CEO compensation data. For example, Aggarwal and Samwick (1999*b*, p. 104) suggest that "relative performance evaluation considerations are not incorporated into executive compensation contracts." Further, Murphy (1999, p. 40) states that "the paucity of RPE in options and other components of executive compensation remains a puzzle worth understanding."

A puzzle seemingly unrelated to the RPE Puzzle is the "Mediocrity Puzzle": sometimes there are stronger incentives for a mediocre performance than for a high performance. The mechanisms underlying such nonmonotonic rewards can both be informal, through peer pressure, and formal, through explicit work contracts. For peer pressure promoting mediocrity, Levine (1992) reports several illustrating cases. For example, Frederick Taylor, the creator of Scientific Management, was, in his youth, threatened with shooting by his coworkers for being too productive and innovative. Mui (1995) reports a much-publicized case in China where a successful village entrepreneur was haunted by several misfortunes after becoming rich; the timber of his house was stolen, a pregnant cow was stabbed to death, and several animals were poisoned. There are also examples where formal rewards work in favor of mediocrity. For example, fund manager compensation schemes sometimes have an outlier effect: a very low portfolio return and a very high portfolio return both yield a lower reward from the principal than performances in the middle.[1]

The purpose of this article is to provide explanations of the RPE Puzzle and the Mediocrity Puzzle, based on extending the Lazear-Rosen (1981) tournament model to accommodate for risk taking. In a tournament, a

[1] To illustrate this point, Skandia Fund Management (SFM), which manages approximately $50 billion in the Scandinavian market, first selects an initial pool of fund managers and then gradually terminates the relationship with the managers whose return are too high or too low as compared with an index return. SFM engages in a long(er) term relationship with the remaining managers. I am grateful to the CEO of SFM, Harald Troye, for providing this information, which motivated this work.

principal sets a prize, and several agents compete to attain the highest observed output and to win the prize. This article departs from the existing tournament literature by assuming that agents can influence both the spread of distribution of output and the distribution mean. For example, CEOs can affect whether their firm has a safe or a risky research and development profile, which type of workers to employ, whether to enter emerging markets, in addition to deciding how hard to work themselves. And fund managers can choose the riskiness of their portfolio, in addition to choosing how much resources to spend on providing and analyzing relevant stock information.

Intuitively, there are two types of combinations of effort and risk taking that are consistent with equilibrium. If the equilibrium risk taking is high, then the marginal increase in the probability of winning from increasing effort is low, and equilibrium effort must be low. And conversely, if the equilibrium risk taking is low, the marginal increase in the probability of winning from increasing effort is high, and the equilibrium effort must be high. Hence, for a given prize structure, equilibrium must either have a high level of risk and a low effort level or a low level of risk and a high level of effort. However, it is not obvious which of these configurations will be consistent with equilibrium, since we treat both risk taking and effort as endogenous variables.

In proposition 1, it is shown that, with no limits to possible risk taking, agents exert zero effort and choose an infinite risk in equilibrium. Since the expected output is zero in this case, the tournament breaks down as a reward scheme. This result is somewhat modified in proposition 2, where possible risk taking is assumed to be limited, but the moral hazard problem is still serious. Propositions 1 and 2 together indicate that a reason why CEO compensation to a small extent depends on the relative performance of the firm is that a compensation scheme putting weight on the rank of the CEO in the industry may induce CEOs to choose risky projects (and consequently a low effort).

Given this negative result, we ask whether the tournament reward scheme can be modified to avoid the risky-lazy “trap” of the standard tournament. To this end, a scheme where agents are ranked according to the relative closeness of their output to a benchmark k is considered. The idea behind this scheme, labeled k-contracts, is that excessive risk taking can be avoided, which in turn can provide incentives for working hard.

The second main result, proposition 3, states that there exist intermediate values of the benchmark k such that the first-best level of effort can be implemented under risk neutrality. The empirical value of proposition 3 is that it sheds light on why sometimes higher rewards are given to agents with a modest performance than to agents with a very high performance: a norm that gives the highest informal status to agents that have a moderately high relative performance can be more efficient than a norm that gives highest informal status to agents with the highest relative status. Or, a fund management company that wants to reward their fund managers according to their relative output (to, e.g., protect the managers against common risk factors) may consider giving the highest reward to the manager with a portfolio return that comes closest to some benchmark or index return, rather than giving the highest reward to the manager with the highest portfolio return.

The article is structured as follows. In Section II, the related literature is discussed. Section III sets up the model and presents the analysis, while Section IV concludes. Some of the proofs are relegated to appendices A and B.

II. Related Literature

As outlined by Lazear (1995, 1999), tournament theory is one of the cornerstones of personnel economics. While there is a growing empirical literature on tournaments (Ehrenberg and Bognanno 1990; Brown, Harlow, and Starks 1996; Chevalier and Ellison 1997; Eriksson 1999), this literature overview focuses on the theoretical literature on tournaments before it briefly discusses some alternative explanations of the RPE Puzzle and the Mediocrity Puzzle.

Tournaments were first studied in the classic work by Lazear and Rosen (1981), who, in a model with effort as the only choice variable, showed that individualistic schemes (e.g., piece rates) and tournament schemes both can implement first-best levels of effort. Later contributions to the "effort" strand of the tournament literature include Nalebuff and Stiglitz (1983) on correlated output, Rosen (1988) on knock-out tournaments, Lazear (1989) on sabotage, Clark and Riis (1998) and Moldovanu and Sela (2001) on the case with multiple prizes, and Fullerton and McAfee (1999) on tournaments with entry fees. Tournaments with risk taking as a choice variable were first considered by Bronars (1987), who showed that, in sequential tournaments, followers have incentive to choose risky strategies, while leaders have incentive to "lock in" the gains. Other papers in the risk taking strand of the literature include Cabral (1997) on the endogenous choice of covariance, Dekel and Scotchmer (1999) on tail dominance, and Hvide and Kristiansen (forthcoming) on the selection properties of tournaments.

While the received literature on tournaments considers either effort or risk taking as choice variables for the agents, the current article considers the interaction between effort and risk taking. By including both as choice variables, we can highlight how the equilibrium choice of effort depends on the equilibrium choice of risk taking, and vice versa.

On the RPE Puzzle, Aggarwal and Samwick (1999*a*) argue that the RPE effect on compensation schemes can be neutralized by a delegation effect stemming from imperfect competition in the product markets. However, their delegation argument is ambiguous; if Cournot competition, rather than Bertrand competition, prevails in the product markets, delegation would strengthen the prediction of the RPE hypothesis rather than weaken it. We point out harmful effects of RPE in compensation schemes assuming product markets to be competitive.[2]

On the Mediocrity Puzzle, which is much less established in the literature than the RPE Puzzle, Heinkel and Stoughton (1994) derive an optimal reward scheme for fund managers where managers with a "too" high return will be replaced. However, this result refers to the solution of an adverse selection problem, while the scheme proposed in the current article solves an unrelated moral hazard problem. Gibbons (1987) argues that piece rate schemes necessarily involve nonmonotonicity if a high current performance yields a less lucrative future piece rate (the ratchet effect). However, since Gibbons (1987) considers environments where the payoff of an agent is independent of the productivity of other agents, it is mute on the link between relative performance and nonmonotonic rewards. Hence, this argument is unable to explain the Mediocrity Puzzle, where relative performance is an essential ingredient. Levine (1992), building on Jones (1984), argues that a norm to punish "ratebusters" may be efficient for the work group, from the same type of argument as in Gibbons (1987). However, since the firm realizes a low level of profit under this norm, the overall level of welfare is suboptimal, and it is not clear why a norm promoting mediocrity should survive in their setting. In contrast, my findings indicate that a norm promoting mediocrity can realize higher levels of overall welfare.

III. Analysis

Subsection IIIA sets up a standard tournament model with effort as the only choice variable. Subsection IIIB adds a notion of risk taking to that model, and Subsection IIIC introduces *k*-contracts.

A. The Tournament Model

In this subsection, we review the model of Lazear and Rosen (1981). There is one risk-neutral principal and several risk-neutral agents, for

[2] An older argument against RPE is that compensation schemes that put too much weight on relative performance are sensitive to collusion between the agents that are compared. For illustration, if the sum of compensation for two workers is constant, then both workers would be better off if they could collude in slacking their effort. However, since collusion typically requires a long-term relationship, this argument seems more applicable to explain lack of intrafirm RPE than lack of interfirm RPE. Another argument against the use of RPE is that such compensation may give agents incentive to sabotage each other's output, as in Lazear (1989). This argument also seems more fit to explain scarcity of intrafirm RPE than scarcity of interfirm RPE.

convenience assumed to be only two.[3] The value of agent i's output equals $Y_i = \mu_i + \varepsilon_i$, where μ_i is agent i's choice of effort and where ε_i is an independently and identically distributed (i.i.d.) shock with $E(\varepsilon_i) = 0$ and $E(\varepsilon_i^2) = \sigma^2$. The only contractible variables are Y_i and Y_j. The cost of effort is symmetric, $V_1(.) = V_2(.) = V(.)$, and $V(.)$ is assumed to satisfy $V(0) = V'(0) = 0$, and V', $V'' > 0$ for $\mu > 0$. The first-best level of effort for agent i, denoted μ_i^*, is the μ_i that solves $V'(\mu_i) = 1$. For clarity of exposition, we deviate from Lazear and Rosen (1981) by considering the special case when ε_i is normally distributed. A more general case is considered in appendix A.

Under a rank-order scheme, the principal fixes the prizes W_1 and W_2 (where $W_1 > W_2$), and the agents then compete in winning the first prize W_1, which is awarded to the agent with the highest Y_i. Expected utility for agent i, U_i, equals

$$U_i = P_i W_1 + (1 - P_i)W_2 - V(\mu_i) = P_i \Delta W + W_2 - V(\mu_i), \tag{1}$$

where $\Delta W = W_1 - W_2$ and $P_i = \Pr(Y_i > Y_j) = \Pr(\mu_i - \mu_j > \varepsilon_j - \varepsilon_i)$. For agent 1, we get $P_1 = \Pr(Y_1 > Y_2) = \Pr(\mu_1 - \mu_2 > \varepsilon) = G(\mu_1 - \mu_2)$, where $G(.)$ is the cumulative density function (c.d.f.) of ε [$\varepsilon \equiv \varepsilon_2 - \varepsilon_1$]. Clearly, ε is normally distributed with $E(\varepsilon) = 0$ and $E(\varepsilon^2) = 2\sigma^2$. Notice that

$$\frac{\partial P_1}{\partial \mu_1} = \frac{\partial G(\mu_1 - \mu_2)}{\partial \mu_1} = g(\mu_1 - \mu_2) = \frac{\partial P_2}{\partial \mu_2} = \frac{\partial [1 - G(\mu_1 - \mu_2)]}{\partial \mu_2}. \tag{2}$$

The first order condition for optimal provision of effort then becomes

$$\frac{\partial U_i}{\partial \mu_i} = g(\mu_1 - \mu_2)\Delta W - \frac{\partial V}{\partial \mu_i} = 0, \quad i = 1, 2. \tag{3}$$

Due to the option-like structure of the prizes, only the difference between the first and the second prize, ΔW, enters the first order conditions. Since it follows from (3) that $V'(\mu_1) = g(\mu_1 - \mu_2)\Delta W = V'(\mu_2)$, then $\mu_1 = \mu_2$ in equilibrium, and the outcome of the tournament is purely random, that is, $P_i = 1/2$, since $G(0) = 1/2$. By substituting $\mu_1 = \mu_2$ in (3), equilibrium effort, μ_i^*, can be characterized by

$$\frac{\partial V}{\partial \mu_i} = \Delta W g(0), \quad i = 1, 2. \tag{4}$$

Inserting for the normal density,

$$\frac{\partial V}{\partial \mu_i} = \frac{\Delta W}{2\sqrt{\sigma^2 \pi}}, \quad i = 1, 2. \tag{5}$$

From inspecting (5), it can easily be seen that μ_i^* is implementable with

[3] All results can easily be generalized to hold for an arbitrary number of agents.

an appropriate choice of ΔW. Since ΔW can be made independent of the total outlays, $W_1 + W_2$, the positive implementation result is consistent with any division of profits. Notice also that it follows from (5) that the equilibrium effort is decreasing in σ. Intuitively, a higher σ makes the outcome of the tournament more noisy, which decreases the marginal gain of increasing effort and hence reduces equilibrium effort.

B. Risk Taking in the Tournament Model

By an agent "increasing risk," it is meant that the agent induces a mean-preserving spread of Y_i through increasing the variance of ε_i (in practice, either through choice of projects or through manipulating the principal's measurement error of Y_i). In particular, the assumptions of the previous section are adhered to, except that the variance of the shock ε_i is now an endogenous variable. The variance of ε_i equals η_i^2, where $\eta_i^2 = \sigma^2 + s_i^2$, with $\sigma > 0$ and $s_i \in \Re_+$. We interpret σ as the level of (nondiversifiable) background noise, and s_i is the degree of voluntary spread in the output distribution. Thus, s_i is a choice variable for agent *i*, while σ is, as before, a parameter. The cost of adjusting s_i is assumed to be uniformly zero. As before, output is assumed to be the only contractible variable.

Notice that risk taking added to the agents' choice set in this manner has the convenient property that increased risk has no direct effect either on utility or on profits (expected output), and first-best levels of effort are identical to that in the previous section.[4] Although there is no direct link between risk taking and welfare of neither principal nor agents, the following result shows that the indirect effect can be dramatic.

Proposition 1. The unique equilibrium induces infinite variance and zero effort from both agents.

Proof. We first show that $X^* = \{s_i = s_j = \infty \text{ and } \mu_i = \mu_j = 0\}$ is a Nash equilibrium (NE) and then show uniqueness. Suppose that $\mu_i = 0$ and $s_i = \infty$. Then agent *j* wins with probability 1/2 irrespective of his choice of μ_j and s_j. Therefore, $\mu_j = 0$ and $s_j = \infty$ is a best reply to $\mu_i = 0$ and $s_i = \infty$, and hence X^* is a NE. To prove that X^* is a unique NE, first consider strategy tuples with (i)$\mu_i < \mu_j$. For (i) to be a Nash equilibrium, clearly $s_i = \infty$, since that choice of s_i maximizes P_i. That implies that $\mu_i = 0$. But, in that case, $\mu_j = 0$ is a best reply for agent *j*, which contradicts (i). So, in any NE, we have that $\mu_i = \mu_j$. Tuples with (ii)$\mu_i = \mu_j > 0$ are now excluded. If $\mu_i = \mu_j$, then $P_i = 1/2$. But since both players have a positive cost of effort, player *i* can gain by changing μ_i (one obvious im-

[4] We are well aware of cases where it can be difficult to separate the expectation of Y_i and its variance. For example, in the classic Capital Asset Prices Model (e.g., Mossin 1969), a portfolio with a higher systematic risk will also generate a higher expected return. To treat such a case, we would need to have a risk-averse principal, which would harden the computations significantly.

provement is to set $\mu_i = 0$ and $s_i = \infty$). But then we are in case (i). Hence, neither (i) nor (ii) is consistent with Nash behavior. Finally, (iii) $\{s_i, s_j$ finite, and $\mu_i = \mu_j = 0\}$ cannot be an NE, since both agents would have an incentive to increase effort. Hence, X^* is a unique NE. Q.E.D.

Thus, if agents can choose their level of risk taking in addition to their effort, a tournament induces extremely risky and lazy behavior from the agents. The intuition for the result is that the agents have a common incentive to increase the level of noise in the tournament to thereby lessen the importance of differences in means (effort) to the win probability. And, in turn, when effort becomes less detrimental to the win probability, the agents have less incentive to expend effort, which makes the equilibrium levels of effort more comfortable to them.

It should be emphasized that the intuition for the result is not that tournament rewards are convex in performance, which, in turn, would give incentives for an extreme degree of risk taking. The reason why this intuition is false is that whether an agent has incentives for risk taking or not depends on whether he exerts more effort than the other agents. If an agent exerts more effort than the other agent, he has an incentive to choose a low level of risk rather than a high level of risk. So this intuition does not take into account that effort is an endogenous variable.

Proposition 1 contradicts the intuition of Lazear and Rosen (1981), which states, "In this paper the worker has no choice over [the variance of individual output]. This does not affect the risk-neutral solution but does have an effect if workers are risk averse, since they tend to favor overly cautious strategies" (n. 1, p. 843).[5]

Since proposition 1 is obtained under rather special assumptions, let us discuss its robustness. First, notice that exactly the same argument and the same negative result go through if risk-averse agents play the tournament. Hence, agents choose infinite variance and zero effort in equilibrium even if they are risk averse. In fact, the only requirement for the result to go through is that $U(.)$ is monotonic. This can be seen by replacing ΔW by ΔU in equation (5), where $\Delta U = U(W_1) - U(W_2)$. Second, although normality of the shocks is convenient for illustration, weaker distributional assumptions can be made. In appendix A, we generalize proposition 1 to hold for ε_i unimodal and symmetric. Third, since lack of independence in the sense of a positively correlated shocks is one of the main justifications for applying tournaments (see, e.g., Nalebuff and Stiglitz 1983), it is worth noticing that proposition 1 holds for any degree

[5] Also Murphy (1999, p. 41) seems to be overly optimistic with respect to the optimality properties of RPE when the agent has additional choice variables to effort: "RPE remains a strong prediction of the model after expanding the managerial action set, since paying based on relative performance provides essentially the same incentives as paying based on absolute performance, while insulating risk-averse managers from common shocks."

of correlation between the shocks. Recall that, for the normal distribution, the coefficient of correlation, ρ, can be determined independently of the variances. When the variances go to infinity, P_i goes to 1/2 independently of μ_i and ρ. Hence proposition 1 is robust to introducing risk averse or risk loving preferences and to having a more general stochastic structure.

However, since the meaning of "infinite variance" is somewhat unclear, it is useful to consider the case where there are limits to risk taking. It is now assumed that $s_i \in [s^{\min}, s^{\max}]$, $\forall i$, where $0 < s^{\min} < s^{\max}$, with $s^{\max}$ finite. Hence, risk taking of the agents is bounded by a lower limit $s^{\min}$ and an upper limit $s^{\max}$. To avoid problems with nonexistence of equilibria, we consider subgame perfect equilibria of the game where the agents first choose level of risk taking and then, after observing each other's choice of risk, decide how hard to work.

Proposition 2. In the unique subgame perfect equilibrium, both agents choose $s_i = s^{\max}$ in the first stage and the corresponding low effort in the second stage.

Proof. First notice that by exactly the same argument as in Section IIIA, in the unique equilibrium at stage 2, $P_i = 1/2$, $\forall i$. This result holds independently of level of risk taken at stage 1 because the agents take (the sum) of risk as given in stage 2. Since the equilibrium effort in the subgame at stage 2 is a decreasing function in the sum of s_1 and s_2, both agents choose $s_i = s^{\max}$ at stage 1 in dominant strategies. The equilibrium risk taking at stage 1 being high, the equilibrium effort at stage 2 is consequently low. Q.E.D.

Proposition 2 shows that, even when there are limits to risk taking, the moral hazard problem induced by a tournament reward structure can be serious: equilibrium behavior by the agents is risky and lazy. As with proposition 1, proposition 2 can be generalized to a situation with risk-averse agents and a more general stochastic structure (see app. A). Furthermore, a comparative statics exercise on $s^{\max}$ yields a simple result; the equilibrium effort is monotonically decreasing in $s^{\max}$. This can be interpreted as the greater opportunity of taking risk, the less efficient is a tournament reward structure.

However, it is true that, given risk neutrality, first best can be implemented for any finite $s^{\max}$ with an appropriate choice of ΔW. Therefore, to make the efficiency argument clearer, we need to move to risk-averse agents. In the following, we compare the relative efficiency of piece rates schemes and tournament schemes under risk aversion.

Suppose that the principal sets the piece rate and that the agent then responds by choosing a level of risk and a level of effort. First, notice that the efficiency of piece rates schemes is independent of $s^{\max}$, since the agents will choose $s_i = s^{\min}$ in dominant strategies. This is a consequence of the well-known fact that a risk-averse agent prefers a lower variance to a higher variance for a given coefficient of the incentive scheme. On the other hand, in tournaments with risk-averse agents, an increase in $s^{\max}$ implies that ΔW must increase to induce the same level of effort. But then the welfare of the agents is reduced since the variability of payment increases. Let us summarize this in a remark.

REMARK 1. Under risk aversion, the relative efficiency of tournaments versus piece rates is decreasing in $s^{\max}$.

A different, natural concern is whether asymmetries between agents can reverse the risk-taking results, the intuition being that when a weak agent and a strong agent play, the stronger player will have incentive to reduce risk, to preserve the asymmetry (as in Bronars 1987). However, for the stronger player, there are two effects from decreased risk. The positive effect is that decreased risk will increase his probability of winning because differences in effort have a larger impact. The second, negative, effect is that his equilibrium effort will increase. With large asymmetries, the first effect can dominate the second effect, so that the strong player chooses a low-risk strategy in equilibrium. With smaller asymmetries, however, the probability of winning is close to 1/2 anyhow, the first effect is dominated by the second effect, and the strong agent chooses a high-risk strategy in equilibrium. The following example illustrates this point.[6]

EXAMPLE 1. Strong versus weak. Let there be two agents, with the following cost functions,

$$c_1(\mu_1) = \frac{\mu_1^2}{2},$$

$$c_2(\mu_2) = \frac{t\mu_2^2}{2}. \tag{6}$$

We assume that agent 2 has a cost advantage over agent 1, that is, $t < 1$. Assume that a two-stage game as in proposition 2 is played, and, for simplicity, let $\Delta W = 1$. The first order conditions for equilibrium at stage 2 are then

$$\mu_1 = g(\mu_1 - \mu_2),$$

$$\mu_2 = \frac{g(\mu_1 - \mu_2)}{t}, \tag{7}$$

which implies that $\mu_2 = \mu_1/t$ in equilibrium. Let $g(.)$ be a normal density function, and assume that each agent can choose either safe ($s = 0$) or risky ($s = 1$). The nondiversifiable risk is assumed to have variance equal to 1. We wish to show that, when t is low, risky-safe is the equilibrium, but if t is high (but less than 1), then the risky-risky equilibrium is preserved.

[6] The computations of the example are available from the author.

As can easily be shown, agent 1 will always choose risky. We, hence, focus on the risk taking of agent 2 as a function of t. First, consider $t = 1/2$, a large cost advantage. The equilibrium efforts when choosing high risk at stage 1 equal $\mu_1 = .13$, $\mu_2 = .26$, with corresponding utility equal to $U_2^{t=1/2}(\text{risky}) = .49$. The equilibrium efforts when choosing safe at stage 1 equal $\mu_1 = .2$, $\mu_2 = .4$, with corresponding utility equal to $U_2^{t=1/2}(\text{safe}) = .5 > U_2^{t=1/2}(\text{risky})$. Hence, when the cost advantage is large, the strong player chooses a safe strategy and risky-risky is not an equilibrium. Now consider $t = 3/4$, a small cost advantage. The equilibrium efforts when choosing risky at stage 1 equal $\mu_1 = 2$, $\mu_2 = 2.7$, with corresponding utility equal to $U_2^{t=3/4}(\text{risky}) = .494$. The equilibrium efforts when choosing safe at stage 1 equal $\mu_1 = .13$, $\mu_2 = .18$, with corresponding utility equal to $U_2^{t=3/4}(\text{safe}) = .487 < U_2^{t=3/4}(\text{risky})$. Hence, when t is closer to 1, the equilibrium is risky-risky, as in the symmetric case.

Hence, even when asymmetric agents compete, high-risk strategies are played in equilibrium, provided that the asymmetries between the agents are not too large. To conclude, we have shown that, in tournaments where risk taking is an option, the equilibrium level of risk taking will be high and the level of effort will be low, provided that asymmetries between agents are not too large. Moreover, we have shown that, under risk aversion, the relative efficiency of tournaments versus piece rates is decreasing in the level of possible risk taking.

An application of these results is that they shed light on the RPE Puzzle, that is, why Relative Performance Evaluation is used less in CEO compensation than what standard agency theory suggests. Specifically, if risk taking is a choice variable for a CEO, then the principal (e.g., the board) should be careful in conditioning rewards on the performance of other CEO's, since such schemes induce risky and lazy behavior from CEOs.

For example, a potentially important component of CEO pay is the use of Relative Performance Evaluation in annual bonus plans (Murphy 1999). Such plans could specify a bonus for the CEO if the performance of the firm (measured in, e.g., stock returns) exceeds that of the competitors. Potentially, there is a gain in such plans, since it insulates the CEO for common risk factors like market demand. For simplicity, assume that there are only two firms in the industry and that the annual bonus of the CEO can take two values, 0 if the firm has a worse performance than the competitor (e.g., with respect to return on capital) and 1 if the firm is more successful than the competitor. If the CEOs of the competing firms also have such a bonus package as an important ingredient of the compensation scheme, then the CEOs in the industry can be viewed as competing in a tournament where the winner is the CEO whose performance is the highest. We can then predict that, in equilibrium, the CEO will choose risky projects and work less ardently than if he would have chosen less risky projects, since increasing the mean profit through hard work pays less. The board can offset this effect by increasing the bonus size, but such a maneuver would add risk to the CEOs compensation and reduce his welfare under managerial risk aversion. In view of this argument, the board should be cautious with conditioning the CEO compensation on relative performance. Caution with basing pay on relative performance is exactly what the findings behind the RPE Puzzle tell us goes on in real life executive compensation.

More generally, the choice of risk taking from an agent's standpoint will be a trade-off between the reduced positive effect of increasing risk on the relative component of the compensation tournament (decreased effort) and the negative effect of increasing risk on the absolute component of the compensation package (increased variance of payment). With this trade-off in mind, a conjecture is that if the CEO is risk averse and faces a mixture of relative and absolute rewards, the optimal contract when risk taking is a choice variable relies less on relative factors than when risk taking is not a choice variable. Since little is known about optimal contracts even when the principal can condition payment on only the agent's own output, to prove this conjecture is presently out of reach.

We now turn to discussing whether the tournament reward structure can be modified to avoid the risky-lazy "trap" of standard tournaments. That will shed light on the Mediocrity Puzzle.

C. *k*-Contracts

The idea behind the contract form proposed in this subsection is that, if agents are motivated to achieve a moderately high output instead of a very high output, they can get an incentive to choose a moderate level of risk taking, which, in the next turn, can create incentives to work hard.[7] Consider a modified tournament reward structure, where the winner of the tournament is the agent with output closest to a finite benchmark k. To avoid confusion with standard tournaments, this modified tournament structure is labeled k-contracts.

The distance between k and agent i's observed output, M_i, equals

$$M_i = |Y_i - k|. \tag{8}$$

Denote by $Q_i(.)$ agent i's probability of having an observed output closer

[7] A criticism of nonmonotonic schemes is that they give incentives to dispose with parts of the output (if output falls in the nonmonotonic range). However, since disposal is equivalent to theft (from the principal's point of view), this criticism applies to all compensation schemes with marginal reward less than marginal output, which gives the agents incentives to steal the output. Contracts in practice prescribe punishment for theft (or "disposal"), if detected. Here, we simply assume that disposal is not an option for the agents.

to k than agent j and hence winning. Formally, $Q_i(.) = \Pr(M_i < M_j)$. The expected utility for a risk-neutral agent i under a k-contract then equals

$$U_i = Q_i W_1 + (1 - Q_i)W_2 - V(\mu_i) = Q_i \Delta W + W_2 - V(\mu_i). \qquad (9)$$

The following remark clarifies the relation between k-contracts and standard tournaments.

REMARK 2. If $k = \infty$, the agents play a standard tournament game. If $k < \infty$, the reward to agent i is nonmonotonic in own performance.

Proof. Recall that $P_i(.)$ is the probability of agent i winning in the standard tournament case, where $P_i = \Pr(Y_i - Y_j > 0)$. We show that $Q_i(.)$ and $P_i(.)$ converge when k goes to infinity. Since $M_i > 0$, we have that

$$Q_i(.) = \Pr(M_i < M_j) = \Pr(M_i^2 < M_j^2)$$
$$= \Pr[(Y_i - Y_j)(Y_i + Y_j - 2k) < 0]. \qquad (10)$$

When k tends to infinity, $(Y_i - Y_j)(Y_i + Y_j - 2k) < 0$ occurs i.i.f. $(Y_i - Y_j) > 0$. Hence, from (8), $Q_i(.) = \Pr[(Y_i - Y_j)(Y_i + Y_j - 2k) < 0]$ converges to $\Pr(Y_i - Y_j > 0) = P_i(.)$ when k tends to infinity. To see that k-contracts are nonmonotonic in own performance, observe that, for any Y_j, the reward to agent i is increasing up to the point $Y_i = k + |Y_j - k|$ and then decreasing. Q.E.D.

Remark 2 shows that the standard tournament reward structure is a special case of k-contracts; when k tends to infinity, a k-contract and a standard tournament are identical. However, for finite k, k-contracts differ from standard tournaments in that they give a higher reward to agents with performance in "the middle" than to agents with a top performance.

To solve for equilibrium levels of risk and effort under k-contracts, a lemma, which we believe is novel, will be very useful. First, a standard definition is given.

DEFINITION 1. Let $G_i(m;\ .)$ and $H_i(m;\ .)$ be c.d.f.'s of M_i. $G_i(m;\ .)$ first order stochastically dominates (FOSD) $H_i(m;\ .)$ if $G_i(m;\ .) \geq H_i(m;\ .)$ for all m, with $G_i(m;\ .) > H_i(m;\ .)$ for some m.

Let $F(m;\ \eta_i)$ be the c.d.f. of M_i as a function of η_i, holding μ_i and k constant at $\hat{\mu}_i$ and $\hat{k}$, respectively, where $\hat{k} > \hat{\mu}_i$. Furthermore, define $\eta_i^* = \hat{k} - \hat{\mu}_i$. Now choose two values of η_i, denoted η_i^1 and η_i^2, where $\eta_i^1 < \eta_i^2$. Then we have the following.

LEMMA 1. $F(m;\ \eta_i^1)$ FOSD $F(m;\ \eta_i^2)$, for $\eta_i^* \leq \eta_i^1 < \eta_i^2$.

Proof. See appendix B.

Lemma 1 puts an upper bound on the risk taking of agent i, in that any choice of standard deviation larger than η_i^* generates a distribution of M_i that is dominated. The intuition for lemma 1 is that η_i^* is the choice of standard deviation that maximizes the probability of hitting very close to the benchmark k. If η_i is set larger than η_i^*, then the distribution generated will perform worse with respect to the probability of hitting very close to k, and the potential gains from an increased probability of hitting only quite close to k does not offset this effect.

COROLLARY 1. Suppose that $\sigma > k$. Then $s_i = 0$ is a (strictly) dominating choice for agent i.

Proof. First, notice that regardless of η_i, it is never optimal for agent i to choose $\mu_i > k$. Now fix μ_i at $\hat{\mu}_i$ and k at $\hat{k}$, where $\hat{\mu}_i \leq \hat{k}$, and recall that $\eta_i^* = \hat{k} - \hat{\mu}_i$. By a simple transformation, it follows that a choice of s_i^2 larger than s_i^{*2} is dominated, where $s_i^{*2} = (\hat{k} - \hat{\mu}_i)^2 - \sigma^2 = (k^2 - \sigma^2) + \mu_i(\mu_i - 2k)$, which is negative for $\sigma > k$. It follows from lemma 1 that $s_i = 0$ is a dominating choice for agent i. Q.E.D.

The corollary shows that $\sigma > k$ is a sufficient condition for agents to choose $s_i = 0$ in equilibrium. Equipped with these results, we have the following.

PROPOSITION 3. For a sufficiently large σ, the first-best provision of effort is implementable with a k-contract.

Proof. See appendix B.

Hence, in contrast to the standard tournament scheme, k-contracts and individual schemes are equivalent under risk neutrality: they both implement first best.[8] The intuition behind proposition 3 is that, to avoid excessive risk taking (and hence a low level of effort), the principal rewards the agent with output closest to a positive constant k rather than rewarding the highest output. Reduced risk taking, in turn, makes it possible to give incentives for effort by increasing the prize spread, ΔW.

It is sufficient for proposition 3 that the distribution of the shocks has the FOSD property described in lemma 1. In addition to the normal, a simple distribution as the uniform also has this property. However, it is unknown whether more general distributions have the FOSD property of lemma 1, so in this section we rely more on the normality assumption than in the previous section. Second, since linear schemes can also implement first best in the case where agents choose both effort and risk, it is not obvious why k -contracts should be preferred to linear schemes. The downside with individual schemes as compared with k-contracts, however, is that they do not exploit commonality of the shocks, which may be important, for example, in the market for fund managers. Hence, k-contracts can insure risk-averse agents as well as linear schemes and provide stronger incentives. Numerically, we have obtained examples with risk-averse agents (with constant absolute risk aversion) where k-contracts dominate linear schemes when agents are not too risk averse and the shocks are sufficiently correlated. While the first condition mirrors the findings of Lazear and Rosen (1981), the second condition is intuitive

[8] Notice that the condition ρ sufficiently large is the same type of condition that Lazear and Rosen (1981) need to ensure implementation of first best.

since tournaments cancel common factors of the shocks. The more important common shocks are, the more effective tournaments become in relative terms (Nalebuff and Stiglitz 1983).

The empirical value of proposition 3 is that it gives an explanation for the Mediocrity Puzzle. If the rewards that accrue to an agent are such that moderately high relative output is more highly rewarded than a very high relative output, that gives agents incentive to choose a low level of risk and hence gives incentives to work hard.

For example, a fund management company that wants to reward their fund managers according to their relative output to, for example, insulate the managers against common risk factors, may consider to give the highest reward to the manager with an output that comes closest to some benchmark or index return.[9] In addition to insulating the managers against common risk factors, such a scheme avoids giving incentives for excessive risk taking, which a scheme rewarding the highest relative performance would. Moreover, giving the managers incentives for low levels of risk will, in the next turn, give them incentives to hard work on their portfolio, for example, in collecting and assessing financial data.

As indicated in the introduction, there are many examples where informal reward structures are nonmonotonic; agents are encouraged to do not too well as compared with a peer group. For such cases, proposition 3 can be interpreted as saying that a norm for mediocrity, or more precisely a norm for a quite high performance—but not very high—can be more beneficial to a group than a norm for excellence.[10] Informal settings are perhaps particularly relevant because here the reward system is necessarily based on relative features. For example, as emphasized by Frank (1985), social status is a positional good, in fixed supply. Hence, the status of an agent must be based on some comparison between him and other agents.

IV. Conclusion

In brief, the moral of this article is that, in tournaments where risk taking is an option, the principal gets what he does not pay for. Rewarding a high relative performance yields low levels of effort and expected output,

[9] This index return should clearly be higher than the market portfolio return; if not, hitting close to k would be a trivial problem.

[10] It is interesting to note that, in his satiric description of the Norwegian society, "The Laws of Jante," Aksel Sandemose (1933) describes a society where excellence is strongly discouraged. For example, two of the ten laws of Jante are "Thou should not believe you are better than anyone else," and "Thou should not believe you *are* something." Although the laws of Jante tend to focus on self-beliefs rather than accomplishment, it seems fair to say that they strongly discourage outlier achievements to the right. Another issue is whether Norwegian society can be said to have performed well (controlling for the stroke of luck called North Sea oil).

while rewarding a "mediocre" relative performance yield high levels of effort.

We first showed that if a high reward in a group goes to the agent with the highest output, this creates incentives for the agents in that group to take risks. Although risk taking is not necessarily harmful in itself, high risk taking is associated with low effort, which is harmful to production. Hence, if the reward to CEOs depends strongly on how well its firm performs as compared with other firms in an industry, for example, through bonus plans anchored in relative performance, the CEOs in the industry take high risks and put in low work effort in equilibrium. This argument provides an explanation for why real world corporate boards are careful with putting too much weight on relative factors in CEO compensation schemes.

Second, we show that, if the highest reward in a group goes to an agent with a moderately high output ("mediocre") instead of to the agent with the highest output, the agents in the group may be provided with an incentive to take a low level of risk and to work hard. Hence, a norm, or a formal contract, that approves very high relative performances can be self-defeating, while a norm that approves of a "mediocre" relative performance rather than a very high relative performance can yield an efficient outcome.

One extension of the current work would be to reconsider the data underlying the RPE Puzzle. The idea is that, in industries where risk taking by management to a large extent is observable (and contractible), pay should be made conditional on relative performance, as indicated by the RPE hypothesis, since risk taking can be controlled. In industries where risk taking by management to a lesser extent is observable, pay should to a lesser degree be made conditional on relative performance, or not at all, because of the risk taking problems illustrated in the current article. For example, compensation for top managers in the public sector, where the degree of observability of risk taking presumably is quite high, may conform with the RPE hypothesis, while compensation in industries where observability presumably is lower, like the hi-tech industry, may to a lesser extent conform with the RPE hypothesis.

Appendix A

Generalization of Proposition 1 and Proposition 2

Recall that the output of agent i is given by $Y_i = \mu_i + \varepsilon_i$, where ε_i has full support. In proposition 1 and proposition 2, it was shown that, if ε_i is normally distributed and agent i controls the variance of ε_i, then in equilibrium agent i chooses to let the variance of ε_i be as high as possible, since that minimizes $g(0)$ and hence equilibrium effort. In this appendix, we generalize these results to a setting where ε_i is only required to be unimodal and symmetric. In particular, we show that adding an i.i.d.

(nondegenerate) random variable to Y_i will reduce $g(0)$, and, hence, equilibrium effort. That generalizes proposition 2. Moreover, we show that, in the limit, when the agent adds infinitely many i.i.d. variables to ε_i, then $g(0)$ tends to zero. That generalizes proposition 1.

Denote the density function of ε_i by $f(x)$. For simplicity, $f(x)$ is assumed to be differentiable. By symmetry, $f(x) = f(-x)$, $\forall x$, and unimodality and symmetry imply that $f'(0) = 0$ and that $f'(-x) > 0, f'(x) < 0$ for $x > 0$. Now construct the variable $\varepsilon = \varepsilon_i + \delta_i$, where ε_i and δ_i are i.i.d. symmetric and unimodal random variables, and denote the density function of ε by $h(y)$, with corresponding c.d.f. $H(y)$. As can easily be shown, $h(y)$ is unimodal and symmetric. We now show that $f(0) > h(0)$, from which it follows that equilibrium effort decreases when an agent increases risk.

First observe that

$$H(y) = \int_{-\infty}^{\infty}\int_{-\infty}^{y-z} f(x)f(z)dxdz. \tag{A1}$$

Differentiating with respect to y we get

$$h(y) = \int_{-\infty}^{\infty} f(y-z)f(z)dz. \tag{A2}$$

Inserting for $y = 0$ into (A2) and by symmetry, we get

$$h(0) = \int_{-\infty}^{\infty} f(-z)f(z)dz = \int_{-\infty}^{\infty} f(z)^2dz. \tag{A3}$$

It now remains to show that $\int_{-\infty}^{\infty} f(z)^2dz < f(0)$. First, observe that $\int_{-\infty}^{\infty} f(z)^2dz = 2\int_0^{\infty} f(z)^2dz$ by symmetry. Integrating by parts, we have that

$$\int_0^{\infty} f(z)^2dz = -\frac{1}{2}f(0) - \int_0^{\infty} F(z)f'(z)dz. \tag{A4}$$

Using this expression, we get that

$$f(0) - h(0) = 2\left[f(0) + \int_0^{\infty} F(z)f'(z)dz\right]. \tag{A5}$$

Hence, $f(0) - h(0) > 0$ if and only if $f(0) > -\int_0^{\infty} F(z)f'(z)dz$. Substituting for $F(z) = 1 - F(-z)$ and integrating by part once more, we get that

$$\int_0^{\infty} F(z)f'(z)dz = -f(0) - \int_0^{\infty} F(-z)f'(z)dz. \tag{A6}$$

Simplifying, we get that $f(0) + \int_0^{\infty} F(z)f'(z)dz = \int_0^{\infty} F(-z)f'(z)dz < 0$, since $f'(z) < 0$ for $z > 0$. Hence, $f(0) > h(0)$, and we have shown that adding an i.i.d. random variable to ε_i reduces equilibrium effort. That generalizes proposition 2. Furthermore, it can easily be shown, and is hence skipped, that in the limit, as the number of added i.i.d. variables goes to infinity, the density at zero goes to zero. That generalizes proposition 1 and completes the proof. Q.E.D.

Appendix B

Proof of Proposition 3

We start out with a remark establishing some distributional properties of the stochastic variable M_i, the distance between agent i's output Y_i and the benchmark k. Then lemma 1 is proved, and finally proposition 3. Throughout this appendix, subscripts are skipped when possible.

REMARK 3. M has c.d.f. equal to $F(m;\ .) = (1/\sqrt{\pi})\int_{\alpha}^{\beta} e^{-t^2}dt$, where $\alpha = [\sqrt{2}(\mu - k - m)]/2\eta$ and $\beta = [\sqrt{2}(\mu - k + m)]/2\eta$.

Proof. Recall that $M = |k - Y|$, where Y is normally distributed with mean $k - \mu$ and variance η^2. Hence the c.d.f. of M equals

$$F(m;\ .) = \frac{1}{\sqrt{2\pi\eta^2}}\int_{k-m}^{k+m} e^{-(m-\mu)^2/2\eta^2}dm, \tag{B1}$$

where $m \geq 0$. This is just the probability that a single realization of normally distributed variable with expectation μ and variance η^2 falls within a distance m of a benchmark k. By standard procedures, the integral simplifies to

$$F(m;\ .) = \frac{1}{\sqrt{\pi}}\int_{\alpha}^{\beta} e^{-t^2}dt, \text{ where } \alpha = \frac{\sqrt{2}(\mu - k - m)}{2\eta}$$
$$\text{and } \beta = \frac{\sqrt{2}(\mu - k + m)}{2\eta}. \tag{B2}$$

It is easily checked that $F(m;\ .)$ indeed induces a probability distribution, that is, that

$$\lim_{m\to\infty} F(m;.) = \lim_{m\to\infty}\frac{1}{\sqrt{\pi}}\int_{\alpha}^{\beta} e^{-t^2}dt$$
$$= \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-t^2}dt = 1.$$

It can be noted that, since M_i^2 is χ^2-distributed, M_i is distributed as the square root of a χ^2 variable.

Differentiating $F(m;\ .)$ with respect to m, we obtain the density $f(m;\ .)$,

$$f(m;\ .) = \frac{\partial F(m;\ .)}{\partial m} = \frac{e^{-(\mu-k-m)^2/2\eta^2} + e^{-(\mu-k+m)^2/2\eta^2}}{\sqrt{2\pi\eta^2}}. \tag{B3}$$

Q.E.D.

Proof of lemma 1. Recall that, by definition, $\eta^2 = \sigma^2 + s^2$, and $\eta^* = \hat{k} - \hat{\mu}$. We show that any choice of η greater than η^* is dominated in the sense of FOSD. Substitute $\mu = \hat{\mu}$ and $k = \hat{k}$ into $F(m; .)$ from remark 1, substitute for η^*, and differentiate with respect to η to obtain

$$\frac{\partial F(m; .)}{\partial \eta} = \frac{1}{\sqrt{2\pi}\eta^2}\left[(\eta^* - m)e^{-(\eta^*-m)^2/2\eta^2} - (\eta^* + m)e^{-(\eta^*+m)^2/2\eta^2}\right]. \quad (B4)$$

If we can show that the expression in (B4) is negative for any $\eta > \eta^*$, lemma 1 would follow. Denote the first term of the right side of (B4) by A_1 and the second term by A_2. Moreover, substitute in $\eta^* + \gamma$ for η, where $\gamma > 0$ is a constant. Hence,

$$A_1 = (\eta^* - m)e^{-(\eta^*-m)^2/2(\eta^*+\gamma)^2},$$

and

$$A_2 = (\eta^* + m)e^{-(\eta^*+m)^2/2(\eta^*+\gamma)^2}.$$

Since $A_2 > 0$, $\partial F(m; .)/\partial\eta < 0$ is equivalent to $A_1/A_2 < 1$ for $m > 0$. We finish the proof by showing that $A_1/A_2 < 1$ for $m > 0$.

$$\frac{A_1}{A_2} = \frac{(\eta^* - m)e^{-(\eta^*-m)^2/2(\eta^*+\gamma)^2}}{(\eta^* + m)e^{-(\eta^*+m)^2/2(\eta^*+\gamma)^2}}$$

$$= \frac{(\eta^* - m)}{(\eta^* + m)}e^{(2m\eta^*)/(\eta^*+\gamma)^2} = \frac{(\eta^* - m)}{(\eta^* + m)}e^{(2m\eta^*)/(\eta^*+\gamma)^2}. \quad (B5)$$

Notice that, from (B5), it follows that $A_1/A_2 = 1$ when $m = 0$. We show that $A_1/A_2 < 1$ for any $m > 0$. Differentiating (B5) with respect to m yields

$$\frac{\partial\left(\frac{A_1}{A_2}\right)}{\partial m} = -2\frac{e^{2m/(\eta^*+\gamma)^2}\eta^*(2\eta^*\gamma + \gamma^2 + m^2)}{(\eta^* + m)^2(\eta^* + \gamma)^2}, \quad (B6)$$

which is negative for $m > 0$. Hence, $A_1/A_2 < 1$ for $m, \gamma > 0$, and, consequently, $\partial F(m; .)/\partial\eta < 0$ for any $\eta > \eta^*$ and $m > 0$, and lemma 1 follows. Q.E.D.

Proof of proposition 3. Suppose σ is larger than the first-best level of effort, μ_i^*. We show that this condition is sufficient for first best to be implementable. First, notice that, for a given k, to choose effort level μ_i larger than k is a dominated choice for agent i. Hence, we can restrict attention to $\mu_i \in [0, k]$, $i = 1, 2$. Moreover, choose k such that $\mu_i^* < k < \sigma$. Then, by corollary 1, $s_i = 0$ is a dominating strategy for agent i, and we can restrict attention to solve for equilibrium in choice of effort. The first order conditions are

$$\frac{\partial U_i}{\partial \mu_i} = \frac{\partial Q_i}{\partial \mu_i}\Delta W - \frac{\partial V_i}{\partial \mu_i} = 0, i = 1, 2. \quad (B7)$$

The probability of agent i winning under a k-scheme, $Q_i(.)$, equals

$$Q_i = \int_0^\infty F_i(m) f_j(m) dm$$

$$= \int_0^\infty \frac{\mathbf{e}^{-(\mu_j - k - m)^2/2\sigma^2} + \mathbf{e}^{-(\mu_j - k + m)^2/2\sigma^2}}{\sqrt{2\pi\sigma}} \left[\frac{1}{\sqrt{\pi}} \int_\alpha^\beta \mathbf{e}^{-t^2} dt\right] dm. \qquad \text{(B8)}$$

Define $\operatorname{erf}(x) = \int_{-\infty}^{x} \mathbf{e}^{-t^2} dt$ and $\operatorname{erfc}(x) = 1 - \operatorname{erf}(x)$. Differentiate (B8) by μ_1 and normalize by setting $\sigma = 1$ to obtain

$$\frac{\partial Q_1}{\partial \mu_{1_{\mu_1<\mu_2}}} = -\frac{1}{2\sqrt{\pi}} \left(\left\{ \left(\mathbf{e}^{\frac{1}{4}(\mu_1+\mu_2-2k)^2} \right) \left[\operatorname{erfc}\left(k - \frac{1}{2}\mu_1 - \frac{1}{2}\mu_2 \right) \right] - \mathbf{e}^{\frac{1}{4}(\mu_1-\mu_2)^2} \right.\right.$$

$$\left.\left. + \mathbf{e}^{\frac{1}{4}(\mu_1-\mu_2)^2} \left[\operatorname{erfc}\left(\frac{1}{2}\mu_1 - \frac{1}{2}\mu_2 \right) \right] \right\} \mathbf{e}^{\mu_1+\mu_2-k-\frac{1}{2}\mu_1^2-\frac{1}{2}\mu_2^2} \right), \qquad \text{(B9)}$$

while

$$\frac{\partial Q_1}{\partial \mu_{1_{\mu_1>\mu_2}}} = -\frac{1}{2\sqrt{\pi}} \left(\left\{ \left(\mathbf{e}^{\frac{1}{4}(\mu_1+\mu_2-2k)^2} \right) \left[\operatorname{erfc}\left(k - \frac{1}{2}\mu_1 - \frac{1}{2}\mu_2 \right) \right] + \mathbf{e}^{\frac{1}{4}(\mu_1-\mu_2)^2} \right.\right.$$

$$\left.\left. - \mathbf{e}^{\frac{1}{4}(\mu_1-\mu_2)^2} \left[\operatorname{erfc}\left(\frac{1}{2}\mu_1 - \frac{1}{2}\mu_2 \right) \right] \right\} \mathbf{e}^{\mu_1+\mu_2-k-\frac{1}{2}\mu_1^2-\frac{1}{2}\mu_2^2} \right). \qquad \text{(B10)}$$

Substitute for $\mu_1 = \mu_2$ to obtain the symmetric equilibrium,

$$\frac{\partial Q_i}{\partial \mu_{i_{\mu_i=\mu_j}}} = \frac{\operatorname{erf}(k - \mu_i)}{2\sqrt{\pi}}, \qquad \text{(B11)}$$

which is continuous and increasing in k. Therefore, since the cost of effort $V(.)$ is convex, the symmetric equilibrium is increasing in k. From equation (3) and equation (B11) it is evident that the symmetric equilibrium is increasing (continuously) in ΔW, where equilibrium effort equals k, in the limit as ΔW tends to infinity. Hence, for $\mu_i^* < \sigma$ and for any k such that $\mu_i^* < k < \sigma$, there exists a ΔW such that μ_i^* is implemented in Nash equilibrium. Q.E.D.

References

Aggarwal, Rajesh K., and Samwick, Andrew A. "Executive Compensation, Strategic Competition, and Relative Performance Evaluation: Theory and Evidence." *Journal of Finance* 54 (December 1999): 1999–2043. (*a*)

———. "The Other Side of the Trade-Off: The Impact of Risk on Ex-

ecutive Compensation." *Journal of Political Economy* 107 (February 1999): 65–105. (*b*)
Bronars, Stephen. "Risk Taking in Tournaments." Working paper. Austin: University of Texas, 1987.
Brown, Keith; Harlow, William; and Starks, Laura. "Of Tournaments and Temptations: An Analysis of Managerial Incentives in the Mutual Fund Industry." *Journal of Finance* 51 (March 1996): 85–110.
Cabral, Luis. "Sailing, Tennis, and R&D: The Choice of Variance and Covariance in Sequential Tournaments." Working paper. London: London Business School, 1997.
Chevalier, Judith, and Ellison, Glenn. "Risk Taking by Mutual Funds as a Response to Incentives." *Journal of Political Economy* 105 (December 1997): 1167–1200.
Clark, Derek, and Riis, Christian. "Contests with More than One Prize." *American Economic Review* 88 (March 1998): 276–89.
Dekel, Eddie, and Scotchmer, Suzanne. "On the Evolution of Preferences in Winner-Takes-All Games." *Journal of Economic Theory* 87 (July 1999): 125–43.
Ehrenberg, Ronald G., and Bognanno, Michael. "Do Tournaments Have Incentive Effects?" *Journal of Political Economy* 98 (December 1990): 1307–24.
Eriksson, Tor. "Executive Compensation and Tournament Theory: Empirical Tests on Danish Data." *Journal of Labor Economics* 17 (April 1999): 262–80.
Frank, Robert. *Choosing the Right Pond.* New York: Oxford University Press, 1985.
Fullerton, Richard L., and McAfee, Preston. "Auctioning Entry into Tournaments." *Journal of Political Economy* 107 (June 1999): 573–606.
Gibbons, Robert. "Piece-Rate Incentive Schemes." *Journal of Labor Economics* 5 (October 1987): 413–29.
Heinkel, Robert, and Stoughton, Neal. "The Dynamics of Portfolio Management Contracts." *Review of Financial Studies* 7 (Summer 1994): 351–87.
Holmstrom, Bengt. "Moral Hazard and Observability." *Bell Journal of Economics* 10 (Spring 1979): 74–91.
Hvide, Hans K., and Kristiansen, Eirik G. "Risk Taking in Selection Contests." *Games and Economic Behavior,* forthcoming.
Jones, Stephen R. G. *The Economics of Conformism.* New York: Blackwell, 1984.
Lazear, Edward. "Pay Inequality and Industrial Politics." *Journal of Political Economy* 97 (June 1989): 561–80.
———. *Personnel Economics.* Cambridge, MA: MIT Press, 1995.
———. "Personnel Economics: Past Lessons and Future Directions." *Journal of Labor Economics* 17 (April 1999): 199–236.
Lazear, Edward, and Rosen, Sherwin. "Rank-Order Tournaments as Optimum Labor Contracts." *Journal of Political Economy* 89 (October 1981): 841–64.

Levine, David I. “Piece Rates, Output Restriction, and Conformism.” *Journal of Economic Psychology* 13 (September 1992): 473–89.

Moldovanu, Benny, and Sela, Aner. “The Optimal Allocation of Prizes in Contests.” *American Economic Review* 91 (June 2001): 542–58.

Mossin, Jan. “Security Pricing and Investment Criteria in Competitive Markets.”*American Economic Review* 59 (December 1969): 749–56.

Mui, Vai-Lam. “The Economics of Envy.” *Journal of Economic Behavior and Organization* 26 (May 1995): 311–36.

Murphy, Kevin J. “Executive Compensation.” In *Handbook of Labor Economics,* vol. 3, edited by Orley Ashenfelter and David Card, pp. 2485–2563. Amsterdam: North Holland, 1999.

Nalebuff, Barry, and Stiglitz, Joseph. “Prizes and Incentives: Towards a General Theory of Compensation and Competition.” *Bell Journal of Economics* 14 (Spring 1983): 21–43.

Prendergast, Canice. “The Provision of Incentives in Firms.” *Journal of Economic Literature* 37 (March 1998): 7–63.

Rosen, Sherwin. “Prizes and Incentives in Elimination Tournaments.” *American Economic Review* 76 (September 1986): 701–15.

Sandemose, Aksel. *A Refugee Crosses His Tracks.* Oslo: Gyldendal Forlag, 1933.