Chapter 5: How to Value Bonds and Stocks

5.1 The present value of any pure discount bond is its face value discounted back to the present.

a. PV $= F / (1+r)^{10}$
$= \$1,000 / (1.05)^{10}$
$=$ **\$613.91**

b. PV $= \$1,000 / (1.10)^{10}$
$=$ **\$385.54**

c. PV $= \$1,000 / (1.15)^{10}$
$=$ **\$247.19**

5.2 First, find the amount of the semiannual coupon payment.

Semiannual Coupon Payment = Annual Coupon Payment / 2
$= (0.08 \times \$1,000) / 2$
$= \$40$

a. Since the stated annual interest rate is compounded semiannually, simply divide this rate by two in order to calculate the semiannual interest rate.

Semiannual Interest Rate $= 0.08 / 2$
$= 0.04$

The bond has 40 coupon payments (=20 years × 2 payments per year). Apply the annuity formula to calculate the PV of the 40 coupon payments. In addition, the \$1,000 payment at maturity must be discounted back 40 periods.

P $= C\ A^{T}_{r} + F / (1+r)^{40}$
$= \$40\ A^{40}_{0.04} + \$1,000 / (1.04)^{40}$
$=$ **\$1,000**

The price of the bond is \$1,000. Notice that whenever the coupon rate and the market rate are the same, the bond is priced at par. That is, its market value is equal to its face value.

b. Semiannual Interest Rate $= 0.10 / 2$
$= 0.05$

P $= \$40\ A^{40}_{0.05} + \$1,000 / (1.05)^{40}$
$=$ **\$828.41**

The price of the bond is \$828.41. Notice that whenever the coupon rate is below the market rate, the bond is priced below par.

c. Semiannual Interest Rate $= 0.06 / 2$
$= 0.03$

P $= \$40\ A^{40}_{0.03} + \$1,000 / (1.03)^{40}$
$=$ **\$1,231.15**

The price of the bond is \$1,231.15. Notice that whenever the coupon rate is above the market rate, the bond is priced above par.

5.3 Since the payments occur semiannually, discount them at the semiannual interest rate. Convert the effective annual yield (EAY) to a semiannual interest rate.

Semiannual Interest Rate $= (1+EAY)^{1/T} - 1$
$= (1.12)^{1/2} - 1$
$= 0.0583$

a. Calculate the semiannual coupon payment.

Semiannual Coupon Payment = Annual Coupon Payment / 2
$= (0.08 \times \$1,000) / 2$
$= \$40$

Apply the annuity formula to calculate the PV of the 40 coupon payments (=20 years × 2 payments per year). In addition, the \$1,000 payment at maturity must be discounted back 40 periods. The appropriate discount rate is the semiannual interest rate.

P $= C\, A^T_r + F / (1+r)^{40}$
$= \$40\, A^{40}_{0.0583} + \$1,000 / (1.0583)^{40}$
= **\$718.65**

The price of the bond is \$718.65.

b. Calculate the semiannual coupon payment.

Semiannual Coupon Payment $= (0.10 \times \$1,000) / 2$
$= \$50$

Apply the annuity formula to calculate the PV of the 30 coupon payments (=15 years × 2 payments per year). In addition, the \$1,000 payment at maturity must be discounted back 30 periods. The appropriate discount rate is the semiannual interest rate.

P $= \$50\, A^{30}_{0.0583} + \$1,000 / (1.0583)^{30}$
= **\$883.64**

The price of the bond is \$883.64.

5.4 First, calculate the semiannual interest rate.

Semiannual Interest Rate $= (1+EAY)^{1/T} - 1$
$= (1.10)^{1/2} - 1$
$= 0.04881$

Next, find the semiannual coupon payment.

Semiannual Coupon Payment $= (0.08 \times \$1,000) / 2$
$= \$40$

The bond has 40 payments (=20 years × 2 payments per year). Apply the annuity formula to find the PV of the coupon payments. In addition, discount the $1,000 payment at maturity back 40 periods. The appropriate discount rate is the semiannual interest rate.

$$P = C\,A^{T}_{r} + F / (1+r)^{40}$$
$$= \$40\,A^{40}_{0.04881} + \$1{,}000 / (1.04881)^{40}$$
$$= \mathbf{\$846.33}$$

The price of the bond is $846.33.

5.5 First, calculate the semiannual interest rate.

$$\text{Semiannual Interest Rate} = 0.10 / 2 = 0.05$$

Set the price of the bond equal to the sum of the PV of the 30 semiannual coupon payments (=15 years × 2 payments per year) and the PV of the payment at maturity. The PV of the semiannual coupon payments should be expressed as an annuity. Solve for *C*, the semiannual coupon payment.

$$P = C\,A^{T}_{r} + F / (1+r)^{30}$$
$$\$923.14 = C\,A^{30}_{0.05} + \$1{,}000 / (1.05)^{30}$$
$$[\$923.14 - \$1{,}000 / (1.05)^{30}] / A^{30}_{0.05} = C$$
$$\$45 = C$$

To find the coupon rate on the bond, set the semiannual coupon payment, $45, equal to the product of the coupon rate and face value of the bond, divided by two.

Semiannual Coupon Payment	= (Coupon Rate × Face Value) / 2
$45	= (Coupon Rate × $1,000) / 2
$90	= Coupon Rate × $1,000
$90 / $1,000	= Coupon Rate
0.09	= Coupon Rate

The annual coupon rate is 9 percent.

5.6 a. The market interest rate and the coupon rate are equal because the bond is selling at par. Since the face value of the bond is $1,000 and the semiannual coupon payment is $60, the semiannual coupon rate is six percent (=$60 / $1,000). Thus, the semiannual interest rate is also six percent. Calculate the yield, expressed as an effective annual yield, by compounding the semiannual interest rate over two periods.

$$\text{Yield} = (1+r)^2 - 1$$
$$= (1.06)^2 - 1$$
$$= \mathbf{0.1236}$$

The yield is 0.1236.

b. You are willing to pay a price equal to the PV of the bond's payments. To find the PV of the 12 coupon payments, apply the annuity formula, discounted at the semiannual rate of return. Also, discount the $1,000 payment made at maturity back to the present. The discount rate, *r*, is the same as calculated in part (*a*).

$$P = C\,A^{T}_{r} + F / (1+r)^{12}$$
$$= \$30\,A^{12}_{0.06} + \$1{,}000 / (1.06)^{12}$$
$$= \mathbf{\$748.49}$$

The price of the bond is $748.49.

c. If the five-year bond pays $40 in semiannual payments and is priced at par, the semiannual rate of return will be different from that in part (*a*). Since the face value of the bond is $1,000 and the semiannual coupon payment is $40, the semiannual interest rate is four percent (=$40 / $1,000). To calculate the price of the bond, apply the annuity formula, discounted at the semiannual interest rate. In addition, discount the $1,000 payment made at maturity back 12 periods.

$P = C\,A^{T}_{r} + F / (1+r)^{12}$
$= \$30\,A^{12}_{0.04} + \$1{,}000 / (1.04)^{12}$
$= \mathbf{\$906.15}$

The price of the bond is $906.15.

5.7 a. Since the coupon rates of the bonds are equal to the market interest rate, the bonds are priced at face value. Both bonds have face values of $1,000.

$P_A = \mathbf{\$1{,}000}$

$P_B = \mathbf{\$1{,}000}$

b. Discount the cash flows of the bonds at 12 percent. Since the coupon rates of both bonds are less than the market interest rate, the bonds will be priced at a discount.

$P_A = \$100\,A^{20}_{0.12} + \$1{,}000 / (1.12)^{20}$
$= \mathbf{\$850.61}$

$P_B = \$100\,A^{10}_{0.12} + \$1{,}000 / (1.12)^{10}$
$= \mathbf{\$887.00}$

c. Discount the cash flows of the bonds at eight percent. Since the coupon rates of both bonds are greater than the market interest rate, the bonds will be priced at a premium.

$P_A = \$100\,A^{20}_{0.08} + \$1{,}000 / (1.08)^{20}$
$= \mathbf{\$1{,}196.36}$

$P_B = \$100\,A^{10}_{0.08} + \$1{,}000 / (1.08)^{10}$
$= \mathbf{\$1{,}134.20}$

5.8 a. The prices of long-term bonds should fall. The price of any bond is the PV of the cash flows associated with the bond. As the interest rate increases, the PV of those cash flows falls. This can be easily seen by looking at a one-year, pure discount bond.

$P = \$1{,}000 / (1+i)$

As *i* increases, the denominator, (1 + *i*), rises, thus reducing the value of the numerator ($1,000). The price of the bond decreases.

b. The effect on stocks is not as clear-cut as the effect on bonds. The nominal interest rate is a function of both the real interest rate, *r*, and the inflation rate, i.e.,

$(1+i) = (1+r)\,(1+\text{Inflation})$

From this relationship it is easy to conclude that, as inflation rises, the nominal interest rate, *i*, rises. However, stock prices are a function of dividends and future prices as well as the interest rate. Those dividends and future prices are determined by the earning power of the firm. Inflation may increase or decrease firm earnings. Thus, a rise in interest rates has an uncertain effect on the general level of stock prices.

5.9 Set the price of the bond equal to the PV of its cash flows, discounted at the yield to maturity, *r*. Solve for *r*.

a. $P = C\,A^{T}_{r} + F / (1+r)^{20}$

$\$1,200 = \$80\,A^{20}_{r} + \$1,000 / (1+r)^{20}$

r = **0.0622**

The yield to maturity is 6.22 percent.

b. $\$950 = \$80\,A^{10}_{r} + \$1,000 / (1+r)^{10}$

r = **0.0877**

The yield to maturity is 8.77 percent.

5.10 The appropriate discount rate is the semiannual interest rate because the bond makes semiannual payments. Thus, calculate the appropriate semiannual interest rate for both bonds *A* and *B*.

Semiannual Interest Rate $= 0.12 / 2$
$= 0.06$

a. The price of Bond *A* is the sum of the PVs of each of its cash flow streams. Apply the delayed annuity formula to calculate the PV of the 16 payments of $2,000 that begin in year 7 as well as to calculate the PV of the 12 payments of $2,500 that begin in year 15. Because the payments are made semiannually, the delayed annuities begin in periods 13 and 29, respectively. Applying the annuity formula will yield the PV of a stream as of one period prior to its first payment. Thus, applying the annuity formula will yield the PV of the streams as of periods 12 and 28, respectively. To find the PV as of today (year 0) discount those streams back 12 and 28 periods, respectively. Also, discount the payment made at maturity back 40 periods.

$P_A = C\,A^{T}_{r} / (1+r)^{12} + C\,A^{T}_{r} / (1+r)^{28} + F / (1+r)^{40}$
$= \$2,000\,A^{16}_{0.06} / (1.06)^{12} + \$2,500\,A^{12}_{0.06} / (1.06)^{28} + \$40,000 / (1.06)^{40}$
= **$18,033.86**

The price of Bond *A* is $18,033.86.

b. Discount Bond *B*'s face value back 40 periods at the semiannual interest rate.

$P_B = \$40,000 / (1.06)^{40}$
= **$3,888.89**

The price of Bond *B* is $3,888.89.

5.11 a. **True.** The bond with the shortest maturity is the ATT 5 1/8, which matures in 2003. Its closing price is 100, or 100 percent of the $1,000 face value.

b. **True.** The coupon rate of the bond maturing in 2018 is nine percent. The coupon payment is $90 (=$1,000 × 0.09).

c. **True.** The price of the bond on February 10, 2002 was 107 3/8. Since that price marked a 1/8 decline from the day before, the price on February 9, 2002 was 107 4/8, or $1,075.

d. **False.** The current yield is the annual coupon payment divided by the price of the bond. For the AT&T bond maturing in 2002, the current yield is 6.84 percent (=$71.25 / $1,041.25).

e. **True.** Since the bond is priced at a premium, the coupon rate must be higher than the current yield to maturity.

5.12 a. **True.** Since the bond is priced at a discount, the yield to maturity must be greater than the bond's coupon rate.

b. **False.** The closing price of the bond on Thursday, April 22, 2002 is 100 3/8, or $1,003.75. Since that price marked a 1/8 decline from the day before, the close on April 21, 2002 is 100 1/2, or $1,005.

c. **True.** The coupon rate is 7.5 percent. Thus, the annual coupon payment is $75 (=$1,000 × 0.075).

d. **True.** The current yield is the annual coupon payment divided by the price of the bond. The current yield is 0.0729 (=$75 / $1,028.85).

5.13 The price of a share of stock is the PV of its dividend payments. Since a dividend of $2 was paid yesterday, the next dividend payment, to be received one year from today, will be $2.16 (=$2 × 1.08). The dividend for each of the two successive years will also grow at eight percent.

$$\text{PV(Year 1 – 3)} = Div_1 / (1+r) + Div_2 / (1+r)^2 + Div_3 / (1+r)^3$$
$$= \$2.16 / (1.12) + \$2.33 / (1.12)^2 + \$2.52 / (1.12)^3$$
$$= \$5.58$$

The dividend at year 4 is $2.62 since the $2 dividend that occurred yesterday has grown three years at eight percent and one year at four percent [=$2 × $(1.08)^3$ × 1.04]. Applying the perpetuity formula to the dividends that begin in year 4 will generate the PV as of the end of year 3. Discount that value back three periods to find the PV as of today, year 0.

$$\text{PV(Year 4 - } \infty) = [Div_4 / (r – g)] / (1+r)^3$$
$$= [\$2.62 / (0.12 – 0.04)] / (1.12)^3$$
$$= \$23.31$$

The price of the bond is the sum of the PVs of the first three dividend payments and the PV of the dividend payments thereafter.

$$P = Div_1 / (1+r) + Div_2 / (1+r)^2 + Div_3 / (1+r)^3 + [Div_4 / (r – g)] / (1+r)^3$$
$$= \$2.16 / (1.12) + \$2.33 / (1.12)^2 + \$2.52 / (1.12)^3 + [\$2.62 / (0.12 – 0.04)] / (1.12)^3$$
$$= \mathbf{\$28.89}$$

The price of the stock is $28.89.

5.14 a. **True.** The dividend yield is the dividend payment divided by the price of the stock.

$$\text{Dividend Yield} = Div_1 / P_0$$
$$= \$1.8 / \$115$$
$$= \mathbf{0.0156}$$

b. **False.** On February 11, 2002, the stock closed at \$115, marking a \$1.25 decline from the previous day's close. Thus, on February 10, 2002, the stock's closing price was \$116.25.

c. **True.** The closing price of the stock was \$115 on February 11, 2002.

d. **True.** Set the price-earnings ratio (P/E) of 30 equal to the stock's price (P) divided by the earnings per share (EPS). Solve for earnings.

$$P/E = P_0 / EPS$$
$$30 = \$115 / EPS$$
$$EPS = \$115 / 30$$
$$EPS = \mathbf{\$3.83}$$

5.15 Use the growing perpetuity formula to price the stock. The first dividend payment is \$1.39 (=\$1.30 × 1.07). The dividend of \$1.30 was paid yesterday, and thus, does not figure into today's stock price. Solve for the discount rate, *r*.

$$P = Div_1 / (r - g)$$
$$\$98.13 = \$1.39 / (r - .07)$$
$$r = \mathbf{0.084}$$

The required return is 8.4 percent.

5.16 To find the number of shares you own, divide the total value of your shares (\$100,000) by the price per share. The price of each share is the PV of its cash flows, which include after-tax dividend payments and capital gains. You will receive pre-tax dividend payments of \$2 and \$4 in years 1 and 2, respectively.

$$PV(Dividends) = (1 - T)\ Div_1 / (1+r) + (1 - T)\ Div_2 / (1+r)^2$$
$$= (0.72)\ \$2 / (1.15) + (0.72)\ \$4 / (1.15)^2$$
$$= \$3.43$$

At the end of year 3, you will sell the stock at \$50.

$$PV(Cap.\ Gain) = C / (1+r)^3$$
$$= \$50 / (1.15)^3$$
$$= \$32.88$$

The price per share is the sum of the PV of the dividend payments and the PV of the capital gain.

$$P = (1 - T_c)\ Div_1 / (1+r) + (1 - T_c)\ Div_2 / (1+r)^2 + C / (1+r)^3$$
$$= (0.72)\ \$2 / (1.15) + (0.72)\ \$4 / (1.15)^2 + \$50 / (1.15)^3$$
$$= \$36.31$$

Divide the total value of your position by the price per share to find the number of shares held.

$$Shares = Total\ Value / Price\ Per\ Share$$
$$= \$100{,}000 / \$36.31$$
$$= \mathbf{2{,}754}$$

You own 2,754 shares.

5.17 a. Apply the constant-dividend growth model to find the price of the stock.

$$\begin{aligned} P &= Div_1 / (r - g) \\ &= \$2 / (0.12 - 0.05) \\ &= \mathbf{\$28.57} \end{aligned}$$

The price of the stock is $28.57.

b. To determine the price of the stock 10 years from today, find the PV of the stock's dividends as of year 10. The first relevant dividend is paid at year 11. That payment is equal to the original $2 dividend compounded at five percent over 10 years, $3.26 [=$(1.05)^{10} \times \$2$]. Apply the growing perpetuity formula, discounted at 12 percent and growing at five percent. Remember that the growing perpetuity formula values the cash flows as of one year prior to the first cash flow. Therefore, the result is the PV of the dividend payments as of year 10, the year at which you are valuing the stock.

$$\begin{aligned} P_{10} &= Div_{11} / (r - g) \\ &= (1.05)^{10}\ \$2 / (0.12 - 0.05) \\ &= \mathbf{\$46.54} \end{aligned}$$

The price of the stock in 10 years from today will be $46.54.

5.18 Find the PV of the dividend payments. Since the dividend of $1.15 was just paid yesterday, the dividend payment in year 1 is $1.36 (=$1.15 × 1.18). Remember to adjust the dividend payment each year for the appropriate growth rate.

$$\begin{aligned} Div_1 &= \$1.15 \times 1.18 &&= \$1.36 \\ Div_2 &= \$1.15 \times 1.18^2 &&= \$1.60 \\ Div_3 &= \$1.15 \times 1.18^2 \times 1.15 &&= \$1.84 \\ Div_4 &= \$1.15 \times 1.18^2 \times 1.15 \times 1.06 &&= \$1.95 \end{aligned}$$

Apply the growing perpetuity formula to find the PV of the dividend payments starting in year 4 and growing at six percent forever. Since the perpetuity formula yields the PV of the cash flows as of year 3, discount the perpetuity back three periods to find its value as of today.

$$\begin{aligned} P &= Div_1 / (1+r) + Div_2 / (1+r)^2 + Div_3 / (1+r)^3 + [Div_4 / (r - g)] / (1+r)^3 \\ &= \$1.36 / (1.12) + \$1.60 / (1.12)^2 + \$1.84 / (1.12)^3 + [\$1.95 / (0.12 - 0.06)] / (1.12)^3 \\ &= \mathbf{\$26.93} \end{aligned}$$

The price of the stock is $26.93.

5.19 Apply the growing perpetuity formula, discounted at 14 percent and declining at 10 percent per year, to find the PV of all the dividend payments beginning a year from now. The dividend payment a year from now is $4.50 [=$5 × (1 – 0.10)]. Add the dividend payment of $5 that is about to be paid. Since it occurs tomorrow, do not discount this payment. The PV of the dividend payments is the value of the firm's stock.

$$\begin{aligned} P &= Div_1 / (r - g) + Div_0 \\ &= \$4.50 / [0.14 - (-0.10)] + \$5 \\ &= \mathbf{\$23.75} \end{aligned}$$

The value of the firm's stock is $23.75.

5.20 The dividend payments must be discounted at the quarterly interest rate.

Quarterly Interest Rate $= 0.1 / 4$
$= 0.025$

Apply the annuity formula, discounted at 0.025, to find the PV of the first 12 quarterly payments of $1.

$$PV_{1\text{-}12} = Div_1 \, A^T_r$$
$$= \$1 \, A^{12}_{0.025}$$
$$= \$10.26$$

Next, apply the perpetuity formula to find the PV of the dividend payments that start at quarter 13 and grow at 0.5 percent each quarter. The dividend payment at quarter 13 is $1.005 (=$1 × 1.005). Since the perpetuity formula calculates the PV of the payments as of quarter 12, discount that value back 12 quarters to find the value as of today.

$$PV_{13\text{-}\infty} = [Div_{13} / (r - g)] / (1+r)^{12}$$
$$= [\$1.005 / (0.025 - 0.005)] / (1.025)^{12}$$
$$= \$37.36$$

The price of the stock is the sum of the present values of the dividend payments.

$$P = Div_1 \, A^T_r + [Div_{13} / (r - g)] / (1+r)^{12}$$
$$= \$1 \, A^{12}_{0.025} + [\$1.005 / (0.025 - 0.005)] / (1.025)^{12}$$
$$= \mathbf{\$47.62}$$

The price of the stock is $47.62.

5.21 The price of the stock is the PV of the dividend payments. Apply the discounted-dividend model to find the price of a share of stock. Since the $1.40 dividend was just paid, the first dividend payment will be $1.47 (=$1.4 × 1.05). Apply the growing perpetuity formula, discounted at 10 percent and growing at five percent per year.

$$P = Div_1 / (r - g)$$
$$= \$1.47 / (0.1 - 0.05)$$
$$= \mathbf{\$29.40}$$

The share price is $29.40.

5.22 The price of the stock is the PV of the dividend payments. Discount the dividends paid in years 3 and 4 back to year 0. Next, apply the growing perpetuity formula to the dividend payments that start in year 5. Since the growing perpetuity formula values the dividend payment as of the end of year 4, discount that value back 4 years to calculate the PV of the dividend payment as of today, year 0. Remember that the dividend in year 5 is $2.12 (=$2 × 1.06).

$$P = Div_3 / (1+r)^3 + Div_4 / (1+r)^4 + [Div_5 / (r - g)] / (1+r)^4$$
$$= \$2.00 / (1.16)^3 + \$2.00 / (1.16)^4 + [\$2.12 / (0.16 - 0.06)] / (1.16)^4$$
$$= \mathbf{\$14.09}$$

The share price is $14.09.

5.23 Discount each future dividend payment. The dividend payment at the end of year 1 is $5.99 (=$5.25 × 1.14). The dividend at the end of year 2 is $6.64 (=$5.25 × 1.14 × 1.11), and so on. After the annual dividend growth rate reaches 5 percent, apply the growing perpetuity formula to find the PV of the future payments. Remember to discount the value of the growing perpetuity back four periods because it values the stream as of one year before the first payment at date 5.

$Div_1 = \$5.25 \times 1.14 = \$5.99$
$Div_2 = \$5.25 \times 1.14 \times 1.11 = \$6.64$
$Div_3 = \$5.25 \times 1.14 \times 1.11 \times 1.08 = \$7.17$
$Div_4 = \$5.25 \times 1.14 \times 1.11 \times 1.08 \times 1.05 = \$7.53$
$Div_5 = \$5.25 \times 1.14 \times 1.11 \times 1.08 \times 1.05 \times 1.05 = \$7.91$

$$P = Div_1 / (1+r)^1 + Div_2 / (1+r)^2 + Div_3 / (1+r)^3 + Div_4 / (1+r)^4 + [Div_5 / (r - g)] / (1+r)^4$$
$$= (\$5.99) / (1.14) + (\$6.64) / (1.14)^2 + (\$7.17) / (1.14)^3 + (\$7.53) / (1.14)^4 + [(\$7.91) / (0.14 - 0.05)] / (1.14)^4$$
$$= \mathbf{\$71.70}$$

A share of Webster stock is $71.70.

5.24 Express the price of Allen's stock as the PV of the dividend payments.

The PV of the first two dividend payments can be expressed as follows.

$$PV = Div / (1+r) + Div / (1+r)^2$$
$$= Div / (1.12) + Div / (1.12)^2$$

Apply the growing perpetuity formula, discounted at 12 percent and growing at four percent, to find the PV of the dividend payments that begin in year 3. Remember that the dividend paid at year 3 is four percent greater than the previous year's dividend. Thus, the payment at year 3 is (1.04) × *Div*. Since the growing perpetuity formula values a stream as of one year prior to the first cash flow, discount the value of the growing perpetuity back two years to find the PV as of today.

$$PV = [(1+g)\ Div / (r - g)] / (1+r)^2$$
$$= [(1.04)\ Div / (0.12 - 0.04)] / (1.12)^2$$

Set the current market price of the stock equal to the PV of all of the stock's future dividend payments. Solve for *Div*.

$$P = Div / (1+r) + Div / (1+r)^2 + [(1+g)\ Div / (r - g)] / (1+r)^2$$
$$\$30 = Div / (1.12) + Div / (1.12)^2 + [(1.04)\ Div / (0.12 - 0.04)] / (1.12)^2$$
$$Div = \mathbf{\$2.49}$$

The expected dividend payment next year is $2.49.

5.25 a. The growth rate of a firm's earnings is equal to the retention ratio of the firm times the return on equity. Applying this formula, we find a growth rate of:

$$g = \text{Retention Ratio} \times \text{Return on Equity}$$
$$= (0.60)\ (0.14)$$
$$= \mathbf{0.084}$$
$$= \mathbf{8.4\%}$$

The firm's growth rate is 8.4 percent.

b. Multiply the firm's current earnings ($20,000,000) by the growth rate calculated in part (*a*) to find next year's earnings.

Next Year's Earnings = Current Earnings × (1+g)
= $20,000,000 (1.084)
= **$21,680,000**

The firm's earnings next year will be $21,680,000.

5.26 Express the rate of return in terms of the dividend yield and *g*, the growth rate of dividends. The dividend yield is the next dividend payment, Div_1, divided by the current stock price, *P*. The rate of return, *r*, is equal to the sum of the dividend yield and *g*.

$$r = Div_1 / P + g$$

To solve for the growth rate, *g*, apply the formula for the growth rate of a firm's earnings.

g = Retention Ratio × Return on Retained Earnings
= (0.75) (0.12)
= 0.09

Find the dividend payment per share made yesterday. Since the retention ratio is 75 percent, the firm pays out 25 percent of its $10 million earnings as dividend payments. Thus, the total dividend payment made yesterday was $2,500,000 [=$10,000,000 × (1 – 0.75)]. To find the dividend paid per share, divide the total dividend payment by the total number of shares outstanding.

Dividend per Share = [Total Earnings × (1 – Retention Ratio)] / Number of Shares
= [$10,000,000 × (1 – 0.75)] / 1,250,000
= $2

Take note that the dividend payment *next* year is needed to solve for the rate of return, *r*. Next year, the firm's earnings and dividend will grow at the annual growth rate of nine percent, as calculated above. Thus, the dividend will be $2.18 (=$2 × 1.09). Solve for the discount rate, *r*.

$$r = Div_1 / P_0 + g$$
= $2.18 / $30 + 0.09
= **0.1627**

The rate of return on the stock is 16.27 percent.

5.27 First, determine the annual dividend growth rate over the past four years. The following equation relates the dividend paid yesterday to the dividend paid four years ago. Solve for the growth rate, *g*.

$$Div_0 = Div_{-4} (1+g)^4$$
$$\$1.66 = \$0.80 (1+g)^4$$
g = **0.2002**

For years 1 through 5, the dividend payment will grow at an annual rate of 20.02 percent. Dividends will grow at eight percent per year for the next two years.

$$Div_7 = Div_0 \times (1+g_{1\text{-}5})^5 \times (1+g_{6\text{-}7})^2$$
$$= \$1.66 \times (1.2002)^5 \times (1.08)^2$$
= **$4.82**

The dividend payment in year 7 is $4.82.

5.28 a. The price of the stock is the net present value of the company's cash flows. Apply the growing perpetuity formula to find the total PV of the firm's revenues and expenses. Remember to multiply last year's revenues and costs by the growth rate since the revenues and costs given in the problem represent last year's cash flows.

PV(Revenues) = C / (r – g)
= ($3,000,000 × 1.05) / (0.15 – 0.05)
= $31,500,000

PV (Costs) = C / (r – g)
= ($1,500,000 × 1.05) / (0.15 – 0.05)
= $15,750,000

NPV = PV(Revenues) – PV(Costs)
= C / (r – g) – C / (r – g)
= ($3,000,000 × 1.05) / (0.15 – 0.05) – ($1,500,000 × 1.05) / (0.15 – 0.05)
= $15,750,000

Divide the NPV by the number of shares to find the price per share.

P = Value of Firm / Number of Shares
= $15,750,000 / 1,000,000
= **$15.75**

The price of the stock is $15.75.

b. The value of a company is the NPV of its current operations plus the NPV of its growth opportunities (NPVGO).

The second cash flow from the new project is discounted back one period. The perpetuity formula is used to find the PV of the cash inflows. Since the perpetuity formula values the cash flows as of the end of year 1, this PV must be discounted back one year.

PV = $C_0 + C_1 / (1+r) + [C_2 / r] / (1+r)$
= -$15,000,000 – $5,000,000 / (1.15) + [$6,000,000 / 0.15] / (1.15)
= $15,434,782.61

Divide the NPV by the number of shares to find the per-share effect of the new project.

Per Share Effect = $15,434,782.61 / 1,000,000
= **$15.43**

The share price will increase by the per-share NPV of the growth opportunity.

Share Price = PV(EPS) + NPVGO
= **PV(EPS) + $15.43**

Since the NPVGO per share is $15.43, the price per share will increase by $15.43.

5.29 a. Value the firm as a "cash cow," ignoring future projects. Apply the perpetuity formula to calculate the PV of the firm's revenues. The price per share is the PV of the revenues divided by the number of shares outstanding.

PV = C_1 / r
= $100,000,000 / 0.15
= $666,666,666.67

Share Price $= \$666,666,666.67 / 20,000,000$
$= \mathbf{\$33.33}$

The share price, excluding future growth opportunities, is $33.33.

b. Calculate the NPV of the growth opportunity (NPVGO). The initial cash outlay occurs today and does not need to be discounted. Discount the cash outlay in the second year back to today (year 0). Apply the perpetuity formula to find the PV of the annual earnings as of the end of year 1. Discount that value back one year to find the PV of the annual earnings as of today.

$$NPVGO = C_0 + C_1 + [C_2 / r] / (1+r)^T$$
$$= -\$15,000,000 – \$5,000,000 / 1.15 + [\$10,000,000 / 0.15] / (1.15)$$
$$= \mathbf{\$38,623,188.41}$$

The value of the growth opportunity is $38,623,188.41.

c. Apply the NPVGO model to calculate the price of the stock. The share price is equal to the per-share value of the firm's existing operations, plus the per-share value of growth opportunities. The earnings per share of the firm's existing operations was calculated in part (*a*) and the NPV of the growth opportunity was calculated in part (*b*). Divide the NPVGO by the number of shares to find the per-share NPVGO.

Share Price = Per-Share PV(Existing Operations) + NPVGO / (Number of Shares)
$= \$33.33 + \$38,623,188.41 / 20,000,000$
$= \mathbf{\$35.26}$

The share price will be $35.26 if the firm undertakes the investment.

5.30 a. If Avalanche does not make the investment, the value of a share of stock will be the PV of its current dividend payments. Apply the perpetuity formula.

$$P = Div / r$$
$$= \$4 / 0.14$$
$$= \$\mathbf{28.57}$$

The price per share is $28.57.

b. First, calculate the growth rate of the investment return. The firm will retain 25 percent of its earnings, and will earn a 40 percent return on its investments.

$$g = \text{Retention Ratio} \times \text{Return on Retained Earnings}$$
$$= (0.25)\ (0.4)$$
$$= 0.1$$

Calculate the NPV of the investment. During year 3, twenty-five percent of the earnings will be reinvested. Therefore, $1 is invested (=$4 × .25). One year later, the shareholders receive a 40 percent return on the investment, in perpetuity. The perpetuity formula values that stream as of year 3. Since the investment opportunity will continue indefinitely and grows at 10 percent, apply the growing perpetuity formula to calculate the NPV of the investment as of year 2. Discount that value back two years to today.

$$NPVGO = [(\text{Investment} + \text{Return} / r) / (r – g)] / (1+r)^2$$
$$= [(-\$1 + \$0.40 / .14) / (0.14 – 0.1)] / (1.14)^2$$
$$= \mathbf{\$35.73}$$

The value of the stock is the PV of the firm without making the investment plus the NPV of the investment.

P = PV(EPS) + NPVGO
= $28.57 + $35.73
= **$64.30**

After the announcement of the investment, the share price is $64.30.

5.31 a. Apply the perpetuity formula to find the price of the firm, *P*. The firm currently earns $800,000 from its existing operations and will earn an additional $100,000 when it accepts the project.

P = ($800,000 + $100,000) / 0.15
= $6,000,000

To calculate the price to earnings ratio, divide the total price of the firm ($6,000,000) by its current earnings ($800,000).

P/E = $6,000,000 / $800,000
= **7.5**

The P/E ratio of Pacific Energy is 7.5.

b. Again, calculate the price of the firm. The price, *P*, of the firm is the PV of its current cash flows plus the PV of the cash flows from the project. Apply the perpetuity formula to the current cash flows as well as to the cash flows of the project.

P = Current Earnings / r + Project Earnings / r
= $800,000 / 0.15 + $200,000 / 0.15
= $6,666,666.67

To calculate the price to earnings ratio, divide the total price of the firm ($6,666,666.67) by its current earnings ($800,000).

P/E = $6,666,666.67 / $800,000
= **8.33**

The P/E ratio of U.S. Bluechips is 8.33.

5.32 a. Price = $4/0.14 = $28.57

b. $$\text{Price} = 28.57 + \frac{(-1 + 0.40/0.14)/0.04}{(1.14)^3}$$
= 28.57 + 31.33
= $59.90

c. The expected return of 14% less the dividend yield of 5% provides a capital gain yield of 9%. If there is no investment the yield is 14%.

d. $3/$59.90 = .05 and $4/$28.57 = .14 without the investment.

5.33. Using dividend model, price of a stock can be written as P = D/(k – g)
Or it can be written as P = E*PO/(k – g) where PO is the dividend payout ratio and denotes multiplication
Rearranging terms we get, P/E = PO/(k – g)
Substituting values 12 = .4/(k – g)
→ 1/(k – g) = 12/0.4
→ 1/(k – g) = 30

P = E*PO/(k – g)
Now substituting P = $32, PO = 40%, 1/(k – g) = 30 we get
32 = E*.4*30
→ E = 8/3

If the dividend payout ratio were 60%
P = E*PO/(k – g)
P = (8/3)*.6*30 = **$48**

5.34 In this problem growth is occurring from two different sources:

1. The learning curve
2. New project

We need to separately compute the value from the two difference sources

First compute the growth from *learning curve*

EPS = Earnings/total number of outstanding shares
EPS = $10 million/10 million
Therefore, EPS = $1

From the NPVGO model, P = E/(k – g) + NPVGO

→ P = 1/(0.10 – 0.05) + NPVGO
= $20 + NPVGO

Compute the NPVGO of the new project to be launched two years from now
Earnings (per share) of the firm two years from now = $1*(1 + 0.05)^2$ = 1.1025
Therefore, Initial Investment = 20% of $1.1025
I_0 = $0.22
Present value of investment made two years from now = $–0.22/(1.1)^2$ = –0.18
Earnings from the new project is a perpetuity of $0.5
Value of Earnings perpetuity = $0.5/(0.1)
= $5
Present value of Earnings perpetuity = $5/1.1^2
= $4.13

NPVGO (per share) = –0.18 + 4.13 = $3.95

Plugging in the NPVGO model we get,
P = 20 + 3.95 = **$23.95**