Chapter 9, Review of Statistics

And related content connecting statistics to the stock market

Tools for Chapters 10, 12.

An Introduction to Risk & Return

Single Asset Holding:

Rate of return:

If initial wealth is $1 and final wealth is W, then

$$R = \frac{(W - I)}{I}$$

If W is known with ***certainty,*** then so is the rate of return.

Up until now, we've had certainty

In the next chapters:

We will learn how to discount *risky* cash flows

- discount *expected*, not actual, cash flows
- discount rate makes allowance for risk

We already know how to deal with risk:
the required return increases.

Uncertainty about the future

We will treat future-dated asset prices, returns, etc. as random variables.

Recall:

$$r = \frac{Div_1 + P_1 - P_0}{P_0}$$

But now, future Div_1 and P_1 are *uncertain*.

Expectation

Example 1: Toss a coin.

I give you \$1 if a coin toss comes up heads, and you give me \$2 if the coin comes up tails.

My payoff is a *random variable*:

I can either lose \$1 with probability ½ or I can gain \$2 with probability ½.

If I toss many times, my *expected payoff* would be (-\$1) ½ + (\$2) ½ = \$0.5

A Second example:

<u>Example 2</u>: (In the context of returns)

Say, we know that the one-year holding period return on a stock of Firm A for that last four years were 12%, 15%, 7% and 9%.

What is the expected return?

Generalizing the formula,

Expected Return = Mean Return

$$= \overline{R} = E[R]$$

$$= R_1 \cdot prob_1 + R_2 \cdot prob_2 + R_3 \cdot prob_3 + \ldots$$

Where R_i = return in state *i*

Each return is multiplied by the respective probability of occurrence.

Warning about probabilities:

Case 1:

If all states are *equally likely*, then

$$E[R] = \frac{R_1 + R_2 + \ldots + R_T}{T}$$

Case 2:

The states are *NOT equally likely*.

Here, the probabilities of states are different. Example:

State	Probability	Return
Depression	40%	0.08 (8%)
Boom	60%	0.14 (14%)

$$E[R] = 0.4 \cdot 0.08 + 0.6 \cdot 0.14 = 0.116$$

Variance

Motivating example:

We look at the returns on stocks of Firm A and Firm B. All four states are equally likely.

Firm A's returns:

– 20%, 10%, 30%, 50%

Firm B's returns:

– 12%, 5%, 9%, 20%

Which firm's stock would you prefer to invest in?

Looking at Returns:

Firm A's expected return is 17.5%, while Firm B has an expected return of 5.5%.

Does this mean everyone wants to invest in Firm A?

Notice Firm A's returns are more spread out around the expected return. This means that *Firm A's returns are RISKIER*!

Definition of Variance

$$Var(R) = \sigma^2 = E\left[\left(R - \overline{R}\right)^2\right]$$

$$= \left(R_1 - \overline{R}\right)^2 \cdot prob_1 + \left(R_2 - \overline{R}\right)^2 \cdot prob_2 + \left(R_3 - \overline{R}\right)^2 \cdot prob_3 + \ldots$$

Again, in the case with equally likely states, all probabilities will equal 1/T

Standard Deviation

Variance is the average *squared* deviation from the mean.

If we take the square root, we will get just the average deviation from the mean:

$$\sqrt{\sigma^2} = \sigma = SD = \text{Standard Deviation}$$

(variance and standard deviation are equivalent measures of risk.)

Calculating the statistics

Let's calculate the variance and the standard deviation of Firm A and Firm B's returns:

Firm A: $E[R_A] = 17.5\%$

Firm B: $E[R_B] = 5.5\%$

$$Var\left[R_A\right] = \frac{\left(-20-17.5\right)^2 + \left(10-17.5\right)^2 + \left(30-17.5\right)^2 + \left(50-17.5\right)^2}{4} = 668.75$$

$$SD[R_A] = \sqrt{668.75} = 25.86\%$$

$$Var\left[R_B\right] = \frac{\left(-12-5.5\right)^2 + \left(5-5.5\right)^2 + \left(9-5.5\right)^2 + \left(20-5.5\right)^2}{4} = 132.25$$

$$SD[R_B] = \sqrt{132.25} = 11.5\%$$

Covariance

Covariance between two stocks' returns is a measure of their association. It shows whether two stocks' returns are "moving together" or not. Covariance can be negative, positive or zero.

$$Cov(R_A, R_B) = \sigma_{AB}$$
$$= \left(R_{A1} - \overline{R_A}\right) \cdot \left(R_{B1} - \overline{R_B}\right) \cdot prob_1 +$$
$$+ \left(R_{A2} - \overline{R_A}\right) \cdot \left(R_{B2} - \overline{R_B}\right) \cdot prob_2 +$$
$$+ \left(R_{A3} - \overline{R_A}\right) \cdot \left(R_{B3} - \overline{R_B}\right) \cdot prob_3 + \dots$$

Correlation

Same idea as covariance, however the range is normalized. The sign will be the same, since SD > 0.

$$Corr(R_A, R_B) = \rho_{AB} = \frac{Cov(R_A, R_B)}{SD(R_A) \cdot SD(R_B)}$$

Correlation coefficients will be in the range from -1 to +1.

Note about Covariance

If we have two points, they by definition lie on a straight line, and thus the correlation coefficient is either +1 or -1.

	State 1	State 2
Stock A	25%	10%
Stock B	10%	15%
Stock C	4%	18%

Corr(AB) = -1
Corr(BC) = +1
Corr(AC) = -1

In the example:

$$Cov(R_A, R_B)$$

$$= \frac{1}{4}(-20-17.5)(5-5.5)+\frac{1}{4}(10-17.5)(20-5.5)$$

$$+\frac{1}{4}(30-17.5)(-12-5.5)+\frac{1}{4}(50-17.5)(9-5.5)$$

$$= -48.75$$

$$Corr(R_A, R_B) = \frac{\rho_{AB}}{\sigma_A \cdot \sigma_B}$$

$$= \frac{-48.75}{25.86 \cdot 11.5} = -0.1639$$

What does this mean?

The stock returns are negatively correlated.

Stock A going up tends to mean Stock B goes down

This is like what is called *hedge* in finance: the stock returns are offsetting each other.