Ch. 10, Portfolios of Stocks

Risky Assets Only

Previously…

Risk for a stock: Variance?

Is that the correct measure?

How do we measure the risk and return of a stock when added to a portfolio of multiple stocks?

Expected return of a two stock portfolio

$$\overline{R_{portfolio}} = X_A \overline{R}_A + X_B \overline{R}_B$$

X_A = the proportion of Stock A in the portfolio.
X_B = the proportion of Stock B in the portfolio.

$$X_A + X_B = 1.$$

This is a weighted average of the returns.

Why is that the case?

Suppose we have two stocks and two equiprobable states. X_A=75%

	R_A	R_B	R_{port}
State 1	**12%**	**20%**	**14%**
State 2	**8%**	**4%**	**7%**
AVG	**10%**	**12%**	**10.5%**

$$\overline{R_{port}} = R_{port1} \cdot prob_1 + R_{port2} \cdot prob_2$$
$$= \left(X_A R_{A1} + X_{B1} R_B\right) \cdot prob_1 + \ldots$$
$$= X_A R_{A1} \cdot prob_1 + X_B R_{B1} \cdot prob_1 + \ldots$$
$$= X_A \overline{R_A} + X_B \overline{R_B}$$

From last lecture: R_A=17.5%, R_B=5.5%

X_A	X_B	E[R]
0%	100%	5.5%
25%	75%	8.5%
50%	50%	11.5%
75%	25%	14.5%
100%	0%	17.5%
125%	-25%	20.5%

$$\overline{R_{portfolio}} = X_A \overline{R_A} + X_B \overline{R_B}$$

$$= 25\% \cdot 17.5\% + 75\% \cdot 5.5\% = 8.5\%$$

Variance and SD of two-stock portfolios

$$\sigma^2_{portfolio} = X_A^2 \sigma_A^2 + 2 X_A X_B \sigma_{AB} + X_B^2 \sigma_B^2$$

$$\sigma^2_{portfolio} = X_A^2 \sigma_A^2 + 2 X_A X_B \rho_{AB} \sigma_A \sigma_B + X_B^2 \sigma_B^2$$

$$\sigma_{portfolio} = \sqrt{\sigma^2_{portfolio}}$$

Note the cross term with the covariance (or correlation coefficient).

Variance and SD example:

Recall: σ^2_A=668.75, σ^2_B=132.25, $\sigma_{A,B}$= -48.75

X_A	X_B	$\sigma^2_{portfolio}$	$\sigma_{portfolio}$
0%	100%	132.25	11.5
25%	75%	97.91	9.89
50%	50%	175.86	13.26
75%	25%	336.16	19.14
100%	0%	668.75	25.86

$$\sigma^2_{portfolio} = .25^2\sigma^2_A + 2\cdot .25\cdot .75\cdot \sigma^2_{AB} + .75^2\sigma^2_B$$

$$= .25^2(668.75) + .375(-48.75) + .75^2(132.25)$$

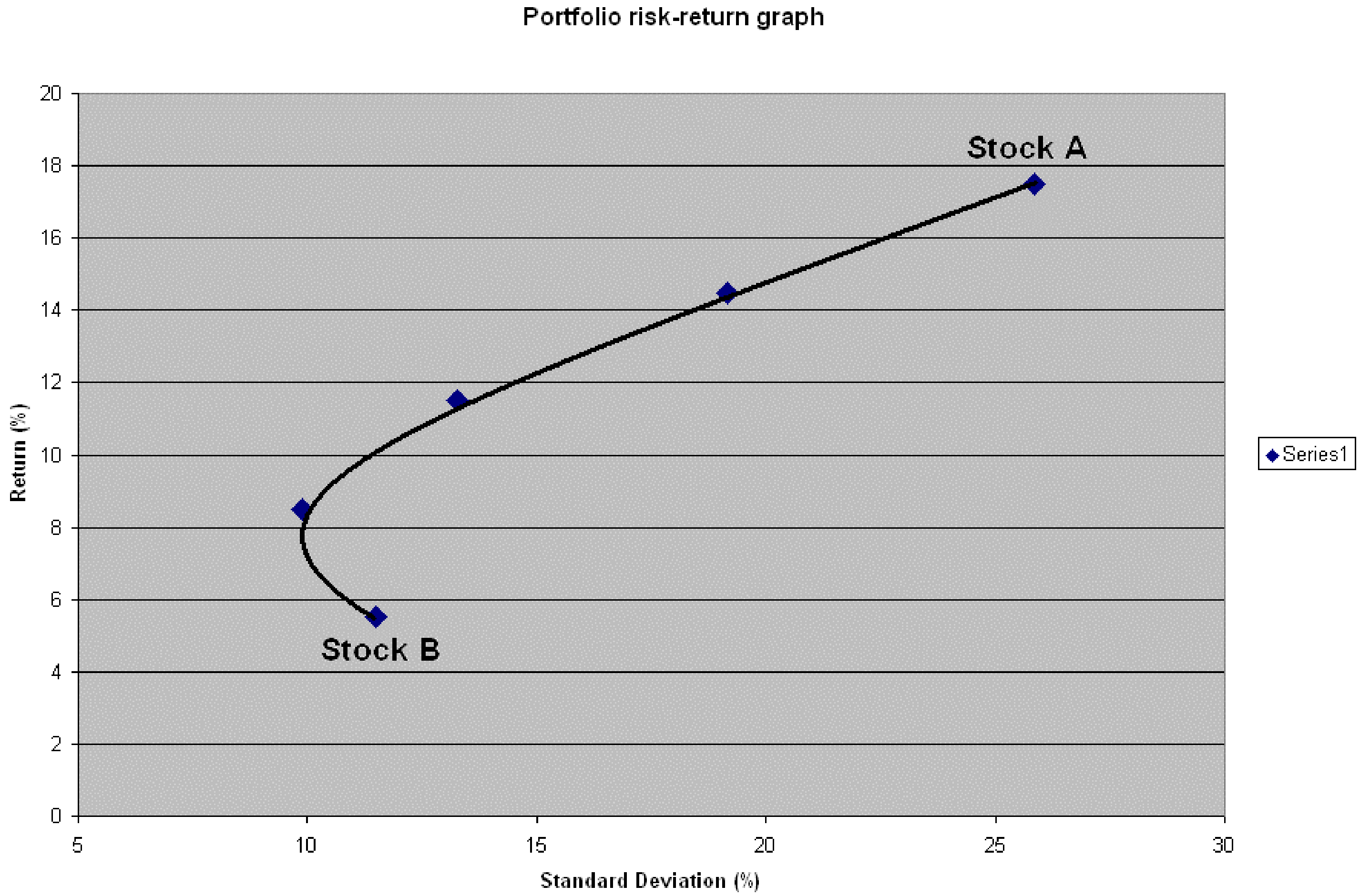
$$= 97.91$$

Summary of results

Average return is the correct measure of return for the portfolio, while the standard deviation is the correct measure of risk for a portfolio.

X_A	X_B	E[R]	SD
0%	100%	5.5	11.5
25%	75%	8.5	9.89
50%	50%	11.5	13.26
75%	25%	14.5	19.14
100%	0%	17.5	25.86

Graphing;



Portfolio risk-return graph
Stock A
Stock B
Return (%)
Standard Deviation (%)
Series1
0
2
4
6
8
10
12
14
16
18
20
5
10
15
20
25
30

Diversification Effects

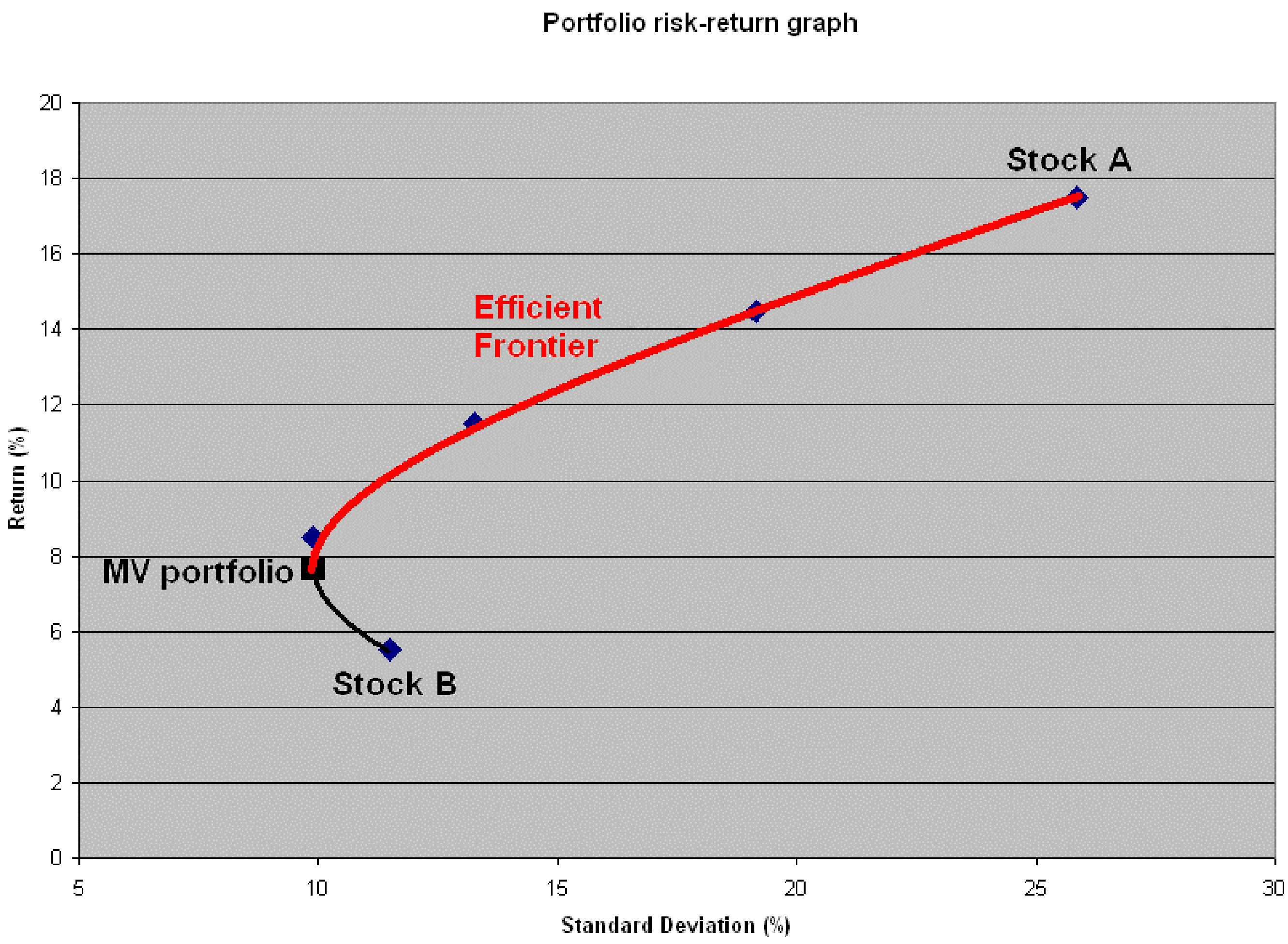
(Or, which stock to pick, finally!)

If you were strongly risk averse, you might think you would be best off selecting the less risky asset, Stock B.

However, that is unambiguously a bad choice.

Why?

Efficient Frontier and MV portfolio

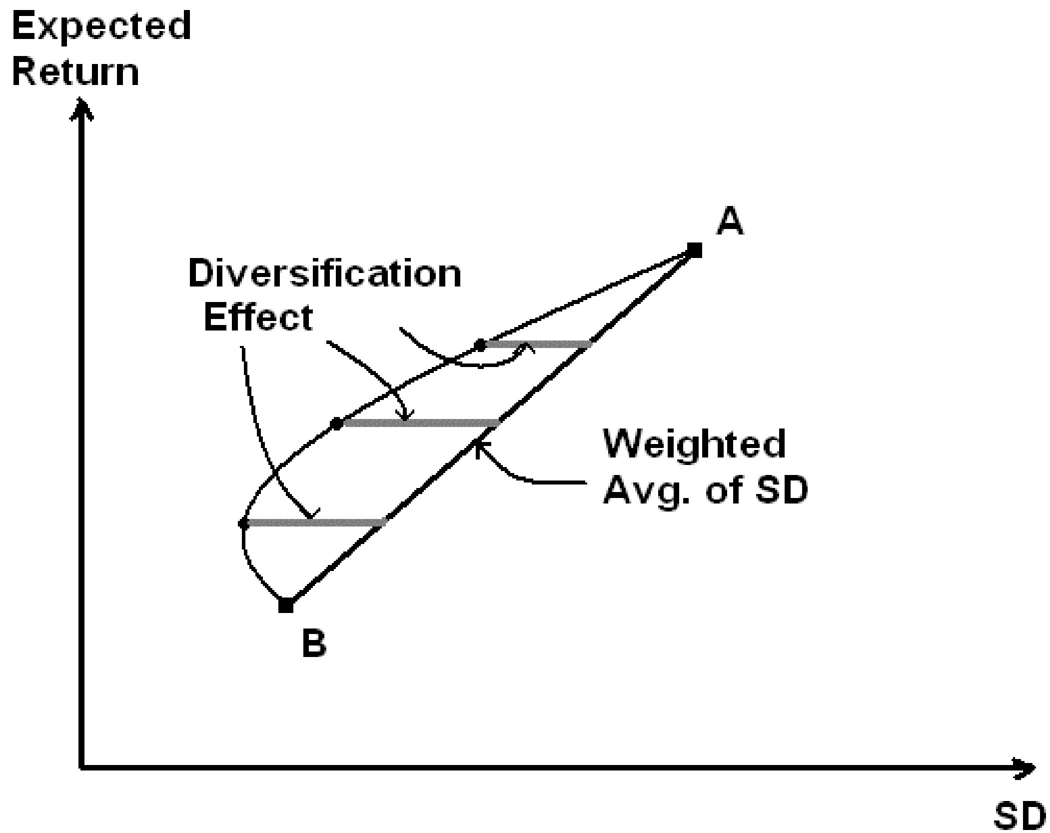


Definitions from the previous graph

- Minimum Variance (MV) portfolio
- Feasible Set (or Opportunity set)
- Efficient Frontier

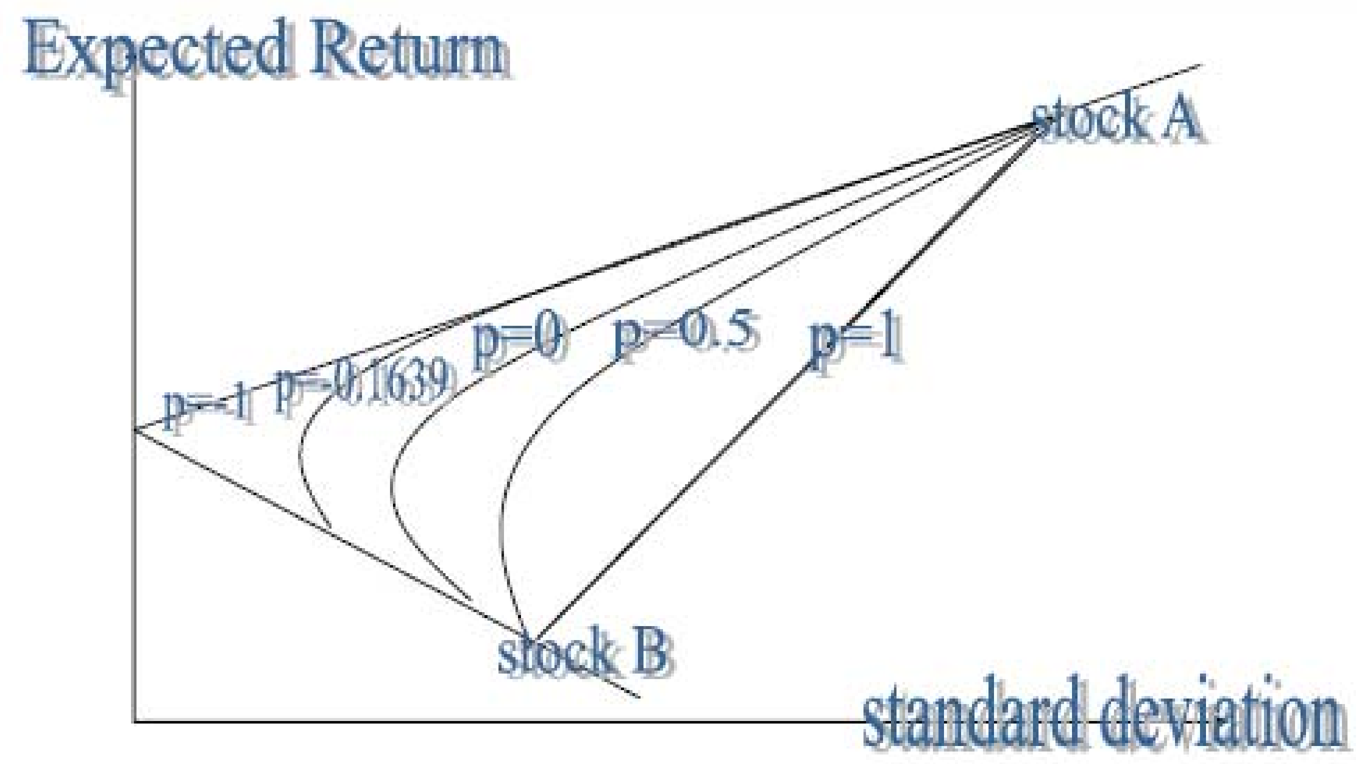
Diversification Effect Take II

The SD of the portfolio is less than the weighted average of the SD of the individual securities.



This happens whenever the correlation between the securities is less than one.

Diversification Effect for various ρ's



Multiple security case:

For a portfolio we would like to know:

- Return
- Variance

'Strange result':

The variance of the return of the portfolio is more dependent on the covariance between the securities than on the variances

Variance of multiple stock portfolios

	Stock A	Stock B	Stock C	Stock D	Stock E
Stock A	**Var(A)**	Cov(AB)	Cov(AC)	Cov(AD)	Cov(AE)
Stock B	Cov(BA)	**Var(B)**	Cov(BC)	Cov(BD)	Cov(BE)
Stock C	Cov(CA)	Cov(CB)	**Var(C)**	Cov(CD)	Cov(CE)
Stock D	Cov(DA)	Cov(DB)	Cov(DC)	**Var(D)**	Cov(DE)
Stock E	Cov(EA)	Cov(EB)	Cov(EC)	Cov(ED)	**Var(E)**

Each entry is weighted by the share of each stock in the portfolio and then summed. If all variances and covariances are equal and have an equal weighting of stocks, we get:

$$Var(port) = \left(n \cdot Var + \left(n^2 - n\right) \cdot Cov\right) \cdot \frac{1}{n^2}$$

$$= \frac{1}{n} Var + \left(1 - \frac{1}{n}\right) Cov$$

$$as\ n \to \infty,$$

$$= Cov$$

Multiple Stock Graph

Same idea, but now the entire shaded region is feasible. There still will be a MV portfolio, and the efficient frontier is the upper edge of the region