

Midterm 1 SOLUTIONS:

1. (25 points) Consider two projects:

1st project: The initial investment is \$5,000. There is one cash flow of \$6,000 a year from now.

2nd project: The initial investment is \$10,000. The annual cash flows equal \$2,000 and are received in perpetuity. The first cash flow is received one year from now.

- a) Which project would you recommend to accept using the IRR rule?
- b) Which project would you recommend if you knew that the interest rate was 10%?

Solutions:

a) 1st project: $NPV = 0 = -\$5,000 + \$6,000/(1+IRR)$. You get: $IRR = 0.2$, or 20%.

2nd project: $NPV = 0 = -\$10,000 + \$2,000/IRR$. You get: $IRR = 0.2$, or 20%.

Both projects have IRRs of 20%, so you could pick either project.

b) NPV of 1st project is $-\$5,000 + \$6,000/(1+0.1) = \$455$.

NPV of 2nd project is $-\$10,000 + \$2,000/0.1 = \$10,000$.

Since NPV is higher for the 2nd project, you pick the second project.

2. (25 points) A project involves an initial investment of \$100,000 today. It will give a cash flow of \$30,000 at the end of every third year for the next 15 years; the first payment is 3 years from now.

What is the value of this project today, if the stated annual interest rate is 8%?

Solutions:

Since annual return is 8%, tri-annual return is $1.08^3 - 1 = 0.26$, or 26.0%. Use this in the annuity formula with $T=5$. You get:

Value of the project = $-\$100,000 + \$30,000 A_{0.26}^5 = -\$100,000 + \$79,052 = -\$20,948$.

3. (25 points) Wei has decided to buy an apartment building in Isla Vista and rent it to students. The current rental income (net of operating expenses) is \$80,000/year, and that figure is expected to increase at an annual rate of 5%.

(a) If the interest rate is 8%, how much should she expect to pay for the apartment?

(b) If Wei borrows 60% of the value of the apartment building in the form of a 10-year mortgage (with equal annual payments), what will be her cash flow (rental income net of mortgage payment) in the first year?

(c) EXTRA CREDIT. Wei's investment is 40% of the value of the apartment building. What will be the annual return on that investment in the first year?

Solutions:

a) Rental income next year will be $\$80,000(1+0.05) = \$84,000$. From the perpetuity formula, the price should be $\$84,000/ (.08 - .05) = \underline{\$2,800,000}$.

b) Wei borrows $0.6 * \$2,800,000 = \$1,680,000$.

From the annuity formula, the annual payment on the mortgage is the solution to $\$1,680,000 = C A_{0.08}^{10}$. Annual mortgage payment equals $\$250,369$. Cash flow = $\$84,000 - \$250,369 = \underline{-\$166,369}$, which is rent minus mortgage payment.

c) (EXTRA CREDIT – extra 10 points)

You need to solve for:

$$\frac{\text{The value of the investment next year} - \text{The amount paid now}}{\text{The amount paid now}}$$

The value of the investment next year = the value of the property next year – the value of the remaining mortgage payments + the cash flow in the first year.

The value of the property next year is $\$2,800,000 * (1+0.05) = \$2,940,000$.

The value of the remaining mortgage payments next year is $\$250,369 A_{0.08}^9 = \$1,564,030$.

The cash flow in the first year is $-\$166,369$ from part b.

The amount paid now is the downpayment, which equals 40% of the purchase price: $0.4 * \$2,800,000 = \$1,120,000$.

So, we get: $(2,940,000 - 1,564,030 - 166,369 - 1,120,000) / 1,120,000 = 0.08$, or 8%.

But you don't have to do any calculating on this one! You knew it had to be 8% from the present value formula!

4. (25 points) Prof. LeRoy and Ms. Chernobai each have \$1000. Prof. LeRoy deposits his money in a bank which pays 6% interest at the end of each year. Ms. Chernobai buys a 6% bond that pays coupons twice annually. She uses the coupon payment that will occur after six months to buy more of the bond (assume there's no problem with fractional bond holdings) and sells the bond portfolio at the end of the year.

Assuming that the bond is purchased at par, and that its price stays equal to par value, how much money will each have at the end of a year?

Solutions:

Prof. LeRoy will have $\$1,000(1 + 0.06) = \underline{\$1,060}$ at the end of the year.

Ms. Chernobai will use the first coupon payment of \$30 half a year from now to buy more bonds. How many more bonds will she buy? Since the bond is purchased at par, the bond price half a year from now will equal \$1,000, so she will buy $\$30/\$1,000 = 0.03$ bonds. So she will have 1.03 bonds after 6 months.

At the end of a year she will sell all her bonds (that is, 1.03 of them) and will get $1.03 * \$1,000 = \$1,030$. She will also get a \$30 coupon on each bond, so from 1.03 bonds she will get $1.03 * \$30 = \30.9 . So, at the end of a year she will get a total of $\$1,030 + \$30.9 = \underline{\$1,060.90}$.