Chapter 4: Net Present Value

4.1 a. Future Value $= C_0 (1+r)^T$
$= \$1,000 (1.05)^{10} =$ **\$1,628.89**

b. Future Value $= \$1,000 (1.07)^{10} =$ **\$1,967.15**

c. Future Value $= \$1,000 (1.05)^{20} =$ **\$2,653.30**

d. Because interest compounds on interest already earned, the interest earned in part (*c*), \$1,653.30 (=\$2,653.30 - \$1,000) is more than double the amount earned in part (*a*), \$628.89 (=\$1,628.89).

4.2 The present value, PV, of each cash flow is simply the amount of that cash flow discounted back from the date of payment to the present. For example in part (*a*), discount the cash flow in year 7 by seven periods, $(1.10)^7$.

a. $PV(C_7) = C_7 / (1+r)^7$
$= \$1,000 / (1.10)^7 =$ **\$513.16**

b. $PV(C_1) = \$2,000 / 1.10 =$ **\$1,818.18**

c. $PV(C_8) = \$500 / (1.10)^8 =$ **\$233.25**

4.3 The decision involves comparing the present value, PV, of each option. Choose the option with the highest PV. Since the first cash flow occurs 0 years in the future, or today, it does not need to be adjusted.

$PV(C_0) =$ **\$1,000**

Since the second cash flow occurs 10 years in the future, it must be discounted back 10 years at eight percent.

$PV(C_{10}) = C_{10} / (1+r)^{10}$
$= \$2,000 / (1.08)^{10}$
= **\$926.39**

Since the present value of the cash flow occurring today is higher than the present value of the cash flow occurring in year 10, you should take the \$1,000 now.

4.4 Since the bond has no interim coupon payments, its present value is simply the present value of the \$1,000 that will be received in 25 years. Note that the price of a bond is the present value of its cash flows.

$P_0 = PV(C_{25})$
$= C_{25} / (1+r)^{25}$
$= \$1,000 / (1.10)^{25}$
= **\$92.30**

The price of the bond is \$92.30.

4.5 The future value, FV, of the firm's investment must equal the $1.5 million pension liability.

FV $= C_0 (1+r)^{27}$

To solve for the initial investment, C_0, discount the future pension liability ($1,500,000) back 27 years at eight percent, $(1.08)^{27}$.

$\$1,500,000 / (1.08)^{27}$ $= C_0$
= **$187,780.23**

The firm must invest $187,708.23 today to be able to make the $1.5 million payment.

4.6 The decision involves comparing the present value, PV, of each option. Choose the option with the highest PV.

a. At a discount rate of zero, the future value and present value of a cash flow are always the same. There is no need to discount the two choices to calculate the PV.

PV(Alternative 1) = **$10,000,000**

PV(Alternative 2) = **$20,000,000**

Choose Alternative 2 since its PV, $20,000,000, is greater than that of Alternative 1, $10,000,000.

b. Discount the cash flows at 10 percent. Discount Alternative 1 back one year and Alternative 2, five years.

PV(Alternative 1) = C / (1+r)
$= \$10,000,000 / (1.10)^1$
= **$9,090,909.10**

PV(Alternative 2) $= \$20,000,000 / (1.10)^5$
= **$12,418,426.46**

Choose Alternative 2 since its PV, $12,418,426.46, is greater than that of Alternative 1, $9,090,909.10.

c. Discount the cash flows at 20 percent. Discount Alternative 1 back one year and Alternative 2, five years.

PV(Alternative 1) = C / (1+r)
$= \$10,000,000 / (1.20)^1$
= **$8,333,333.33**

PV(Alternative 2) $= \$20,000,000 / (1.20)^5$
= **$8,037,551.44**

Choose Alternative 1 since its PV, $8,333,333.33, is greater than that of Alternative 2, $8,037,551.44.

d. You are indifferent when the PVs of the two alternatives are equal.

Alternative 1, discounted at *r* = Alternative 2, discounted at *r*
$\$10,000,000 / (1+r)^1 = \$20,000,000 / (1+r)^5$

Solve for the discount rate, *r*, at which the two alternatives are equally attractive.

$[1 / (1+r)^1] (1+r)^5 = \$20,000,000 / \$10,000,000$
$(1+r)^4 = 2$
$1+r = 1.18921$
$r = 0.18921 =$ **18.921%**

The two alternatives are equally attractive when discounted at 18.921 percent.

4.7 The decision involves comparing the present value, PV, of each offer. Choose the offer with the highest PV.

Since the Smiths' payment occurs immediately, its present value does not need to be adjusted.

PV(Smith) = **$115,000**

The Joneses' offer occurs three years from today. Therefore, the payment must be discounted back three periods at 10 percent.

$$\begin{aligned} PV(Jones) &= C_3 / (1+r)^3 \\ &= \$150,000 / (1.10)^3 \\ &= \mathbf{\$112,697.22} \end{aligned}$$

Since the PV of the Joneses' offer, $112,697.22, is less than the Smiths' offer, $115,000, you should choose the Smiths' offer.

4.8 a. Since the bond has no interim coupon payments, its present value is simply the present value of the $1,000 that will be received in 20 years. Note that the price of the bond is this present value.

$$\begin{aligned} P_0 &= PV(C_{20}) \\ &= C_{20} / (1+r)^{20} \\ &= \$1,000 / (1.08)^{20} \\ &= \mathbf{\$214.55} \end{aligned}$$

The current price of the bond is $214.55.

b. To find the bond's price 10 years from today, find the future value of the current price.

$$\begin{aligned} P_{10} &= FV_{10} \\ &= C_0 (1+r)^{10} \\ &= \$214.55 (1.08)^{10} \\ &= \mathbf{\$463.20} \end{aligned}$$

The bond's price 10 years from today will be $463.20.

c. To find the bond's price 15 years from today, find the future value of the current price.

$$\begin{aligned} P_{15} &= FV_{15} \\ &= C_0 (1+r)^{15} \\ &= \$214.55 (1.08)^{15} \\ &= \mathbf{\$680.59} \end{aligned}$$

The bond's price 15 years from today will be $680.59.

4.9 Ann Woodhouse would be willing to pay the present value of its resale value.

$$PV = \$5{,}000{,}000 / (1.12)^{10}$$
$$= \mathbf{\$1{,}609{,}866.18}$$

The most she would be willing to pay for the property is $1,609,866.18.

4.10 a. Compare the cost of the investment to the present value of the cash inflows. You should make the investment only if the present value of the cash inflows is greater than the cost of the investment. Since the investment occurs today (year 0), it does not need to be discounted.

PV(Investment) = **$900,000**

$$PV(\text{Cash Inflows}) = \$120{,}000 / (1.12) + \$250{,}000 / (1.12)^2 + \$800{,}000 / (1.12)^3$$
$$= \mathbf{\$875{,}865.52}$$

Since the PV of the cash inflows, $875,865.52, is less than the cost of the investment, $900,000, you should not make the investment.

b. The net present value, NPV, is the present value of the cash inflows minus the cost of the investment.

NPV = PV(Cash Inflows) – Cost of Investment
= $875,865.52 – $900,000
= **-$24,134.48**

The NPV is -$24,134.48.

c. Calculate the PV of the cash inflows, discounted at 11 percent, minus the cost of the investment. If the NPV is positive, you should invest. If the NPV is negative, you should not invest.

NPV = PV(Cash Inflows) – Cost of Investment

$$= \$120{,}000 / (1.11) + \$250{,}000 / (1.11)^2 + \$800{,}000 / (1.11)^3 – \$900{,}000$$
$$= \mathbf{-\$4{,}033.18}$$

Since the NPV is still negative, -$4,033.18, you should not make the investment.

4.11 Calculate the NPV of the machine. Purchase the machine if it has a positive NPV. Do not purchase the machine if it has a negative NPV.

Since the initial investment occurs today (year 0), it does not need to be discounted.

PV(Investment) = -$340,000

Discount the annual revenues at 10 percent.

$$PV(\text{Revenues}) = \$100{,}000 / (1.10) + \$100{,}000 / (1.10)^2 + \$100{,}000 / (1.10)^3 + \$100{,}000 / (1.10)^4 + \$100{,}000 / (1.10)^5$$
$$= \$379{,}078.68$$

Since the maintenance costs occur at the beginning of each year, the first payment is not discounted. Each year thereafter, the maintenance cost is discounted at an annual rate of 10 percent.

PV(Maintenance) = -\$10,000 - \$10,000 / (1.10) - \$10,000 / $(1.10)^2$ - \$10,000 / $(1.10)^3$ –
\$10,000 / $(1.10)^4$
= -\$41,698.65

NPV = PV(Investment) + PV(Cash Flows) + PV(Maintenance)
= -\$340,000 + \$379,078.68 - \$41,698.65
= **-\$2,619.97**

Since the NPV is negative, -\$2,619.97, you should not buy the machine.

To find the NPV of the machine when the relevant discount rate is nine percent, repeat the above calculations, with a discount rate of nine percent.

PV(Investment) = -\$340,000

Discount the annual revenues at nine percent.

PV(Revenues) = \$100,000 / (1.09) + \$100,000 / $(1.09)^2$ + \$100,000 / $(1.09)^3$ +
\$100,000 / $(1.09)^4$ + \$100,000 / $(1.09)^5$
= \$388,965.13

Since the maintenance costs occur at the beginning of each year, the first payment is not discounted. Each year thereafter, the maintenance cost is discounted at an annual rate of nine percent.

PV(Maintenance) = -\$10,000 - \$10,000 / (1.09) - \$10,000 / $(1.09)^2$ - \$10,000 / $(1.09)^3$ –
\$10,000 / $(1.09)^4$
= -\$42,397.20

NPV = PV(Investment) + PV(Cash Flows) + PV(Maintenance)
= -\$340,000 + \$388,965.13 - \$42,397.20
= **\$6,567.93**

Since the NPV is positive, \$6,567.93, you should buy the machine.

4.12 a. The NPV of the contract is the PV of the item's revenue minus its cost.

PV(Revenue) = $C_5 / (1+r)^5$
= \$90,000 / $(1.10)^5$
= \$55,882.92

NPV = PV(Revenue) – Cost
= \$55,882.92 - \$60,000
= **-\$4,117.08**

The NPV of the item is -\$4,117.08.

b. The firm will break even when the item's NPV is equal to zero.

NPV = PV(Revenues) – Cost
= $C_5 / (1+r)^5$ – Cost
\$0 = \$90,000 / $(1+r)^5$ - \$60,000
r = 0.08447 = **8.447%**

The firm will break even on the item with an 8.447 percent discount rate.

4.13 Compare the PV of your aunt's offer with your roommate's offer. Choose the offer with the highest PV. The PV of your aunt's offer is the sum of her payment to you and the benefit from owning the car an additional year.

PV(Aunt) = PV(Trade-In) + PV(Benefit of Ownership)
= \$3,000 / (1.12) + \$1,000 / (1.12)
= **\$3,571.43**

Since your roommate's offer occurs today (year 0), it does not need to be discounted.

PV(Roommate) = **\$3,500**

Since the PV of your aunt's offer, \$3,571.43, is higher than your roommate's offer, \$3,500, you should accept your aunt's offer.

4.14 The cost of the car 12 years from today will be \$80,000. To find the rate of interest such that your \$10,000 investment will pay for the car, set the FV of your investment equal to \$80,000.

$$FV = C_0 (1+r)^{12}$$
$$\$80{,}000 = \$10{,}000 (1+r)^{12}$$

Solve for the interest rate, *r*.

$$8 = (1+r)^{12}$$
$$\mathbf{0.18921} = r$$

The interest rate required is 18.921%.

4.15 The deposit at the end of the first year will earn interest for six years, from the end of year 1 to the end of year 7.

$$FV = \$1{,}000 (1.12)^6 = \$1{,}973.82$$

The deposit at the end of the second year will earn interest for five years.

$$FV = \$1{,}000 (1.12)^5 = \$1{,}762.34$$

The deposit at the end of the third year will earn interest for four years.

$$FV = \$1{,}000 (1.12)^4 = \$1{,}573.52$$

The deposit at the end of the fourth year will earn interest for three years.

$$FV = \$1{,}000 (1.12)^3 = \$1{,}404.93$$

Combine the values found above to calculate the total value of the account at the end of the seventh year:

FV = \$1,973.82 + \$1,762.34 + \$1,573.52 + \$1,404.93
= **\$6,714.61**

The value of the account at the end of seven years will be \$6,714.61.

4.16 To find the future value of the investment, convert the stated annual interest rate of eight percent to the effective annual yield, EAY. The EAY is the appropriate discount rate because it captures the effect of compounding periods.

a. With annual compounding, the EAY is equal to the stated annual interest rate.

$$FV = C_0 (1+ EAY)^T$$
$$= \$1,000 (1.08)^3$$
$$= \$1,259.71$$

The future value is \$1,259.71.

b. Calculate the effective annual yield (EAY), where *m* denotes the number of compounding periods per year.

$$EAY = [1 + (r/m)]^m – 1$$
$$= [1 + (0.08 / 2)]^2 – 1$$
$$= 0.0816$$

Apply the future value formula, using the EAY for the interest rate.

$$FV = C_0 [1+EAY]^3$$
$$= \$1,000 (1 + 0.0816)^3$$
$$= \mathbf{\$1,265.32}$$

The future value is \$1,265.32.

c. Calculate the effective annual yield (EAY), where *m* denotes the number of compounding periods per year.

$$EAY = [1 + (r/m)]^m – 1$$
$$= [1 + (0.08 / 12)]^{12} – 1$$
$$= 0.083$$

Apply the future value formula, using the EAY for the interest rate.

$$FV = C_0 (1+ EAY)^3$$
$$= \$1,000 (1 + 0.083)^3$$
$$= \mathbf{\$1,270.24}$$

The future value is \$1,270.24.

d. Continuous compounding is the limiting case of compounding. The EAY is calculated as a function of the constant, *e*, which is approximately equal to 2.718.

$$\begin{aligned} FV &= C_0 \times e^{rT} \\ &= \$1{,}000 \times e^{0.08\times3} \\ &= \mathbf{\$1{,}271.25} \end{aligned}$$

The future value is $1,271.25.

e. The future value of an investment increases as the compounding period shortens because interest is earned on previously accrued interest payments. The shorter the compounding period, the more frequently interest is paid, resulting in a larger future value.

4.17 Continuous compounding is the limiting case of compounding. The future value is a function of the constant, *e*, which is approximately equal to 2.718.

a. $$\begin{aligned} FV &= C_0 \times e^{rT} \\ &= \$1{,}000 \times e^{0.12\times5} \\ &= \mathbf{\$1{,}822.12} \end{aligned}$$

The future value is $1,822.12.

b. $$\begin{aligned} FV &= \$1{,}000 \times e^{0.10\times3} \\ &= \mathbf{\$1{,}349.86} \end{aligned}$$

The future value is $1,349.86.

c. $$\begin{aligned} FV &= \$1{,}000 \times e^{0.05\times10} \\ &= \mathbf{\$1{,}648.72} \end{aligned}$$

The future value is $1,648.72.

d. $$\begin{aligned} FV &= \$1{,}000 \times e^{0.07\times8} \\ &= \mathbf{\$1{,}750.67} \end{aligned}$$

The future value is $1,750.67.

4.18 Convert the stated annual interest rate to the effective annual yield, EAY. The EAY is the appropriate discount rate because it captures the effect of compounding periods. Next, discount the cash flow at the EAY.

$$\begin{aligned} EAY &= [1+(r / m)]^m - 1 \\ &= [1+(0.10 / 4)]^4 - 1 \\ &= 0.10381 \end{aligned}$$

Discount the cash flow back 12 periods.

$$\begin{aligned} PV(C_{12}) &= C_{12} / (1+EAY)^{12} \\ &= \$5{,}000 / (1.10381)^{12} \\ &= \$1{,}528.36 \end{aligned}$$

The problem could also have been solved in a single calculation:

$$PV(C_{12}) = C_T / [1+(r / m)]^{mT}$$
$$= \$5,000 / [1+(0.10 / 4)]^{4\times12}$$
$$= \mathbf{\$1,528.36}$$

The PV of the cash flow is $1,528.36.

4.19 Deposit your money in the bank that offers the highest effective annual yield, EAY. The EAY is the rate of return you will receive after taking into account compounding. Convert each bank's stated annual interest rate into an EAY.

$$EAY(\text{Bank America}) = [1+(r / m)]^m - 1$$
$$= [1+(0.041 / 4)]^4 - 1$$
$$= 0.0416 = \mathbf{4.16\%}$$

$$EAY(\text{Bank USA}) = [1+(r / m)]^m - 1$$
$$= [1+(0.0405 / 12)]^{12} - 1$$
$$= 0.0413 = \mathbf{4.13\%}$$

You should deposit your money in Bank America since it offers a higher EAY (4.16%) than Bank USA offers (4.13%).

4.20 The price of any bond is the present value of its coupon payments. Since a consol pays the same coupon every year in perpetuity, apply the perpetuity formula to find the present value.

$$PV = C_1 / r$$
$$= \$120 / 0.15$$
$$= \mathbf{\$800}$$

The price of the consol is $800.

4.21 a. Apply the perpetuity formula, discounted at 10 percent.

$$PV = C_1 / r$$
$$= \$1,000 / 0.1$$
$$= \mathbf{\$10,000}$$

The PV is $10,000.

b. Remember that the perpetuity formula yields the present value of a stream of cash flows one period before the initial payment. Therefore, applying the perpetuity formula to a stream of cash flows that begins two years from today will generate the present value of that perpetuity as of the end of year 1. Next, discount the PV as of the end of 1 year back one year, yielding the value today, year 0.

$$PV = [C_2 / r] / (1+r)$$
$$= [\$500 / 0.1] / (1.1)$$
$$= \mathbf{\$4,545.45}$$

The PV is $4,545.45.

c. Applying the perpetuity formula to a stream of cash flows that begins three years from today will generate the present value of that perpetuity as of the end of year 2. Thus, use the perpetuity formula to find the PV as of the end of year 2. Next, discount that value back two years to find the value today, year 0.

$$PV = [C_3 / r] / (1+r)^2$$
$$= [\$2,420 / 0.1] / (1.1)^2$$
$$= \mathbf{\$20,000}$$

The PV is $20,000.

4.22 Applying the perpetuity formula to a stream of cash flows that starts at the end of year 9 will generate the present value of that perpetuity as of the end of year 8.

$$PV_8 = [C_9 / r]$$
$$= [\$120 / 0.1]$$
$$= \$1,200$$

To find the PV of the cash flows as of the end of year 5, discount the PV of the perpetuity as of the end of year 8 back three years.

$$PV_5 = PV_8 / (1+r)^3$$
$$= \$1,200 / (1.1)^3$$
$$= \mathbf{\$901.58}$$

The PV as of the end of year 5 is $901.58.

4.23 Use the growing perpetuity formula. Since Harris Inc.'s last dividend was $3, the next dividend (occurring one year from today) will be $3.15 (= $3 × 1.05). Do not take into account the dividend paid yesterday.

$$PV = C_1 / (r - g)$$
$$= \$3.15 / (0.12 - 0.05)$$
$$= \mathbf{\$45}$$

The price of the stock is $45.

4.24 Use the growing perpetuity formula to find the PV of the dividends. The PV is the maximum you should be willing to pay for the stock.

$$PV = C_1 / (r - g)$$
$$= \$1 / (0.1 - 0.04)$$
$$= \mathbf{\$16.67}$$

The maximum you should pay for the stock is $16.67.

4.25 The perpetuity formula yields the present value of a stream of cash flows one period before the initial payment. Apply the growing perpetuity formula to the stream of cash flows beginning two years from today to calculate the PV as of the end of year 1. To find the PV as of today, year 0, discount the PV of the perpetuity as of the end of year 1 back one year.

$$PV = [C_2 / (r - g)] / (1+r)$$
$$= [\$200,000 / (0.1 - 0.05)] / (1.1)$$
$$= \mathbf{\$3,636,363.64}$$

The PV of the technology is $3,636,363.64.

4.26 Barrett would be indifferent when the NPV of the project is equal to zero. Therefore, set the net present value of the project's cash flows equal to zero. Solve for the discount rate, *r*.

NPV	= Initial Investment + Cash Flows
0	= -$100,000 + $50,000 / r
0.5	= r

The discount rate at which Barrett is indifferent to the project is 50%.

4.27 Because the cash flows occur quarterly, they must be discounted at the rate applicable for a quarter of a year. Since the stated annual interest rate is given in terms of quarterly periods, and the payments are given in terms of quarterly periods, simply divide the stated annual interest rate by four to calculate the quarterly interest rate.

Quarterly Interest Rate	= Stated Annual Interest Rate / Number of Periods
	= 0.12 / 4
	= 0.03 = 3%

Use the perpetuity formula to find the PV of the security's cash flows.

PV	= C_1 / r
	= $10 / 0.03
	= **$333.33**

The price of the security is $333.33.

4.28 The two steps involved in this problem are a) calculating the appropriate discount rate and b) calculating the PV of the perpetuity.

Since the payments occur quarterly, the cash flows must be discounted at the interest rate applicable for a quarter of a year.

Quarterly Interest Rate	= Stated Annual Interest Rate / Number of Periods
	= 0.15 / 4
	= 0.0375 = 3.75%

Remember that the perpetuity formula provides the present value of a stream of cash flows one period before the initial payment. Therefore, applying the perpetuity formula to a stream of cash flows that begins 20 periods from today will generate the present value of that perpetuity as of the end of period 19. Next, discount that value back 19 periods, yielding the price today, year 0.

PV	= $[C_{20} / r] / (1+r)^{19}$
	= $[\$1 / 0.0375] / (1.0375)^{19}$
	= **$13.25**

The price of the stock is $13.25.

4.29 Calculate the NPV of the asset. Since the cash inflows form an annuity, you can use the present value of an annuity factor. The annuity factor is referred to as A^T_r, where *T* is the number of payments and *r* is the interest rate.

PV(Investment)	= -$6,200
PV(Cash Inflows)	= $C\ A^T_r$
	= $\$1{,}200\ A^8_{0.1}$
	= $6,401.91

The NPV of the asset is the sum of the initial investment (-$6,200) and the PV of the cash inflows ($6,401.91).

NPV = -Initial Investment + Cash Flows
= -$6,200 + $6,401.91
= **$201.91**

Since the asset has a positive NPV, $201.91, you should buy it.

4.30 There are 20 payments for an annuity beginning in year 3 and ending in year 22. Apply the annuity formula to this stream of 20 annual payments.

PV(End of Year 2) $= C\ A^{T}_{r}$
$= \$2{,}000\ A^{20}_{0.08}$
= $19,636.29

Since the first cash flow is received at the end of year 3, applying the annuity formula to the cash flows will yield the PV as of the end of year 2. To find the PV as of today, year 0, discount that amount back two years.

PV(Year 0) $= \text{PV(End of Year 2)} / (1+r)^{T}$
$= \$19{,}636.29 / (1.08)^{2}$
= **$16,834.95**

The PV of the cash flows is $16,834.95.

4.31 There are 15 payments for an annuity beginning in year 6 and ending in year 20. Apply the annuity formula to this stream of 15 annual payments.

PV(End of Year 5) $= C\ A^{T}_{r}$
$= \$500\ A^{15}_{0.15}$
= $2,923.69

Since the first cash flow is received at the end of year 6, applying the annuity formula to the cash flows will yield the PV as of the end of year 5. To find the PV as of today, year 0, discount that amount back five years at 12 percent.

PV(Year 0) $= \text{PV(End of Year 5)} / (1.12)^{5}$
$= \$2{,}923.69 / (1.12)^{5}$
= **$1,658.98**

The PV of the annuity is $1,658.98.

4.32 Set the price of the note equal to the present value of the annuity of $2,000 per year.

P $= C\ A^{T}_{r}$
$12,800 $= \$2{,}000\ A^{10}_{r}$

The problem can be solved by using a calculator to find the appropriate discount rate.

6.4 $= A^{10}_{r}$
0.090626 = r

The problem can also be solved by using table A.2 in the back of the textbook. In table A.2, scan across the row for 10-year annuity factors until one approximates 6.4. 6.4177, corresponding to a rate of 9%, is close to the above factor, 6.4. Thus, the rate received is slightly more than 9%.

The rate received is 9.0626%.

4.33 a. To calculate the necessary annual payments, first find the PV of the $25,000 which you will need in five years.

$$PV = C_5 / (1+r)^5$$
$$= \$25{,}000 / (1.07)^5$$
$$= \$17{,}824.65$$

Next, compute the annuity that will yield the same PV as calculated above. Solve for the deposit you will make each year.

$$PV = C\ A^T{}_r$$
$$\$17{,}824.65 = C\ A^5{}_{0.07}$$
$$\$17{,}824.65 / A^5{}_{0.07} = \mathbf{\$4{,}347.27}$$

Depositing $4,347.27 into the 7% account each year will provide $25,000 five years from today.

b. The lump sum payment must be the present value of the $25,000 you will need five years from today.

$$PV = C_5 / (1+r)^5$$
$$= \$25{,}000 / (1.07)^5$$
$$= \mathbf{\$17{,}824.65}$$

You must deposit $17,824.65 as a lump sum to have $25,000 in the account at the end of five years.

4.34 First, determine the balance of the loan Nancy must pay.

$$\text{Balance} = \$120{,}000\ (0.85)$$
$$= \$102{,}000$$

Apply the annuity formula since Nancy will pay the balance of the loan in 20 equal, end-of-year, payments. Set the present value of the annuity equal to the balance of the loan. Solve for the annual payment, *C*.

$$\text{Balance} = C\ A^T{}_r$$
$$\$102{,}000 = C\ A^{20}{}_{0.1}$$
$$\$102{,}000 / A^{20}{}_{0.1} = C$$
$$\mathbf{\$11{,}980.88} = C$$

The equal installments are $11,980.88.

4.35 a. The cash flows form a 31-year annuity where the first payment is received today. Remember to use the after-tax cash flows. The first payment of a standard annuity is received one year from today. Therefore, value all after-tax cash flows except the first after-tax payment using the standard annuity formula. Then add back the first after-tax payment to obtain the value of the option. Since the first payment is treated separately from the other payments, the annuity has 30 periods instead of 31 periods.

$$\begin{aligned} PV &= (1 - T_c)\, C_1\, A^T{}_r + (1 - T_c)\, C_0 \\ &= (1 - 0.28)\ \$160{,}000\ A^{30}{}_{0.1} + (1 - 0.28)\ \$160{,}000 \\ &= \mathbf{\$1{,}201{,}180.55} \end{aligned}$$

b. This option pays $446,000, after-tax, immediately. The remaining money is received as a 30-year annuity that pays $101,055, annually before tax. Find the PV of the annuity, discounted at 10 percent. Remember to apply taxes to the annuity.

$$\begin{aligned} PV &= (1 - T_c)\, C_1\, A^T{}_r + C_0 \\ &= (1 - 0.28)\ \$101{,}055\ A^{30}{}_{0.1} + \$446{,}000 \\ &= \mathbf{\$1{,}131{,}898.53} \end{aligned}$$

Choose the first option with a PV of $1,201,180.55 since it has a higher PV than the second option, $1,131.898.53.

4.36 First, use the standard annuity formula to compute the present value of all the payments you must make for each of your children's educations.

$$\begin{aligned} \text{PV(Each Child's Education)} &= C\, A^T{}_r \\ &= \$21{,}000\ A^{4}{}_{0.15} \\ &= \$59{,}954.55 \end{aligned}$$

The annuity formula values any annuity as of one year before the first cash flow. Since the first payment for each child is made when the child enters college, the above value represents the cost of the older child's education 14 years from now and of the younger child's education 16 years from now. To find the PV of the children's education at year 0, discount the above PV back 14 years and 16 years for both the older and younger child, respectively.

$$\begin{aligned} \text{PV(Older Child)} &= \text{PV(Education)} / (1+r)^{14} \\ &= \$59{,}954.55 / (1.15)^{14} \\ &= \$8{,}473.30 \end{aligned}$$

$$\begin{aligned} \text{PV(Younger Child)} &= \$59{,}954.55 / (1.15)^{16} \\ &= \$6{,}407.03 \end{aligned}$$

$$\begin{aligned} \text{PV(Total Cost)} &= \text{PV(Older Child)} + \text{PV(Younger Child)} \\ &= \$8{,}473.30 + \$6{,}407.03 \\ &= \$14{,}880.32 \end{aligned}$$

You will make 15 payments, since your first payment is made one year from today and the last payment is made when your first child enters college, 15 years from now. To find the amount of each payment, set the total PV of the children's education costs equal to a 15-year annuity, discounted at 15 percent. Solve for the annual payment, *C*.

$$\begin{aligned} \text{PV(Total Cost)} &= C\, A^T{}_r \\ \$14{,}880.32 &= C\, A^{15}{}_{0.15} \\ \$14{,}880.32 / A^{15}{}_{0.15} &= C \\ \mathbf{\$2{,}544.79} &= C \end{aligned}$$

The annual payment that will allow you to pay for the total cost of your children's college educations in 15 years is $2,544.79.

4.37 To determine whether or not the policy is worth buying, calculate the NPV of the policy. The parent's six payments are cash outflows and the insurance company's payment is a cash inflow. The PV of the parent's payments can be calculated by applying the annuity formula, discounted at six percent, to both the first three payments (each $750) and the last three payments (each $800).

PV(First 3 Payments) $= C_1 A^T_r$
$= -\$750 A^3_{0.06}$
$= -\$2,004.76$

The annuity formula calculates the PV as of one period prior to the first cash flow. Since the first \$800 payment occurs four years from today, the PV of the annuity of the last three payments must be discounted back three years.

PV(Last 3 Payments) $= [C_4 A^T_r] / (1+r)^3$
$= [-\$800 A^3_{0.06}] / (1.06)^3$
$= -\$1,795.45$

Discount the insurance company's payment back 65 years. Take note that the discount rate is six percent for years 1 through 6 and seven percent for years 7 through 65.

PV(Insurance Payment) $= C_{65} / [(1+r)^{\text{Year 1 - 6}} (1+r)^{\text{Year 7 – 65}}]$
$= \$250,000 / [(1.06)^6 (1.07)^{59}]$
$= \$3,254.33$

NPV = PV(First 3 Payments) + PV(Last 3 Payments) + PV(Insurance Payment)
= -\$2,004.76 + -\$1,795.45 + \$3,254.33
= **-\$545.88**

Since the NPV of the policy is negative, -\$545.88, it is not worth buying.

4.38 Calculate the present value of the lease offer. An annuity in advance is a stream of cash flows beginning today. Since the annual lease payments form an annuity in advance, value all payments except the one made today using the standard annuity formula. Add back the payment made today. The immediate payment is not discounted because it occurs today, year 0. Because the first payment is treated separately, the annuity has nine periods instead of 10 periods.

PV(Payments) $= C_0 + C_1 A^T_r$
$= -\$15,000 + -\$15,000 A^9_{0.08}$
$= -\$108,703.32$

PV(Purchase Option) $= C_T / (1+r)^T$
$= -\$25,000 / (1.08)^{10}$
$= -\$11,579.84$

PV(Lease) = PV(Payments) + PV(Purchase Option)
= -\$108,703.32 - \$11,579.84
= **-\$120,283.16**

Since the PV of the lease offer is greater than \$120,000, the cost of the machine, you should not accept the offer.

4.39 Remember that your salary grows by four percent each year, and you just received a \$50,000 salary payment. Thus, your salary next year will be \$52,000 (=\$50,000 × 1.04). Two percent of next year's salary will be deposited into the account.

C = (Last Year's Salary) (1+g) (Percent Deposited)
= (\$50,000) (1.04) (0.02)
= \$1,040.00

Since your salary will continue to grow at four percent annually, your deposits will also grow at this rate. Apply the growing annuity formula, discounted at eight percent, to calculate the PV of your retirement account.

PV $= C\ GA^T_{r,g}$ *
$= \$1{,}040.00\ GA^{40}_{0.08,\ 0.04}$
$= \$20{,}254.12$

To determine how much will be in the account at your retirement in 40 years, calculate the future value.

FV $= PV\ (1+r)^T$
$= \$20{,}254.12\ (1.08)^{40}$
= **$440,011.02**

At the time of your retirement, the account will have $440,011.02.

*The notation $GA^T_{r,\ g}$ represents a growing annuity consisting of *T* payments growing at a rate of *g* per payment, discounted at *r*.

4.40 Discount the individual cash flows to compute the NPV of the project. The cash flow, C_0, is the initial investment.

$PV(C_0)$	= -$5,000	
$PV(C_1)$	$= \$700 / (1.1)$	= $636.36
$PV(C_2)$	$= \$900 / (1.1)^2$	= $743.80
$PV(C_3)$	$= \$1{,}000 / (1.1)^3$	= $751.32
$PV(C_4)$	$= \$1{,}000 / (1.1)^4$	= $683.01
$PV(C_5)$	$= \$1{,}000 / (1.1)^5$	= $620.92
$PV(C_6)$	$= \$1{,}000 / (1.1)^6$	= $564.47
$PV(C_7)$	$= \$1{,}250 / (1.1)^7$	= $641.45
$PV(C_8)$	$= \$1{,}375 / (1.1)^8$	= $641.45

NPV = -$5,000 + $636.36 + $743.80 + $751.32 + $683.01 + $620.92 + $564.47 + $641.45 + $641.45
= **$282.78**

Purchase the machine since it has a positive NPV.

4.41 a. Engineer:

Apply the annuity formula, discounted at five percent, to calculate the PV of his undergraduate education.

PV(Undergraduate) $= C\ A^T_r$
$= -\$12{,}000\ A^4_{0.05}$
$= -\$42{,}551.41$

To find the PV of his practical experience in years 5 and 6, discount the two cash flows by five years and six years, respectively.

PV(Experience) $= \$20{,}000 / (1.05)^5 + \$25{,}000 / (1.05)^6$
$= \$34{,}325.90$

Discount the corresponding two cash flows for the master's degree by seven years and eight years.

PV(Master) $= -\$15{,}000 / (1.05)^7 + -\$15{,}000 / (1.05)^8$
$= -\$20{,}812.81$

After completion of his master's degree, your brother will earn $40,000 per year for the next 25 years. Since the annuity formula calculates the PV as of one year before the first cash flow, applying the annuity formula to your brother's future earnings will generate the PV as of the end of year 8. Discount that PV back eight years to find the PV as of today, year 0.

PV(Earnings) $= C_9 \, A^T_r / (1+r)^8$
$= \$40{,}000 \, A^{25}_{0.05} / (1.05)^8$
$= \$381{,}573.46$

Thus, the NPV of his decision to become an engineer is:

NPV(Engineer) = PV(Undergraduate) + PV(Experience) + PV(Master) + PV(Earnings)
$= -\$42{,}551.41 + \$34{,}325.90 – \$20{,}812.81 + \$381{,}573.46$
= **$352,535.14**

Accountant:

Apply the annuity formula, discounted at five percent, to calculate the PV of the accounting undergraduate education.

PV(Undergraduate) $= -\$13{,}000 \, A^4_{0.05}$
$= -\$46{,}097.36$

Apply the annuity formula to calculate the PV of the future earnings. Since the annuity formula calculates the PV as of one year before the first cash flow, applying the annuity formula to your brother's future earnings will generate the PV as of the end of year 4. Discount that PV back four periods to find the PV as of today.

PV(Earnings) $= C_5 \, A^T_r / (1+r)^4$
$= \$31{,}000 \, A^{30}_{0.05} / (1.05)^4$
$= \$392{,}055.56$

Thus, the NPV of his decision to become an accountant is:

NPV(Accountant) = PV(Undergraduate) + PV(Earnings)
$= -\$46{,}097.36 + \$392{,}055.56$
= **$345,958.20**

Since the NPV of becoming an engineer, $352,535.14, is higher than the NPV of becoming an accountant, $345,958.20, your brother should study engineering.

b. After your brother announces that the appropriate discount rate is six percent, recalculate the NPVs the same way as above, using a six percent discount rate.

NPV(Engineer) = PV(Undergraduate) + PV(Experience) + PV(Master) +PV(Earnings)
$= -\$12{,}000 \, A^4_{0.06} + \$20{,}000 / (1.06)^5 + \$25{,}000 / (1.06)^6 – \$15{,}000 / (1.06)^7 – \$15{,}000 / (1.06)^8 + \$40{,}000 \, A^{25}_{0.06} / (1.06)^8$
= **$292,418.30**

NPV(Accountant) = PV(Undergraduate) + PV(Earnings)
= -$13,000 $A^{4}_{0.06}$ + $31,000 $A^{30}_{0.06}$ / $(1.06)^4$
= **$292,947.73**

Your brother made a poor decision. At a six percent rate, the NPV of becoming an accountant, $292,947.73, is higher than the NPV of becoming an engineer, 292,418.30. Thus, he should have chosen to study accounting.

4.42 Use the growing annuity formula, discounted at 12 percent and growing at four percent, to find the PV of Tom's annual salary payments.

PV(Salary) = C $GA^{T}_{r,\,g}$ *
= $35,000 $GA^{25}_{0.12,\,0.04}$
= $368,894.18

* The notation $GA^{T}_{r,\,g}$ represents a growing annuity consisting of *T* payments growing at a rate of *g* per payment, discounted at *r*.

The yearly bonuses are equal to 10 percent of his salary. Since his salary grows at four percent, the annual bonus will also grow at four percent. Use the growing annuity formula, discounted at 12 percent and growing at four percent, to find the PV of Tom's annual bonus payments.

PV(Bonus) = (0.1) ($35,000) $GA^{25}_{0.12,\,0.04}$
= $36,889.42

Mr. Adams will also receive a signing bonus today.

PV(Signing) = $10,000

PV(Offer) = PV(Salary) + PV(Bonus) + PV(Signing)
= $368,894.18 + $36,889.42 + $10,000
= **$415,783.60**

The PV of the offer is $415,783.60.

4.43 Apply the growing annuity formula to find the PV of the cash flows.

PV(Cash Flows) = C $GA^{T}_{r,\,g}$ *
= $10,000 $GA^{5}_{0.1,\,0.07}$
= $43,041.91

Subtract the initial cost of $40,000 from the PV of the growing annuity to find the NPV of the revision.

PV(Cost) = -$40,000

NPV = PV(Cash Flows) + PV(Cost)
= $43,041.91 – $40,000
= **$3,041.91**

Since the NPV of the revision is positive, $3,041.91, revise the textbook. The firm's return on the project will be greater than 10 percent since the NPV is positive.

***** The notation $GA^{T}_{r,\,g}$ represents a growing annuity consisting of *T* payments growing at a rate of *g* per payment, discounted at *r*.

4.44 First, find the PV of Ian's retirement income and his cabin purchase. Since the first retirement payment is made at the end of year 31, applying the annuity formula to the cash flows will yield the PV as of the end of year 30. To find the PV as of today, year 0, discount that value back 30 periods.

$$\begin{aligned} \text{PV(Retirement)} &= C_{31}\, A^{T}_{r} / (1+r)^{30} \\ &= \$300{,}000\, A^{20}_{0.07} / (1.07)^{30} \\ &= \$417{,}511.53 \end{aligned}$$

Since the cabin is purchased at the end of year 10, discount that cash flow back 10 periods to find the PV as of today, year 0.

$$\begin{aligned} \text{PV(Cabin)} &= \$350{,}000 / (1.07)^{10} \\ &= \$177{,}922.25 \end{aligned}$$

Next, find the PV of his annual savings from year 1 through year 10, using the annuity formula.

$$\begin{aligned} \text{PV(Savings)} &= \$40{,}000\, A^{10}_{0.07} \\ &= \$280{,}943.26 \end{aligned}$$

Find the difference between the PV of his savings from year 1 through year 10 and the total PV of Ian's two expenditures (retirement and cabin).

$$\begin{aligned} \text{Difference} &= \text{PV(Savings)} - [\text{PV(Retirement)} + \text{PV(Cabin)}] \\ &= \$280{,}943.26 - (\$417{,}511.53 + \$177{,}922.25) \\ &= -\$314{,}491.52 \end{aligned}$$

In present value terms, Ian must save an additional $314,491.52 in order to meet his objectives. To determine the amount he must save from year 11 through year 30, set the PV of his savings over this time period equal to the difference of $314,491.52. Since the annual savings will begin 11 years from today, discount the annuity back 10 periods. Solve for the amount Ian needs to save each year, *C*.

$$\begin{aligned} \text{Difference} &= C\, A^{T}_{r} / (1+r)^{10} \\ \$314{,}491.52 &= C\, A^{20}_{0.07} / (1.07)^{10} \\ [\$314{,}491.52\, (1.07)^{10}] / A^{20}_{0.07} &= C \\ \mathbf{\$58{,}396.23} &= C \end{aligned}$$

Ian needs to save $58,396.23 annually from year 11 to year 30 in order to meet his objectives.

4.45 Since Jack's salary is paid monthly, the payments need to be discounted at the monthly interest rate.

$$\begin{aligned} \text{Monthly Interest Rate} &= \text{Stated Annual Interest Rate / Number of Periods} \\ &= 0.12 / 12 \\ &= 0.01 \end{aligned}$$

Next, calculate the PV of his monthly salary. Since he will receive 36 monthly payments (=12 payments per year × 3 years), the PV is:

$$\begin{aligned} \text{PV(Salary)} &= C\, A^{T}_{r} \\ &= \$5{,}000\, A^{36}_{0.01} \\ &= \$150{,}537.53 \end{aligned}$$

Next, calculate the PV of his bonuses. The end-of-year bonuses must be discounted at the effective annual yield, EAY. The EAY is the appropriate discount rate because it captures the effect of compounding periods. Find the EAY and then apply the annuity formula, discounted at the EAY, to calculate the PV of the bonuses.

EAY $= [1+(r / m)]^m - 1$
$= [1+(0.12 / 12)]^{12} - 1$
$= 0.12683 = 12.683\%$

PV(Bonus) $= C\ A^T_r$
$= \$10,000\ A^3_{0.12683}$
= $23,739.20

PV(Contract) = PV(Salary) + PV(Bonus)
= $150,537.53 + $23,739.20
= **$174,276.73**

The PV of the contract is $174,276.73.

4.46 First, determine the balance of the loan.

Balance = $15,000 (0.80)
= $12,000

Convert the stated annual interest rate to a monthly interest rate since the payments are made monthly.

Monthly Interest Rate = Stated Annual Interest Rate / Number of Periods
= 0.08 / 12
= 0.0067

Set the balance of the loan equal to the present value of the annuity of 48 monthly payments, discounted at 0.0067. Solve for the monthly payment, *C*.

Balance $= C\ A^T_r$
$\$12,000 = C\ A^{48}_{0.0067}$
$\$12,000 / A^{48}_{0.0067} = C$
$293.18 = C

The monthly installments will be $293.18.

4.47 The balance of the loan is:

Balance = $10,000 – $1,000
= $9,000

Because the payments are made on a monthly basis, first calculate the monthly interest rate.

Monthly Interest Rate = 0.096 / 12
= 0.008

Next, to find Susan's monthly payments, set the balance of the loan equal to an annuity with 60 periods (=12 months × 5 years), discounted at the monthly interest rate.

Balance	$= C\ A^{T}_{r}$
\$9,000	$= C\ A^{60}_{0.008}$
\$9,000 / $A^{60}_{0.008}$	$= C$
\$189.46	$= C$

To find the balance of the loan that Susan will prepay, first find the number of remaining payments beginning with the November 1, 2002 payment. Since her first payment was made on October 1, 2000, her payment on October 1, 2002 was her 25th. Thus, she has 35 remaining payments, including the immediate November, 2002 payment.

Each of the remaining 35 payments is \$189.46. To find the PV, value all payments except the impending November payment using the standard annuity formula. Then add back the November payment to obtain the PV of her repayment. Since the November payment is treated separately from the other payments, the annuity has 34 periods instead of 35 periods.

PV(Loan)	$= C_1\ A^{T}_{r} + C_0$
	$= \$189.46\ A^{34}_{0.008} + 189.46$
	= \$5,809.81

Susan also incurs a 1% penalty. Thus, the total repayment is:

Repayment	= PV(Loan) (1+Penalty)
	= \$5,809.81 (1.01)
	= **\$5,867.91**

The total repayment is \$5,867.91.

4.48 To find the PV of the flower purchases, you must first find the weekly interest rate and the weekly growth rate. Divide both the stated annual interest rate and the stated annual growth rate by 52 to calculate the weekly interest rate and weekly growth rate.

Weekly Interest Rate	= 0.104 / 52
	= 0.002
Weekly Growth Rate	= 0.039 / 52
	= 0.00075

Apply the growing annuity formula, discounted at 0.002 with payments growing at a rate of 0.00075 weekly, to find the PV of Joe's commitment. The growing annuity has 1,560 periods (=30 years × 52 weeks).

PV	$= C_1\ GA^{T}_{r,\ g}$ *
	$= \$5\ GA^{1,560}_{0.002,\ 0.00075}$
	= **\$3,429.38**

The PV of Joe DiMaggio's commitment is \$3,429.38.

*The notation $GA^{T}_{r,\ g}$ represents a growing annuity consisting of *T* payments growing at a rate of *g* per payment, discounted at *r*.

4.49 Since Goose receives his first payment on July 1 and all later payments are made in one-year intervals on July 1, discount the cash flows to July 1 of year 0 using the annual discount rate of nine percent. Then use the six-month discount rate (0.044) to discount the cash flows back to January of year 0.

The PVs of the guaranteed payments over the first six years are:

$PV(C_1)$ = \$875,000 / (1.044) = \$838,122.61
$PV(C_2)$ = \$650,000 / [(1.09) (1.044)] = \$571,197.58
$PV(C_3)$ = \$800,000 / [$(1.09)^2$ (1.044)] = \$644,965.51
$PV(C_4)$ = \$1,000,000 / [$(1.09)^3$ (1.044)] = \$739,639.35
$PV(C_5)$ = \$1,000,000 / [$(1.09)^4$ (1.044)] = \$678,568.21
$PV(C_6)$ = \$300,000 / [$(1.09)^5$ (1.044)] = \$186,761.89

Apply the annuity formula, discounted at nine percent, to find the PV of the 17 deferred payments of \$240,000 from 1990 through 2006. The annuity formula will calculate the PV of the cash flows as of July 1, 1989. Discount that value back 5.5 periods to find the PV of the deferred payments as of January 1984.

PV(Deferred Payments) = $C_7\,A^T_r$ / [(1+Six Month Rate) $(1+\text{Annual Rate})^T$]
= \$240,000 $A^{17}_{0.09}$ / [(1.044) $(1.09)^5$]
= \$1,276,499.81

Perform a similar calculation to find the PV of the 10 deferred payments of \$125,000 from 2007 through 2016. The annuity formula will calculate the PV of the cash flows as of July 1, 2006. Discount that value back 22.5 periods to find the PV of the deferred payments as of January 1984.

PV(Deferred Payments 2) = \$125,000 $A^{10}_{0.09}$ / [(1.044) $(1.09)^{22}$]
= \$115,399.28

NPV(Contract) = $PV(C_1) + PV(C_2) + PV(C_3) + PV(C_4) + PV(C_5) + PV(C_6)$ +
PV(Deferred Payments 1) + PV(Deferred Payments 2)
= \$838,122.61 + \$571,197.58 + \$644,965.51 + \$739,639.35 +
\$678,568.21 + \$186,761.89 + \$1,276,499.81 + \$115,399.28
= **\$5,051,154.24**

The PV of the contract is \$5,051,154.24.

To find the equivalent annual salary from year 1984 through 1988, set the PV of the five-year annuity equal to the PV of the contract.

PV(Contract) = C A^T_r
\$5,051,154.24 = C $A^5_{0.09}$
\$5,051,154.24 / $A^5_{0.09}$ = C
\$1,298,613.65 = C

The equivalent annual salary from year 1984 through 1988 is \$1,298,613.65.

4.50 First, calculate the monthly interest rate.

Monthly Interest Rate = 0.08 / 12
= 0.0067

Mike's balloon payment is the PV of the remaining mortgage payments as of the end of year 8. To calculate the PV of the remaining payments, first calculate Mike's monthly payment. Because he had to make a 20 percent down payment, Mike borrows \$320,000 (=\$400,000 × 0.8). Set this amount equal to a 360 period annuity (=30 years × 12 monthly payments), discounted at the monthly interest rate.

Loan	$= C\ A^{360}_{0.0067}$
\$320,000	$= C\ A^{360}_{0.0067}$
\$320,000 / $A^{360}_{0.0067}$	$= C$
\$2,356.98	$= C$

At the end of the eighth year, Mike will have made 96 payments (=8 years × 12 monthly payments). Thus, he will have 264 remaining payments (=360 – 96). His balloon payment is the PV of those remaining payments. Apply the annuity formula, discounted at 0.0067, to the 264 remaining monthly payments to find the PV of Mike's balloon payment, as of the end of year 8.

PV(Balloon)	= \$2,356.98 $A^{264}_{0.0067}$
	= **\$291,439.54**

The value of Mike's balloon payment at the end of year 8 will be \$291,439.54.

4.51 First, calculate the monthly interest rate.

Monthly Interest Rate	= 0.12 / 12
	= 0.01

Then set the PV of the lease payments equal to \$4,000, the retail price of the equipment. Since the first payment is due immediately, value all payments except the payment made today using the standard annuity formula. Because the first payment is treated separately from the others, the annuity has 23 periods instead of 24 periods. Add back the payment made immediately.

PV	$= C\ A^{T}_{r} + C$
\$4,000	$= C\ A^{23}_{0.01} + C$
\$4,000	$= C\ (A^{23}_{0.01} + 1)$
\$4,000 / $(A^{23}_{0.01} + 1)$	$= C$
\$186.43	$= C$

The monthly lease payment will be \$186.43.

4.52 The effective annual yield (EAY) is the appropriate discount rate because it captures the effect of compounding periods. First calculate the EAY.

$$\begin{aligned} EAY &= [1+(r / m)]^{m} - 1 \\ &= [1+(0.08 / 4)]^{4} - 1 \\ &= 0.0824 \end{aligned}$$

Next, discount the payments of the annuity. The first payment is made at the end of year 5. Since the standard annuity formula calculates the PV of the cash flows as of one year before the first payment, applying the formula will yield the PV as of the end of year 4. Discount that value back four years to find the PV of the annuity as of today.

$$\begin{aligned} PV &= C_5\ A^{T}_{r} / (1+r)^{4} \\ &= \$900\ A^{10}_{0.0824} / (1.0824)^{4} \\ &= \$4,352.43 \end{aligned}$$

The PV of the annuity is \$4,352.43.

4.53 For the project to be an attractive investment, the NPV should be >0.

Initial Investment = C_0 = –1,800 (all numbers hereafter are in thousands)
Note: First year of production occurs in year 3
$C_1 = 0$
$C_2 = 0$
$C_3 = 60$
$C_4 = 60*1.04 = 62.4$
$C_5 = 62.4*1.04 = 64.9$
$C_6 = 64.9*1.04 = 67.5$
$C_7 = 67.5*1.04 = 70.2$
$C_8 = 70.2*1.04 = 73$

Starting year 8, cash flows form a perpetuity

Therefore, $-1{,}800 + 0/1.1^1 + 0/1.1^2 + 60/1.1^3 + 62.4/1.1^4 + 64.9/1.1^5 + 67.5/1.1^6 + 70.2/1.1^7 +$

$$\frac{\left(\frac{73}{0.10-g}\right)}{(1.1)^7} = 0$$ (in the limiting case, NPV is at least zero)

Note: Using the formula for PV of perpetuity gives the PV for one year before the year of first cash flow of the perpetuity. In our problem, PV formula for the perpetuity gives the present value in year 7 which needs to be discounted back by 7 years

Solving we get, **g = 7.66%**

(Be wary of truncation or rounding errors)

4.54 In this problem, we need to calculate Paul's annual savings in the 5 years of operation of his health club and compute their future value in year 5.

Savings (S) = Revenue (R) – Cost (C)

Since both annual fee as well as membership are growing, we need to computed the effective growth rate (EGR) for revenues:

EGR = (1 + 10%) (1 + 3%) – 1
EGR = 13.3 % R_0 = 200,000.00

R_1 = 226,600.00
R_2 = 256,737.80
R_3 = 290,883.93
R_4 = 329,571.49
R_5 = 373,404.50

G = 2% C_0 = 80,000.00

C1 = 81,600.00
C2 = 83,232.00
C3 = 84,896.64
C4 = 86,594.57
C5 = 88,326.46

S1 =	145,000.00
S2 =	173,505.80
S3 =	205,987.29
S4 =	242,976.92
S5 =	285,078.03

Using Table A.3, we get future value of savings = $1,203,594.91

Paul buys a boat for $500,000. Therefore, at the end of year 5, Paul is left with $703,594.91

The annual amount that Paul can spend while on his world tour for a remaining life of 15 years is $82,200.61.