# Portfolios of Stocks

## Last lecture:

**We saw that risk of an *individual stock* is measured by the variance (or st. deviation) of its returns.**

**E.g., when choosing between several stocks, that stock is the riskiest which has the highest Var (or SD).**

## Today:

**How should we measure the expected return and risk of a *portfolio* of two or more stocks?**

# Expected Return on a Portfolio consisting of Stocks A and B:

$$\overline{R_{Portfolio}} = X_A \overline{R_A} + X_B \overline{R_B}$$

**Where $X_A$ = the proportion of Stock A in the portfolio.**
**$X_B$ = the proportion of Stock B in the portfolio.**

**$X_A + X_B = 1.$**

**So, the expected return on a portfolio is a weighted average of the expected returns on the individual stocks.**

## ❖ Example from last lecture:

$\overline{R}_A$=17.5% $\overline{R}_B$=5.5%

**Let's calculate the expected return on a portfolio consisting of stocks A and B in the following proportions:**

| Proportion of A ($X_A$) | Proportion of B ($X_B$) | Portfolio Expected Return ($\overline{R}_{portfolio}$) |
|---|---|---|
| 0% | 100% | 5.5 |
| 25% | 75% | 8.5 |
| 50% | 50% | 11.5 |
| 75% | 25% | 14.5 |
| 100% | 0% | 17.5 |

$\overline{R}_B$ (Hold only B) ← 5.5

$\overline{R}_A$ (Hold only A) ← 17.5

For example:

$$\overline{R_{Portfolio}} = X_A \overline{R_A} + X_B \overline{R_B}$$
$$= 25\% \times 17.5\% + 75\% \times 5.5\% = 8.5\%$$

## Variance and Standard Deviation of a Portfolio of 2 Stocks:

$$\sigma^2_{Portfolio} = X_A^2\sigma_A^2 + 2X_A X_B \sigma_{A,B} + X_B^2\sigma_B^2$$

$$\sigma^2_{Portfolio} = X_A^2\sigma_A^2 + 2X_A X_B \rho_{A,B}\sigma_A\sigma_B + X_B^2\sigma_B^2$$

$$\sigma_{Portfolio} = \sqrt{\sigma^2_{Portfolio}}$$

**Q: Is it just a weighted average of the variances of the 2 stocks (i.e., $X_A \sigma^2_A + X_B \sigma^2_B$) ??**

**A: NO!**

## ❖ Example:

**In our example with Stock A and Stock B on the last lecture we had:**

$\sigma^2_A = 668.75$
$\sigma^2_B = 132.25$
$\sigma_{A,B} = -48.75$

**For different proportions of stocks A and B in the portfolio we will get a different Portfolio Variance, $\sigma^2_{portfolio}$ .**

**For example, if we invest 25% in stock A and the rest 75% in stock B:**

$\sigma^2_{portfolio}=$
$= 0.25^2\ \sigma^2_A+2*0.25*0.75*\sigma_{A,B}+0.75^2\ \sigma^2_B$
$=0.25^2 668.75+2*0.25*0.75*(-48.75)+0.75^2 132.25$
$= 97.91$

**We can calculate portfolio variance and standard deviation for other combinations of stock A and stock B, and put the results in the following Table:**

| Proportion of A ($X_A$) | Proportion of B ($X_B$) | Portfolio Variance ($\sigma^2_{portfolio}$) | Portfolio St.Dev. ($\sigma_{portfolio}$) |
|---|---|---|---|
| 0% | 100% | 132.25 | 11.5 |
| 25% | 75% | 97.91 | 9.89 |
| 50% | 50% | 175.86 | 13.26 |
| 75% | 25% | 366.16 | 19.14 |
| 100% | 0% | 668.75 | 25.86 |

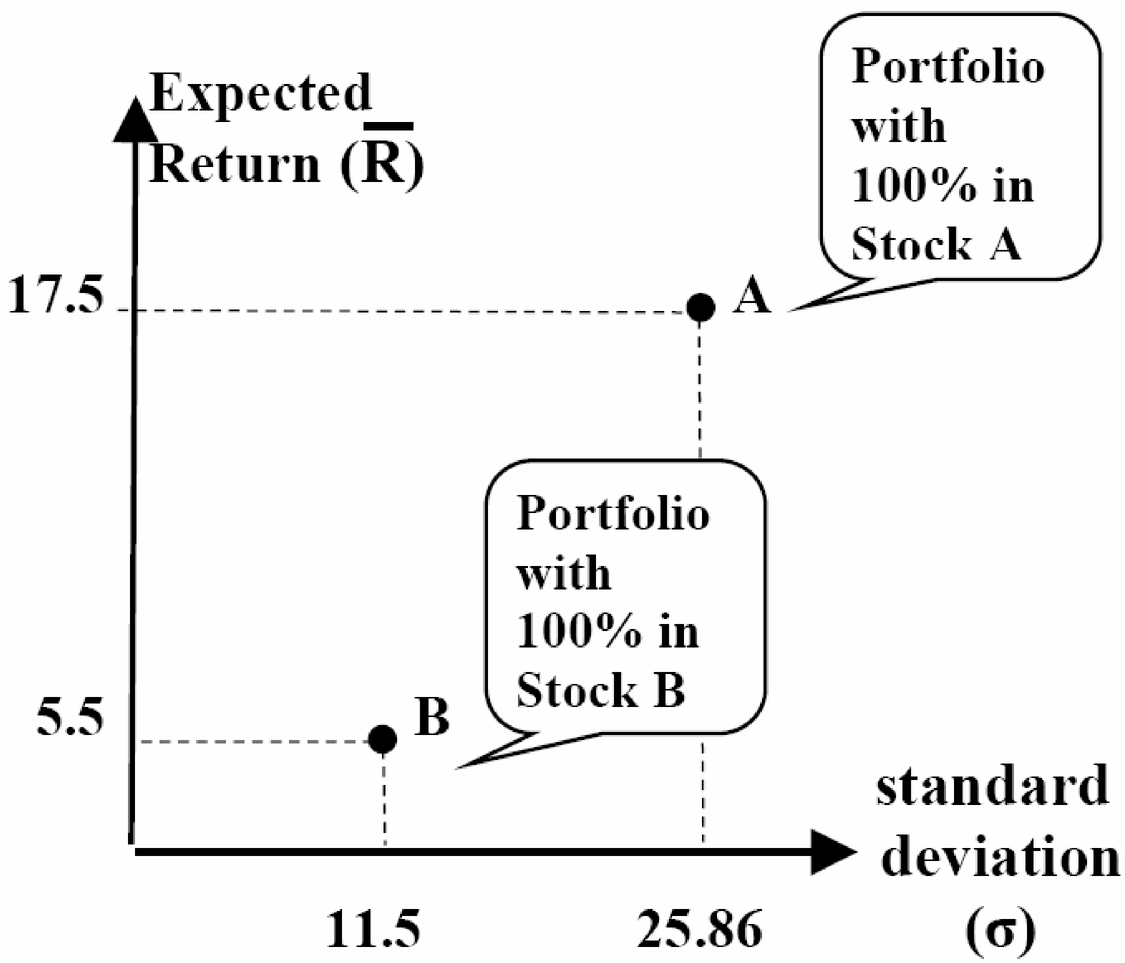
**We will be using these values to measure portfolio risk.**

**We know that for stocks A and B**

$\bar{R}_A = 17.5\%$ $\qquad$ $\bar{R}_B = 5.5\%$

$\sigma_A = 25.86\%$ $\qquad$ $\sigma_B = 11.5\%$

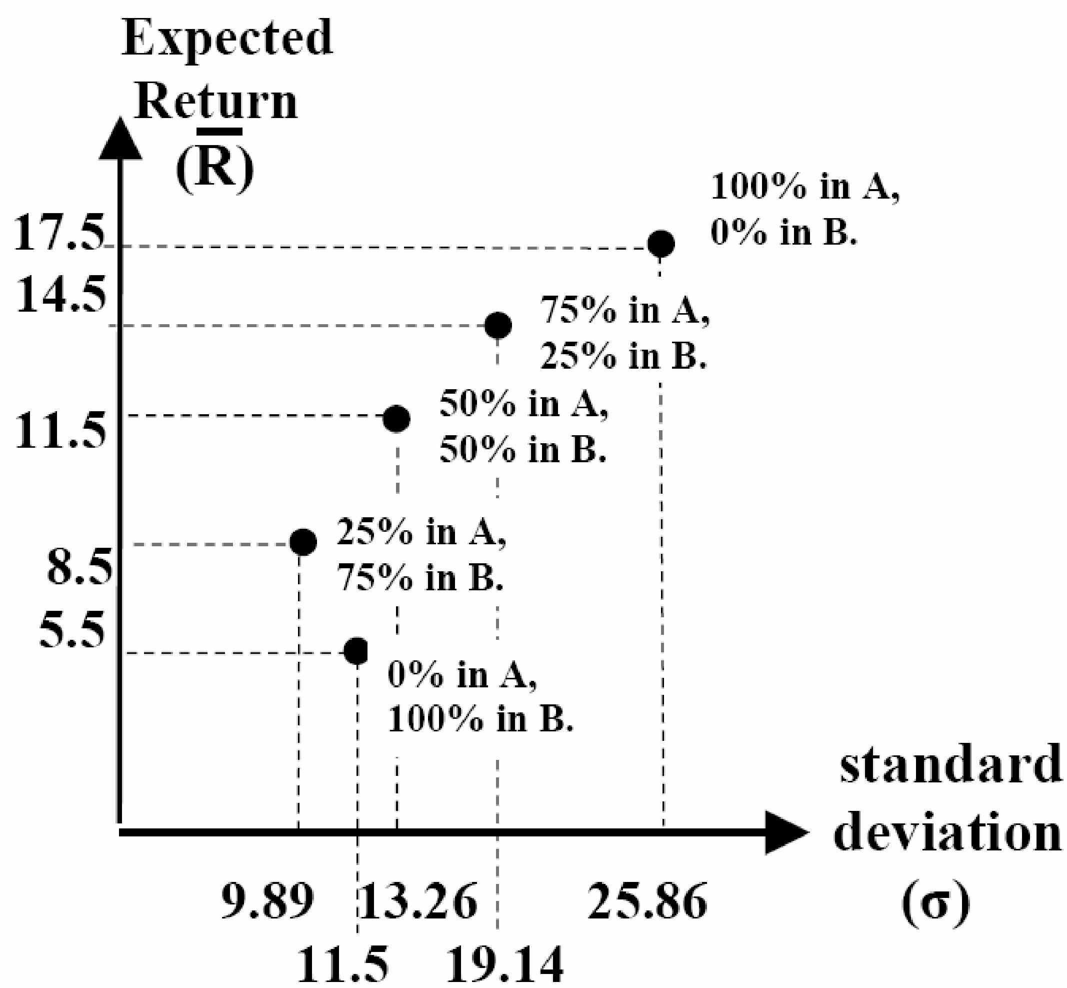
**So we have:**



**Let's plot expected returns against standard deviations for different combinations of A and B in the portfolio.**

| Proportion of A ($X_A$) | Proportion of B ($X_B$) | Portfolio Expected Return ($\overline{R}_{portfolio}$) | Portfolio Standard Deviation ($\sigma_{portfolio}$) |
|---|---|---|---|
| 0% | 100% | 5.5 | 11.5 |
| 25% | 75% | 8.5 | 9.89 |
| 50% | 50% | 11.5 | 13.26 |
| 75% | 25% | 14.5 | 19.14 |
| 100% | 0% | 17.5 | 25.86 |

# We get the following points:



**What is the importance of these mean-SD combinations?**

**– If you are risk-averse, then you would probably think that you would be better off holding just the *less risky* stock B (see the lowest point on the graph).**

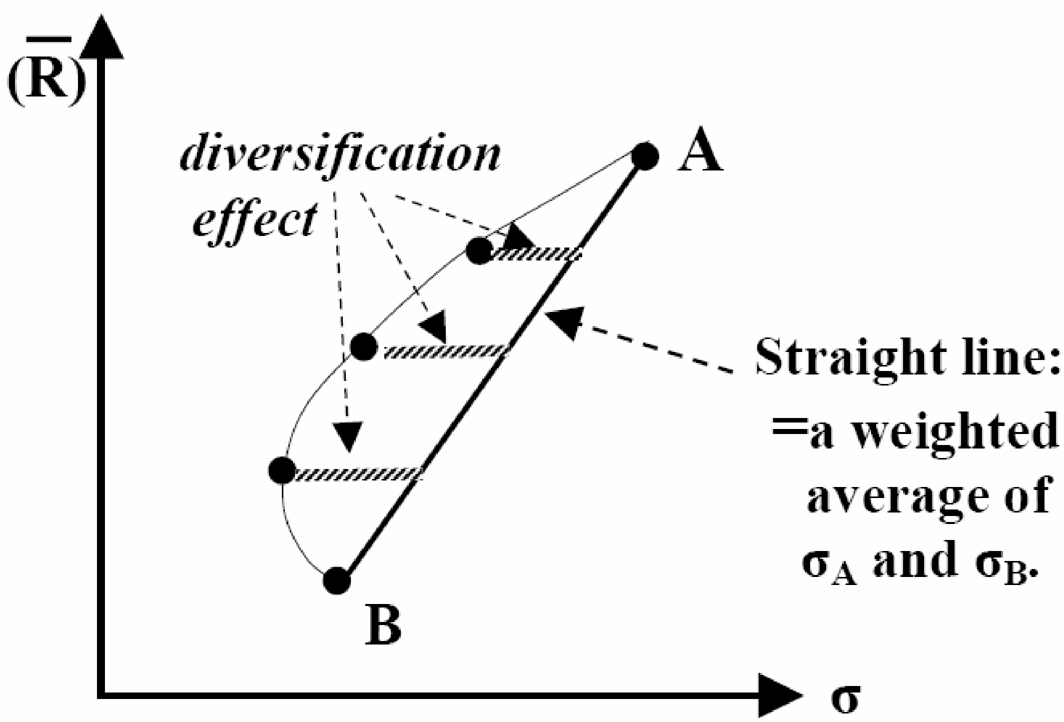
**However… If you initially hold only the less risky stock B, it is possible that when you add a *much riskier* stock A, it actually REDUCES THE OVERALL PORTFOLIO RISK!**

**➔ "Diversification effect"**

# What exactly does the **DIVERSIFICATION EFFECT** mean?

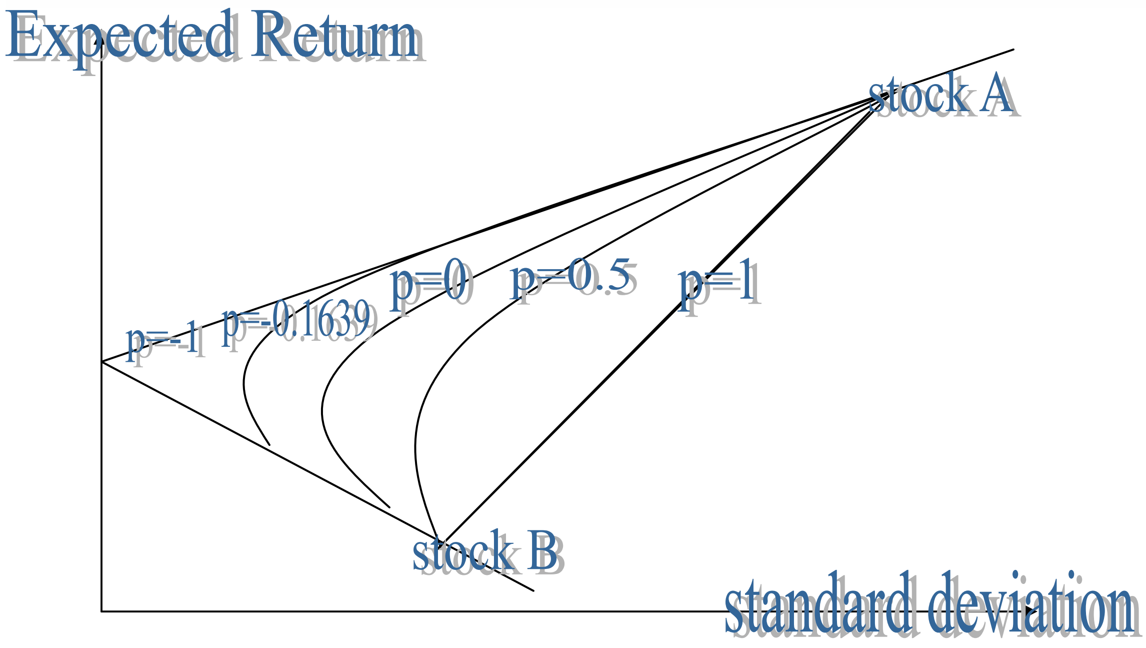
## 1.) It means that the standard deviation of the portfolio is less than a weighted average of the standard deviations of the individual securities.

- **A weighted average of the st.deviations of the individual securities would be on a straight line *between* st.deviation of A (25.86) and st.deviation of B (11.5).**



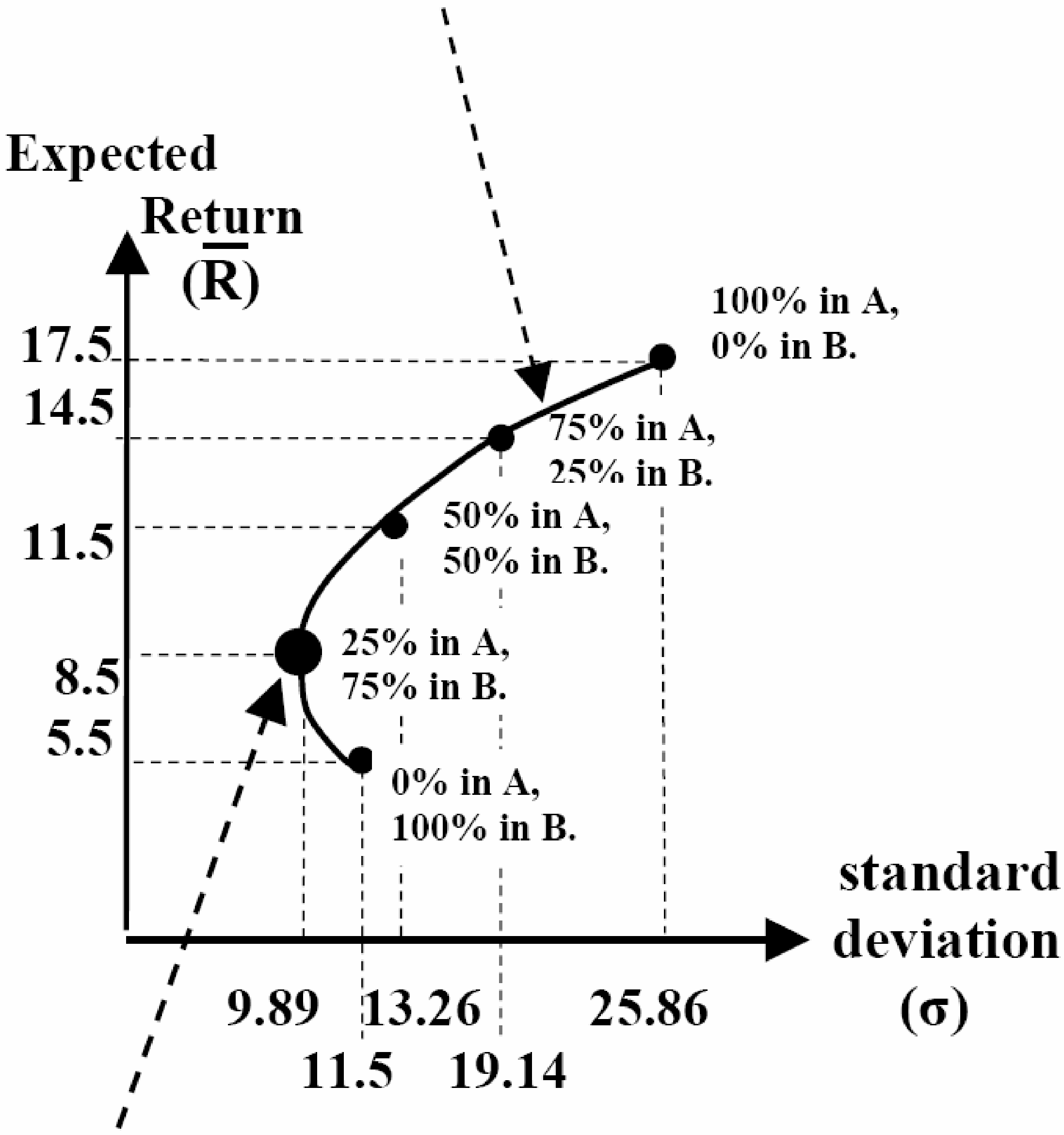
**2.) The diversification effect happens whenever the two securities are *less than perfectly correlated*.**
**In other words, when the correlation coefficient $\rho_{A,B} < 1$.**

**What happens if the correlation coefficient is zero? What if it equals –1?**



- **What else do we see on our graph?**

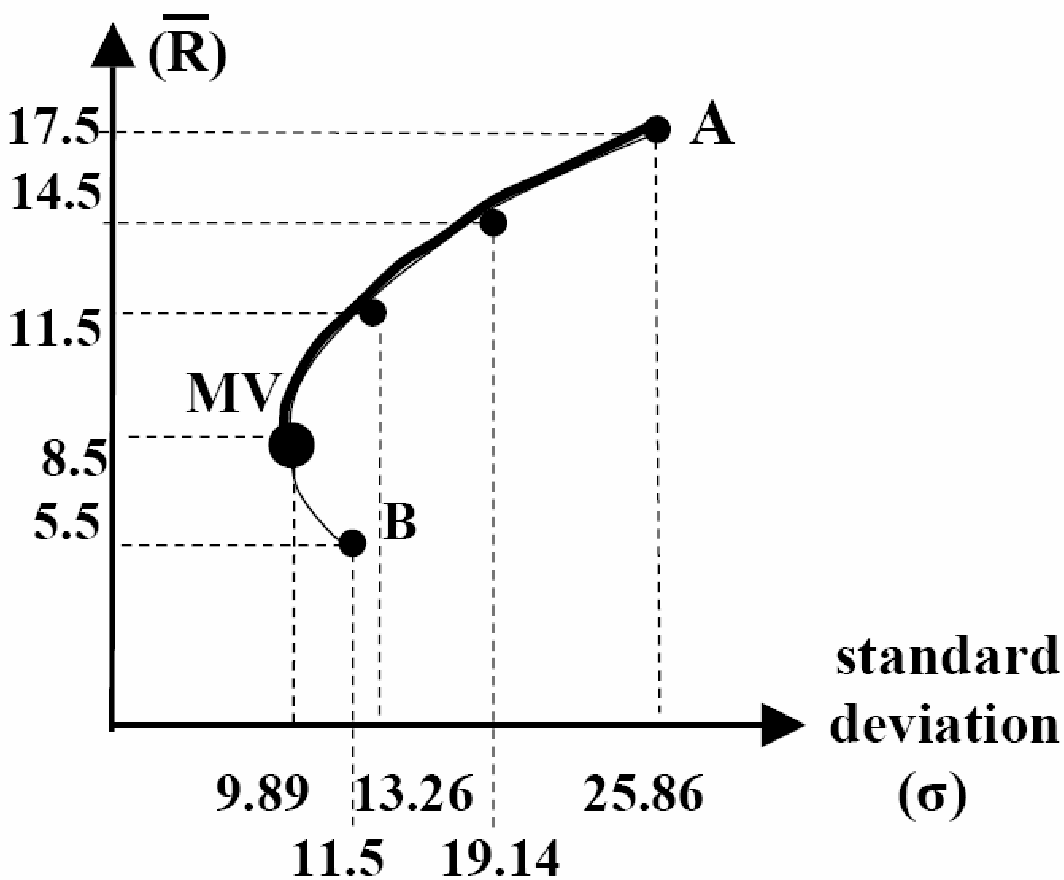
**The curve shows an**
**opportunity set** or **feasible set**




**minimum variance portfolio (MV)**

Which points on the feasible set are **"efficient points"** ?

It's all combinations of the two assets on the upward-sloping portion of the curve above the MV point, called the **"efficient frontier"**:



**Why would you be uninterested in holding a portfolio on the lower part of the curve??**

**Because you want to *maximize the expected return of your portfolio for any given level of risk*.**

**Individuals will select combinations of assets on the efficient frontier, which are also called** efficient (or optimal) portfolios**.**

**Do we know which efficient portfolio an individual would want to hold?**

**– Well, this depends on how *risk-averse* the individual is.**

## Many Securities Case:

**The Expected return of the portfolio is weighted average of the portfolio's component's returns.**

**But the variance is not!!!**
**(See matrix used to calculate the variance on Page 271, Table 10.4)**

**The variance of the return on a portfolio is more dependent on the covariance between the individual securities than on the variances of the individual securities.**

# Graph:

**Same idea.**

**Except that now all possible combinations of securities are represented by the whole area within the feasible set. The efficient set is the upper edge of the entire area.**