Chapter 10, With Risk-free Asset

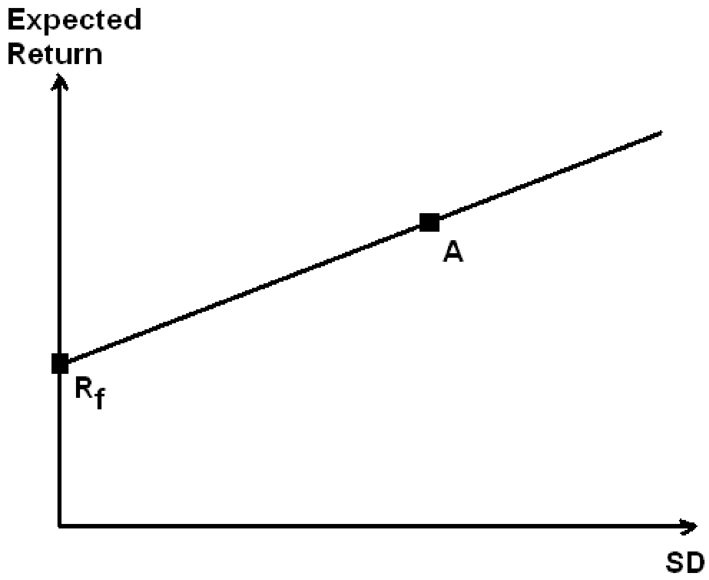
Last lecture…

We found the feasible set of risk-return pairs with two or more *risky* assets.

This lecture: We add in a risk-free asset, and see what happens to the efficient portfolios

Combinations of Risk-free and Risky Assets

Finding the combinations of the risk-free and a single risky asset:



All combinations will lie on a straight line. Why?

Weighted Averages:

The return of the portfolio is the weighted average. What about the standard deviation?

$$\sigma^2_{port} = X_{Rf}^2 \sigma^2_{Rf} + 2 X_{Rf} X_A \sigma_{RF,A} + X^2_A \sigma^2_A$$

But, we know $\sigma_{Rf} = 0,$

$$\sigma_{Rf,A} = 0$$

So the variance is

$$\sigma^2_{port} = X^2_A \sigma^2_A$$

$$\sigma_{port} = X_A \sigma_A$$

Results for the combination

The expected return and the standard deviation are:

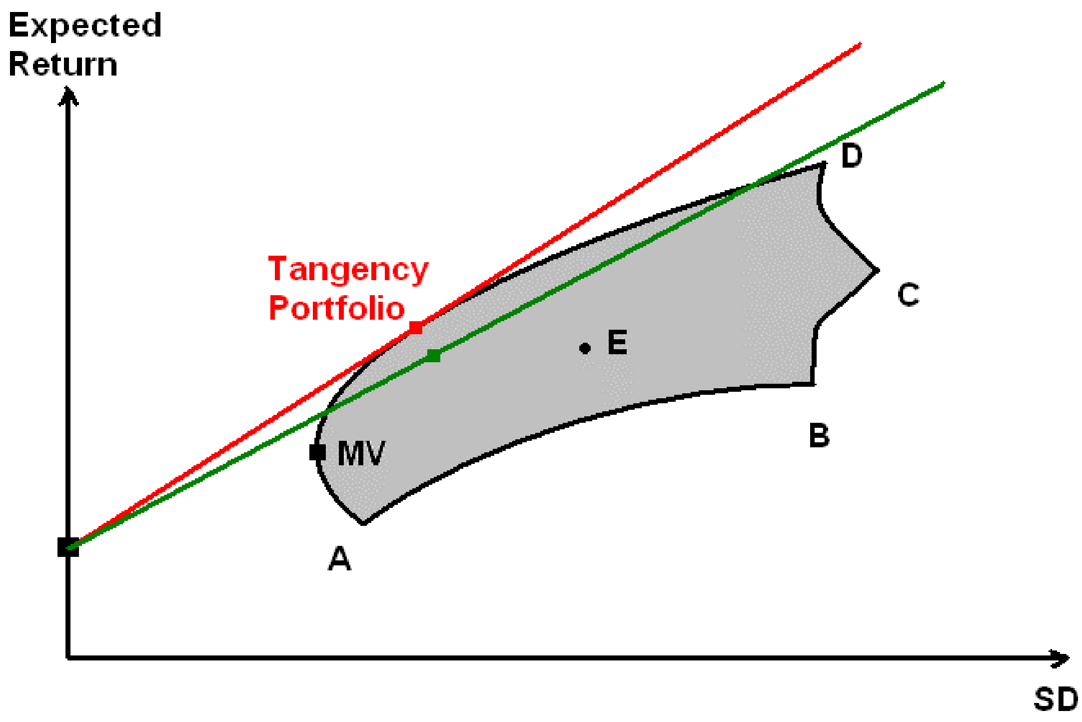
$$\overline{R_{port}} = X_A \overline{R_A} + X_{Rf} \overline{R_{Rf}}$$

$$\sigma_{port} = X_A \sigma_A + X_{Rf} \sigma_{Rf}$$

The expected return *and* the standard deviation (or variance) are the weighted average of the individual assets.

What are the best portfolios available?

As with the risky assets, investors can make a combination of all the assets.

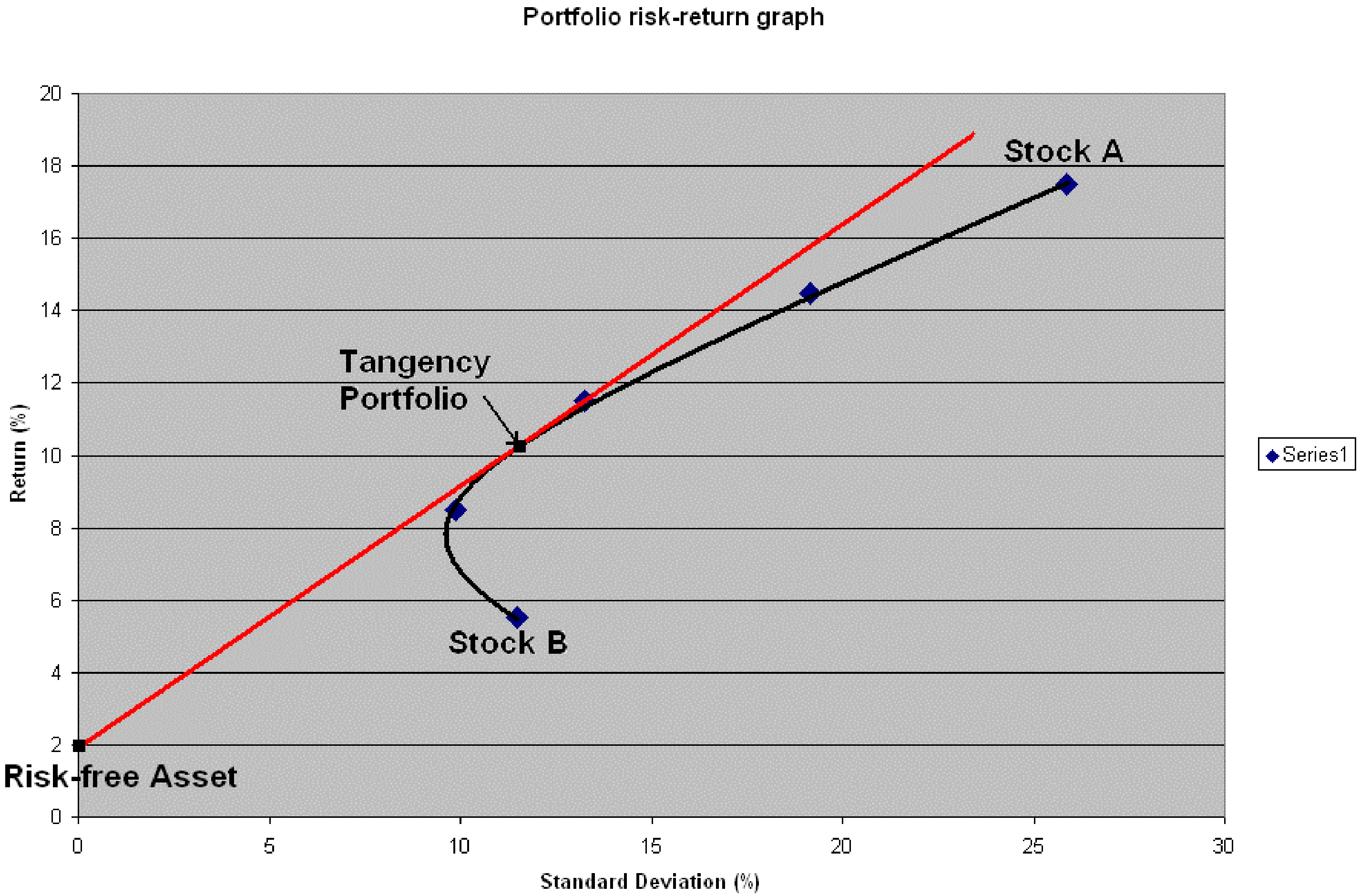


Results/Implications:

- All optimal portfolios are a weighted average (in risk and return) of the tangency portfolio and the risk-free asset.

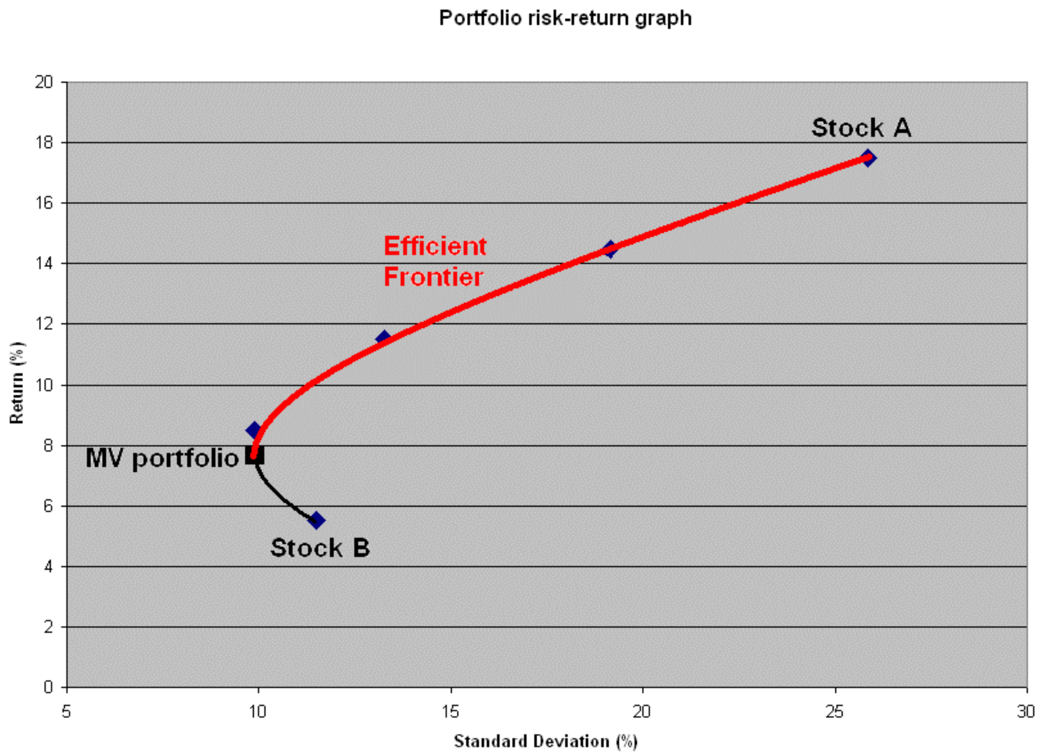
- Borrowing can occur to obtain a return higher than the tangency portfolio.

Stock A or Stock B? Both.



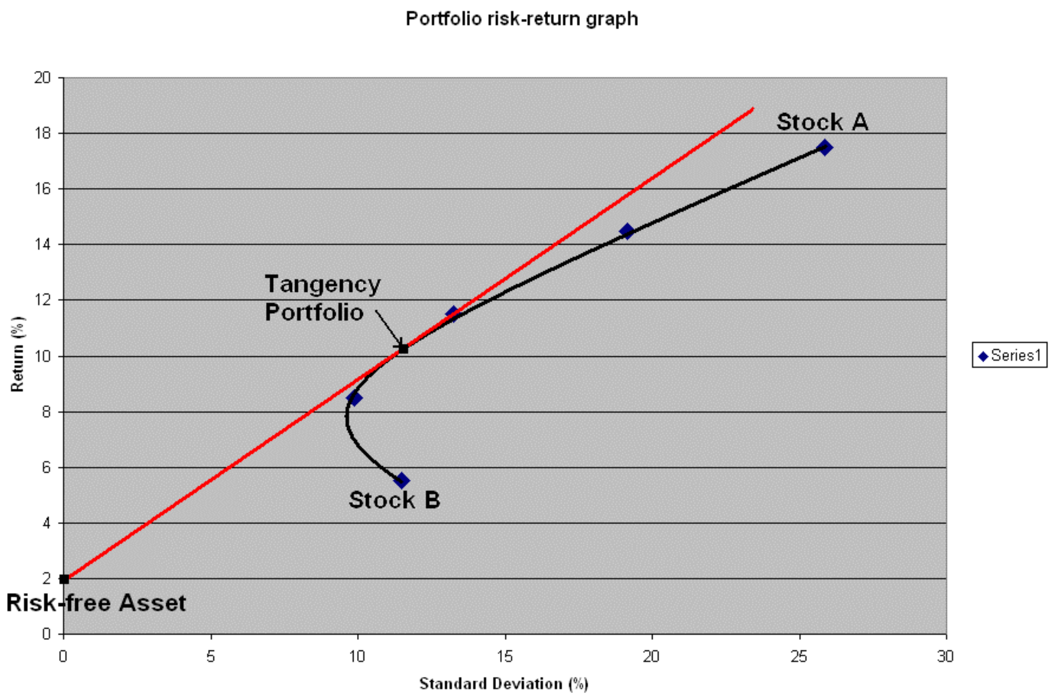
Review

Efficient portfolios with just risky assets: All lie on the feasibility frontier 'above' the MV portfolio



With a risk-free asset

The risk-free asset will help define a tangency portfolio, and all investors will select a portfolio on this straight line.



CAPM (Capital Asset Pricing Model)

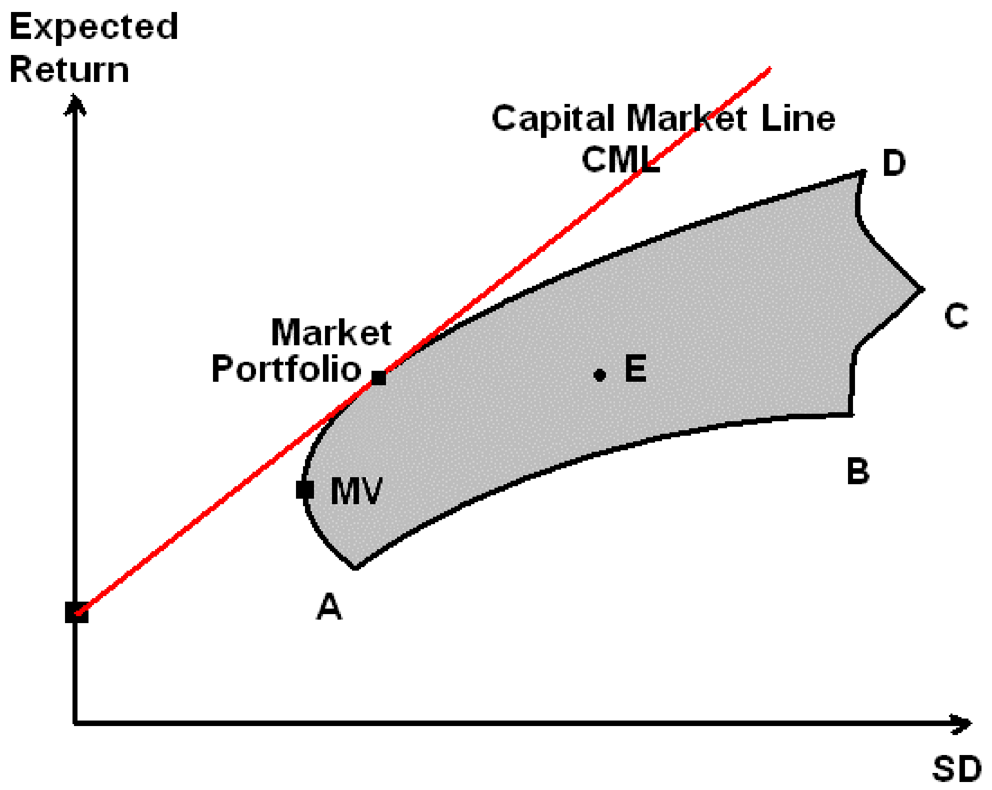
If everybody is following the previous results, the tangency portfolio becomes the market portfolio.

It will be a market weighted average of **all** existing assets. Why?

Proxy: use the Standard and Poor's 500 (S&P 500), or Wilshire 5000, or a similar index.

Problems with this?

Our graphs under this interpretation



Investors hold combinations of the risk-free asset and the market portfolio

The Main Question(s)

The main questions to answer:

- What is the correct measure of risk?
- Given this level of risk, what is the necessary level of return?

Then given these results, we will find the correct discount rate for a project by identifying a similar asset and using that asset's rate.

Risk of an individual security

Is it just the SD of the security? No!

What is more important to an investor is **the *contribution* of an individual security to the risk of the existing portfolio**.

SD is a good measure of portfolio risk, but not of risk on individual securities.

If everyone holds the market portfolio

Covariance of an asset with the market portfolio gives a good indication of the change in risk by adding the asset to the market portfolio

- If the covariance is positive, then the security is risky—it may increase the risk of the portfolio
- If the covariance is negative, then it acts as an insurance policy (reducing the risk)
- If the covariance is 0, it acts like the risk-free asset.

But we still have a scale problem.

$$\beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)}$$

Normalize the covariance by dividing by the variance of the market. This gives the **Beta** of the security.

Beta measures the responsiveness of the security to changes in the market.

Beta = 1? Beta = 0? Negative?

Simple examples/results

Beta on a risk-free asset:

$$\beta_{Rf} = \frac{0}{Var(R_M)} = 0$$

Beta on a market portfolio

$$\beta_{Market} = \frac{Cov(R_M, R_M)}{Var(R_M)}$$

$$= \frac{Var(R_M)}{Var(R_M)} = 1$$

Risk Premium

Investors demand a higher return in exchange for accepting risk. This higher return is known as the risk premium.

$$\text{Risk Premium} = \overline{R_i} - R_{rf}$$

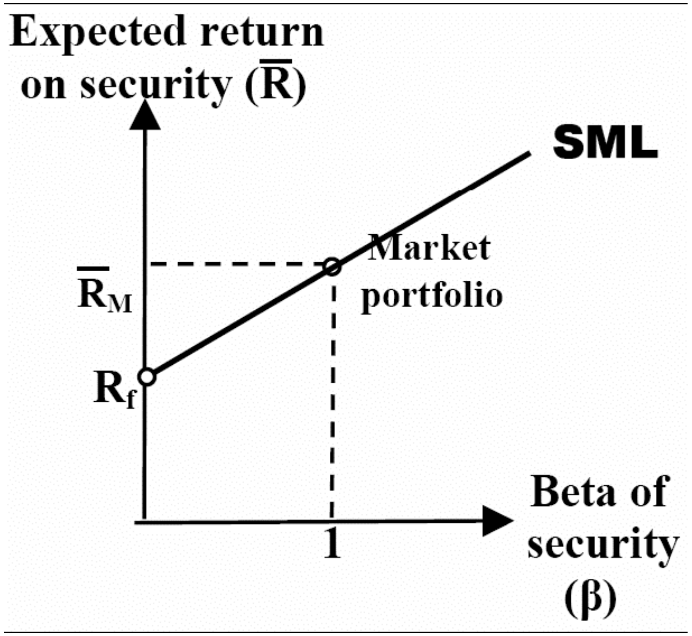
This implies:

$$\overline{R_i} = R_{rf} + \text{risk premium on i}$$

For the market portfolio:

$$\overline{R_M} = R_{rf} + \text{Expected Market Risk Premium}$$

Security Market line



This line is the Security Market Line

It gives the relationship between risk and return

Why is it a straight line?

Beta of a portfolio is the weighted avg.

$$\beta_{port} = \frac{Cov(R_{port}, R_M)}{Var(R_M)}$$

$$\begin{aligned} Cov(R_{port}, R_M) &= \left(R_{port1} - \overline{R_{port}}\right)\left(R_{M1} - \overline{R_M}\right)prob_1 + \ldots \\ &= X_i\left(R_{i1} - \overline{R_i}\right)\left(R_{M1} - \overline{R_M}\right) + X_j\left(R_{j1} - \overline{R_j}\right)\left(R_{M1} - \overline{R_M}\right) + \ldots \\ &= X_i Cov(R_i, R_M) + X_j Cov(R_j, R_M) \end{aligned}$$

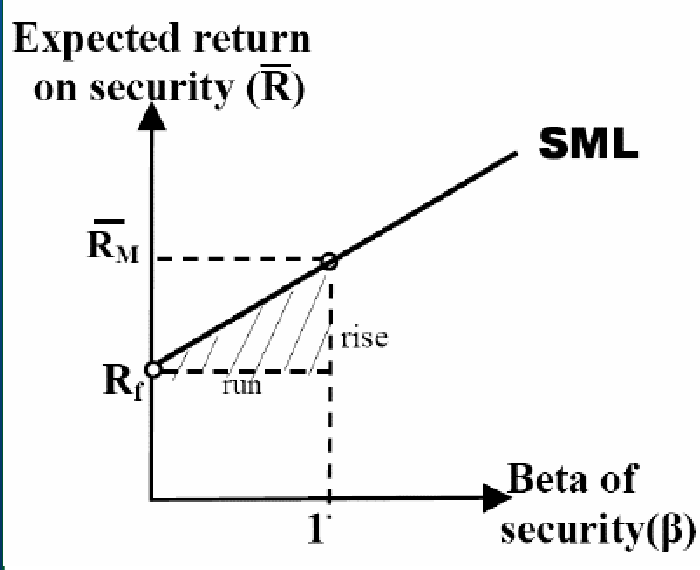
$$\beta_{port} = X_i\beta_i + X_j\beta_j$$

So the beta is a weighted average of the assets, and the return is also a weighted average.

The equation of the SML line

Intercept: R_{rf}

Slope = rise/run = $(E[R_M]-R_{rf})$ / 1 = $E[R_M]-R_{rf}$



CAPM :

$$R_i = R_{rf} + \left(\overline{R_M} - R_{rf}\right)\beta_i$$

Warnings:

- Pay attention to the X-axis variable
  - CML: Standard Deviation
  - SML: Beta

- The Slope for the SML is NOT beta, but the expected market risk premium

Example (10.19)

The market portfolio has an expected return of 12% and a standard deviation of 10%. The risk-free rate is 5%.

a. What is the expected return on a well diversified portfolio with a standard deviation of 7%?
b. If you wanted a return of 20%, what level of risk would you have to minimally accept?

Example (10.33 in text)

The risk-free rate is 7.6%. Potpourri Inc. Stock has a beta of 1.7 and an expected return of 16.7%. Assume CAPM holds

A. What is the Expected Market Risk Premium?
B. Magnolia stock has a beta of 0.8. What is the expected return?
C. Suppose you have $10,000 invested between these two companies. The beta of the portfolio is 1.07. How much did you invest in each stock? What is the expected return?