Static Labor Demand Theory

1. Simplest Case: Single Competitive Firm, One Factor of Production (Labor).

Choose L to maximize:

$$\Pi = pF(L) - wL$$

FOC: $\dfrac{d\Pi}{dL} = pF'(L) - w = 0$; or, VMP=wage

SOC: $\dfrac{d^2\Pi}{dL^2} = pF''(L) < 0$, or, diminishing returns to labor

Totally differentiating the FOC to get comparative statics:

$$pF''(L)dL + F'dp - dw = 0$$

Rearranging:

(holding dw=0): $\dfrac{dL}{dp} = \dfrac{-F'}{pF''} = \dfrac{neg}{neg} > 0$ (higher output prices lead to increased labor demand).

(holding dp=0): $\dfrac{dL}{dw} = \dfrac{1}{pF''} = \dfrac{1}{neg} < 0$

Therefore, labor demand curves are unambiguously downward-sloping.

2. Single Competitive Firm, Multifactor Labor Demand

Now, choose x_1 , … x_n to maximize:

$$\Pi = pF(x_1,...,x_n) - \sum_i w_i x_i$$

According to Varian's graduate micro text, the solution to this problem can be represented by the *profit function*,

$$\Pi(p;\ w_1,...,w_n)$$

which gives the *maximized* level of profits as a function of all the exogenous parameters. Varian also shows that:

$\dfrac{\partial\Pi}{\partial p} = y$, where $y = F(x_1,...,x_n)$, i.e. output supplied

$\dfrac{\partial\Pi}{\partial w_i} = -x_i$, i.e. the demand for input *i*.

These results are sometimes known as *Hotelling's lemma*.

Finally, note that (purely because it represents the *maximized* value of a function) that the profit function must be convex in its arguments (*p* and the vector of *w*'s).

Applying Hotelling's lemma to the Hessian (i.e. the matrix of second derivatives) of the profit function yields:

$$\begin{bmatrix} \pi_{pp} & \pi_{p1} & \cdots & \pi_{pn} \\ \pi_{1p} & \pi_{11} & \cdots & \pi_{1n} \\ \cdots & \cdots & \cdots & \cdots \\ \pi_{np} & \pi_{n1} & \cdots & \pi_{nn} \end{bmatrix} = \begin{bmatrix} \frac{dy}{dp} & \frac{dy}{dw_1} & \cdots & \frac{dy}{dw_n} \\ -\frac{dx_1}{dp} & -\frac{dx_1}{dw_1} & \cdots & -\frac{dx_1}{dw_n} \\ \cdots & \cdots & \cdots & \cdots \\ -\frac{dx_n}{dp} & -\frac{dx_n}{dw_1} & \cdots & -\frac{dx_n}{dw_n} \end{bmatrix}$$

Convexity of Π implies positive definiteness of the above matrices, which in turn implies:

1. $dy/dp>0$. An upward-sloping supply curve.

2. $dx_i/dw_i<0$, for all *i*. Downward-sloping "own" demand curves for every factor *i*. In contrast to the static labor supply case, there is *no* ambiguity here.

3. "Cross-demand" effects, dx_i/dw_j , can in general be either positive or negative. These elasticities sometimes matter a lot for policy purposes; a great deal of empirical work in labor economics has been devoted to estimating them in various contexts.

4. The most surprising result derives not from convexity but from the fact that the matrix of supply and demand responses is a Hessian matrix of some function. Since it doesn't matter in what order we take partial derivatives, Hessian matrices are always *symmetric*.

Thus, for example, theory predicts that:

$\frac{dy}{dw_i} = -\frac{dx_i}{dp}$: the effect of input price *i* on the supply of output equals *minus* the effect of the output price on the demand for factor *i*. AND:

$\frac{dx_i}{dw_j} = \frac{dx_j}{dw_i}$: the effect of input price *j* on the demand for input *i* equals the effect of input price *i* on the demand for factor *j*.

Quite a few attempts have been made to test these truly remarkable predictions; none (in my opinion) all that believable.

Finally, note some (seemingly reasonable) outcomes that are NOT predicted by this model:

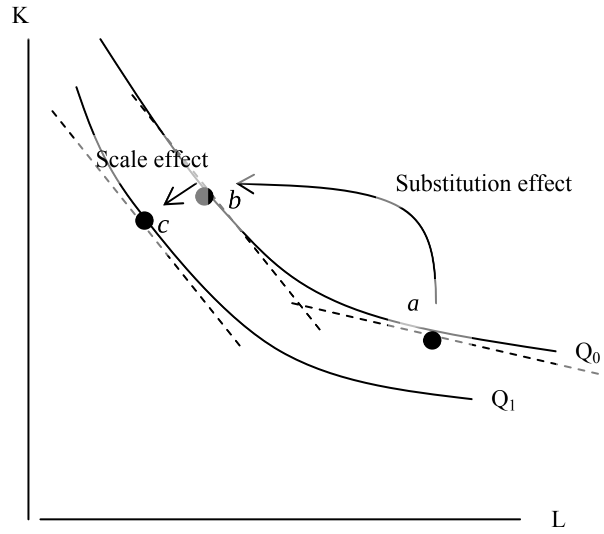
1. The above symmetry restrictions must hold when outputs and inputs are measured in levels but NOT in logs. Thus they don't imply symmetry in the *elasticities* (a fact which is important but easy to forget).

2. The theory does NOT predict that an increase in the price of any one input, *i*, will necessarily reduce the firm's output.

3. The theory does NOT predict that an increase in the price of output will raise the demand for any one input, *i*. (this actually follows from (2) by symmetry).

3. Some Intuition: Why are Own Factor Demand Effects Always Negative?

a) Case 1: "Normal" Inputs (an input is defined as normal iff a cost-minimizing firm uses more of it in response to an exogenous increase in output , assuming all factor prices are unchanged. For example, such an increase in output could be caused by a rise in the price of output, *p*).



The above diagram shows the effects of an increase in the price of labor (*w*) on a firm employing two inputs: labor (*L*) and capital (*K*). Let the isoquant labelled Q_0 represent the profit maximizing output level at the initial wage rate, w_0. The firm's optimal initial input levels must therefore given by point *a*, where an isocost curve with slope $-w_0/r$ is just tangent to the Q_0 isoquant [*r* denotes the price of capital]. Now raise the wage from w_0 to a higher level, w_1. IF the firm were to keep its output unchanged, the new optimal input levels would be at point *b*. This involves *less L* and *more K*. Analogously to consumer theory, we call this the **substitution effect** of the wage increase. BUT when the wage rises it is in general *not* optimal for the firm to keep output constant (after all, costs have increased and product demand has not[1]). To characterize the optimal change in output when *w* rises, we make use of the following result:

FACT: If an input is normal, a profit-maximizing firm will reduce its output if the price of that input rises.

Thus, the new output level must be on a lower isoquant, like Q_1. The new equlibrium after the firm has made all its adjustments must therefore be at point *c*, where the new, steeper isocosts are tangent to the new, lower isoquant. Again analogously to consumer theory, we call the shift from *b* to *c* the **scale effect** (the firm's optimal scale of operations changes when any factor price changes). By definition, point *c* must lie to the left of point *b* if labor is a normal input. Thus, when labor is a normal input, the scale and substitution effects of a wage increase reinforce each other, both tending to reduce labor demand.

The above diagram also illustrates why cross-factor-demand effects are, in general, ambiguous in sign. As shown, the wage increase tends to raise the firm's demand for capital via a substitution effect (*b* is above *a*), but to reduce the firm's demand for capital via a scale effect (*c* is below *b*). Whether the firm's use of capital rises when labor gets more expensive thus depends on which effect predominates. Interestingly, cross-effects get even a little more ambiguous when there are three (or more) inputs (say labor, capital and materials (M)). Now we can't even be sure that the pure substitution effect of a wage increase on the firm's use of capital is positive: while it is clear that –in order to keep output constant along the firm's Q_0 isoquant—the use of *at least* one other input (in the 3-input case, either K or M) must rise when L falls, we cannot be sure that *both* K and M will rise. In fact, there is some evidence that in the case of three inputs --capital, skilled labor and unskilled labor-- the first two are complements even when *Q* is held fixed.

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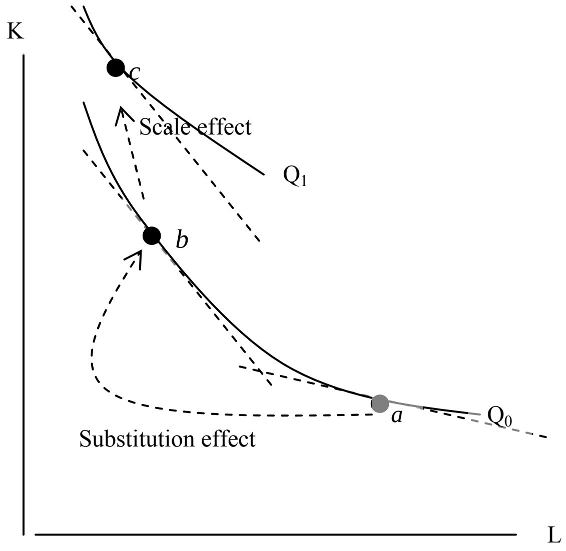
[1] Also notice that, in the one-input example analyzed earlier, output always falls when *w* rises.

a) Case 2: Labor is an "Inferior" Input (an input is defined as inferior iff a cost-minimizing firm uses *less* of it in response to an exogenous increase in output , assuming all factor prices are unchanged). Intuitively, think of inferior inputs as inputs that are best suited to small scales of production. In the case of farming, a roto-tiller might be an example: useful for small plots, but you might replace it by a tractor if your farm got bigger. Just as consumer theory allows for inferior goods, producer theory is perfectly compatible with inferior inputs.

Now, in the consumer theory case, price had unambiguous effects on demand only if the good in question was normal. In producer theory, there is no such qualification: price is always predicted to reduce (own) factor demand. To understand why things are different, we need to look at how scale and substitution effects work for inferior inputs:



The above figure considers the effects of an increase in the price of labor (*w*) on a firm employing two inputs: labor (*L*) and capital (*K*), for the case where *L* is an inferior input[2]. Let the isoquant labelled Q_0 represent the profit maximizing output level at the initial wage rate, w_0. The firm's optimal initial input levels must therefore given by point *a*, where an isocost curve with slope $-w_0/r$ is just tangent to the Q_0 isoquant. Now raise the wage from w_0 to a higher level, w_1. IF the firm were to keep its output unchanged, the new optimal input levels would be at point *b*. Thus, as before, the **substitution effect** says to use *less L* and *more K* when *w* rises. But how will the firm's output change? It turns out that:

FACT: If an input is inferior, a profit-maximizing firm will *increase* its output if the price of that input rises. (If you think about this a bit it actually makes sense: if rototillers doubled in price but the price of tractors didn't change, it might make sense to expand your operations enough to make a tractor practical).

Thus, the new output level must be on a *higher* isoquant, like Q_1. The new equlbrium after the firm has made all its adjustments must therefore be at point *c*, where the new, steeper isocosts are tangent to the new, higher isoquant. By the definition of an inferior input, point *c* must lie to the *left* of point *b*. Thus, when labor is an inferior input, the scale and substitution effects of a wage increase *also* reinforce each other, both tending to reduce labor demand.

Together, the two preceding diagrams explain the complete lack of ambiguity in the predictions of labor demand theory for the effects of (own) factor prices on factor demands: scale and substitution effects always work together, regardless of whether the factor is normal or inferior.

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[2] If a firm employs only two inputs, only one of them can be inferior (you can't increase output while reducing *both* your inputs). Thus, if *K* is inferior, *L* must be normal. We have already considered the case of normal *L* in the previous diagram.