
CHAPTER 10

RISK AND RETURN: THE CAPITAL ASSET PRICING MODEL (CAPM)

Answers to Concepts Review and Critical Thinking Questions

1. Some of the risk in holding any asset is unique to the asset in question. By investing in a variety of assets, this unique portion of the total risk can be eliminated at little cost. On the other hand, there are some risks that affect all investments. This portion of the total risk of an asset cannot be costlessly eliminated. In other words, systematic risk can be controlled, but only by a costly reduction in expected returns.
2.
 - a. systematic
 - b. unsystematic
 - c. both; probably mostly systematic
 - d. unsystematic
 - e. unsystematic
 - f. systematic
3. No to both questions. The portfolio expected return is a weighted average of the asset's returns, so it must be less than the largest asset return and greater than the smallest asset return.
4. False. The variance of the individual assets is a measure of the total risk. The variance on a well-diversified portfolio is a function of systematic risk only.
5. Yes, the standard deviation can be less than that of every asset in the portfolio. However, β_p cannot be less than the smallest beta because β_p is a weighted average of the individual asset betas.
6. Yes. It is possible, in theory, to construct a zero beta portfolio of risky assets whose return would be equal to the risk-free rate. It is also possible to have a negative beta; the return would be less than the risk-free rate. A negative beta asset would carry a negative risk premium because of its value as a diversification instrument.
7. The covariance is a more appropriate measure of a security's risk in a well-diversified portfolio because the covariance reflects the effect of the security on the variance of the portfolio. Investors are concerned with the variance of their portfolios and not the variance of the individual securities. Since covariance measures the impact of an individual security on the variance of the portfolio, covariance is the appropriate measure of risk.

8. If we assume that the market has not stayed constant during the past three years, then the lack in movement of Southern Co.'s stock price only indicates that the stock either has a standard deviation or a beta that is very near to zero. The large amount of movement in Texas Instrument's stock price does not imply that the firm's beta is high. Total volatility (the price fluctuation) is a function of both systematic and unsystematic risk. The beta only reflects the systematic risk. Observing the standard deviation of price movements does not indicate whether the price changes were due to systematic factors or firm specific factors. Thus, if you observe large stock price movements like that of TI, you cannot claim that the beta of the stock is high. All you know is that the total risk of TI is high.
9. The wide fluctuations in the price of oil stocks do not indicate that these stocks are a poor investment. If an oil stock is purchased as part of a well-diversified portfolio, only its contribution to the risk of the entire portfolio matters. This contribution is measured by systematic risk or beta. Since price fluctuations in oil stocks reflect diversifiable plus non-diversifiable risk, observing the standard deviation of price movements is not an adequate measure of the appropriateness of adding oil stocks to a portfolio.
10. The statement is false. If a security has a negative beta, investors would want to hold the asset to reduce the variability of their portfolios. Those assets will have expected returns that are lower than the risk-free rate. To see this, examine the Capital Asset Pricing Model:

$$E(R_S) = R_f + \beta_S[E(R_M) - R_f]$$

If $\beta_S < 0$, then the $E(R_S) < R_f$

Solutions to Questions and Problems

NOTE: All end-of-chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The portfolio weight of an asset is total investment in that asset divided by the total portfolio value. First, we will find the portfolio value, which is:

$$\text{Total value} = 70(\$40) + 110(\$22) = \$5,220$$

The portfolio weight for each stock is:

$$\text{Weight}_A = 70(\$40)/\$5,220 = .5364$$

$$\text{Weight}_B = 110(\$22)/\$5,220 = .4636$$

2. The expected return of a portfolio is the sum of the weight of each asset times the expected return of each asset. The total value of the portfolio is:

$$\text{Total value} = \$1,200 + 1,900 = \$3,100$$

So, the expected return of this portfolio is:

$$E(R_p) = (\$1,200/\$3,100)(0.11) + (\$1,900/\$3,100)(0.16) = .1406 \text{ or } 14.06\%$$

3. The expected return of a portfolio is the sum of the weight of each asset times the expected return of each asset. So, the expected return of the portfolio is:

$$E(R_p) = .50(.11) + .30(.17) + .20(.14) = .1340 \text{ or } 13.40\%$$

4. Here we are given the expected return of the portfolio and the expected return of each asset in the portfolio and are asked to find the weight of each asset. We can use the equation for the expected return of a portfolio to solve this problem. Since the total weight of a portfolio must equal 1 (100%), the weight of Stock Y must be one minus the weight of Stock X. Mathematically speaking, this means:

$$E(R_p) = .122 = .14w_X + .09(1 - w_X)$$

We can now solve this equation for the weight of Stock X as:

$$.122 = .14w_X + .09 - .09w_X$$

$$.032 = .05w_X$$

$$w_X = 0.64$$

So, the dollar amount invested in Stock X is the weight of Stock X times the total portfolio value, or:

$$\text{Investment in X} = 0.64(\$10,000) = \$6,400$$

And the dollar amount invested in Stock Y is:

$$\text{Investment in Y} = (1 - 0.64)(\$10,000) = \$3,600$$

5. The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of the asset is:

$$E(R) = .2(-.05) + .5(.12) + .3(.25) = .1250 \text{ or } 12.50\%$$

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6. The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of each stock asset is:

$$E(R_A) = .10(.06) + .60(.07) + .30(.11) = .0810 \text{ or } 8.10\%$$

$$E(R_B) = .10(-.2) + .60(.13) + .30(.33) = .1570 \text{ or } 15.70\%$$

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the variance and standard deviation of each stock are:

$$\sigma_A^2 = .10(.06 - .0810)^2 + .60(.07 - .0810)^2 + .30(.11 - .0810)^2 = .00037$$

$$\sigma_A = (.00037)^{1/2} = .0192 \text{ or } 1.92\%$$

$$\sigma_B^2 = .10(-.2 - .1570)^2 + .60(.13 - .1570)^2 + .30(.33 - .1570)^2 = .02216$$

$$\sigma_B = (.022216)^{1/2} = .1489 \text{ or } 14.89\%$$

7. The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of the stock is:

$$E(R_A) = .10(-.045) + .20(.044) + .50(.12) + .20(.207) = .1057 \text{ or } 10.57\%$$

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the variance and standard deviation are:

$$\sigma^2 = .10(-.045 - .1057)^2 + .20(.044 - .1057)^2 + .50(.12 - .1057)^2 + .20(.207 - .1057)^2 = .005187$$

$$\sigma = (.005187)^{1/2} = .0720 \text{ or } 7.20\%$$

8. The expected return of a portfolio is the sum of the weight of each asset times the expected return of each asset. So, the expected return of the portfolio is:

$$E(R_p) = .20(.08) + .70(.15) + .1(.24) = .1450 \text{ or } 14.50\%$$

If we own this portfolio, we would expect to get a return of 14.50 percent.

9. a. To find the expected return of the portfolio, we need to find the return of the portfolio in each state of the economy. This portfolio is a special case since all three assets have the same weight. To find the expected return in an equally weighted portfolio, we can sum the returns of each asset and divide by the number of assets, so the expected return of the portfolio in each state of the economy is:

$$\text{Boom: } E(R_p) = (.07 + .15 + .33)/3 = .1833 \text{ or } 18.33\%$$

$$\text{Bust: } E(R_p) = (.13 + .03 - .06)/3 = .0333 \text{ or } 3.33\%$$

To find the expected return of the portfolio, we multiply the return in each state of the economy by the probability of that state occurring, and then sum. Doing this, we find:

$$E(R_p) = .70(.1833) + .30(.0333) = .1383 \text{ or } 13.83\%$$

- b. This portfolio does not have an equal weight in each asset. We still need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

$$\text{Boom: } E(R_p) = .20(.07) + .20(.15) + .60(.33) = .2420 \text{ or } 24.20\%$$

$$\text{Bust: } E(R_p) = .20(.13) + .20(.03) + .60(-.06) = -.0040 \text{ or } -0.40\%$$

And the expected return of the portfolio is:

$$E(R_p) = .70(.2420) + .30(-.004) = .1682 \text{ or } 16.82\%$$

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the variance and standard deviation the portfolio is:

$$\sigma_p^2 = .70(.2420 - .1682)^2 + .30(-.0040 - .1682)^2 = .012708$$

$$\sigma_p = (.012708)^{1/2} = .1127 \text{ or } 11.27\%$$

10. a. This portfolio does not have an equal weight in each asset. We first need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

$$\text{Boom: } E(R_p) = .30(.3) + .40(.45) + .30(.33) = .3690 \text{ or } 36.90\%$$

$$\text{Good: } E(R_p) = .30(.12) + .40(.10) + .30(.15) = .1210 \text{ or } 12.10\%$$

$$\text{Poor: } E(R_p) = .30(.01) + .40(-.15) + .30(-.05) = -.0720 \text{ or } -7.20\%$$

$$\text{Bust: } E(R_p) = .30(-.06) + .40(-.30) + .30(-.09) = -.1650 \text{ or } -16.50\%$$

And the expected return of the portfolio is:

$$E(R_p) = .30(.3690) + .40(.1210) + .25(-.0720) + .05(-.1650) = .1329 \text{ or } 13.29\%$$

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- b. To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the variance and standard deviation the portfolio is:

$$\begin{aligned}\sigma_p^2 &= .30(.3690 - .1329)^2 + .40(.1210 - .1329)^2 + .25(-.0720 - .1329)^2 + .05(-.1650 - .1329)^2 \\ \sigma_p^2 &= .03171\end{aligned}$$

$$\sigma_p = (.03171)^{1/2} = .1781 \text{ or } 17.81\%$$

11. The beta of a portfolio is the sum of the weight of each asset times the beta of each asset. So, the beta of the portfolio is:

$$\beta_p = .25(.6) + .20(1.7) + .15(1.15) + .40(1.34) = 1.20$$

12. The beta of a portfolio is the sum of the weight of each asset times the beta of each asset. If the portfolio is as risky as the market it must have the same beta as the market. Since the beta of the market is one, we know the beta of our portfolio is one. We also need to remember that the beta of the risk-free asset is zero. It has to be zero since the asset has no risk. Setting up the equation for the beta of our portfolio, we get:

$$\beta_p = 1.0 = 1/3(0) + 1/3(1.9) + 1/3(\beta_X)$$

Solving for the beta of Stock X, we get:

$$\beta_X = 1.10$$

13. CAPM states the relationship between the risk of an asset and its expected return. CAPM is:

$$E(R_i) = R_f + [E(R_M) - R_f] \times \beta_i$$

Substituting the values we are given, we find:

$$E(R_i) = .05 + (.14 - .05)(1.3) = .1670 \text{ or } 16.70\%$$

14. We are given the values for the CAPM except for the β of the stock. We need to substitute these values into the CAPM, and solve for the β of the stock. One important thing we need to realize is that we are given the market risk premium. The market risk premium is the expected return of the market minus the risk-free rate. We must be careful not to use this value as the expected return of the market. Using the CAPM, we find:

$$E(R_i) = .14 = .04 + .06\beta_i$$

$$\beta_i = 1.67$$

15. Here we need to find the expected return of the market using the CAPM. Substituting the values given, and solving for the expected return of the market, we find:

$$E(R_i) = .11 = .055 + [E(R_M) - .055](.85)$$

$$E(R_M) = .1197 \text{ or } 11.97\%$$

16. Here we need to find the risk-free rate using the CAPM. Substituting the values given, and solving for the risk-free rate, we find:

$$E(R_i) = .17 = R_f + (.11 - R_f)(1.9)$$

$$.17 = R_f + .209 - 1.9R_f$$

$$R_f = .0433 \text{ or } 4.33\%$$

17. a. Again, we have a special case where the portfolio is equally weighted, so we can sum the returns of each asset and divide by the number of assets. The expected return of the portfolio is:

$$E(R_p) = (.16 + .05)/2 = .1050 \text{ or } 10.50\%$$

- b. We need to find the portfolio weights that result in a portfolio with a β of 0.75. We know the β of the risk-free asset is zero. We also know the weight of the risk-free asset is one minus the weight of the stock since the portfolio weights must sum to one, or 100 percent. So:

$$\beta_p = 0.75 = w_S(1.2) + (1 - w_S)(0)$$

$$0.75 = 1.2w_S + 0 - 0w_S$$

$$w_S = 0.75/1.2$$

$$w_S = .6250$$

And, the weight of the risk-free asset is:

$$w_{Rf} = 1 - .6250 = .3750$$

- c. We need to find the portfolio weights that result in a portfolio with an expected return of 8 percent. We also know the weight of the risk-free asset is one minus the weight of the stock since the portfolio weights must sum to one, or 100 percent. So:

$$E(R_p) = .08 = .16w_S + .05(1 - w_S)$$

$$.08 = .16w_S + .05 - .05w_S$$

$$w_S = .2727$$

So, the β of the portfolio will be:

$$\beta_p = .2727(1.2) + (1 - .2727)(0) = 0.327$$

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d. Solving for the β of the portfolio as we did in part a, we find:

$$\beta_p = 2.4 = w_S(1.2) + (1 - w_S)(0)$$

$$w_S = 2.4/1.2 = 2$$

$$w_{Rf} = 1 - 2 = -1$$

The portfolio is invested 200% in the stock and -100% in the risk-free asset. This represents borrowing at the risk-free rate to buy more of the stock.

18. First, we need to find the β of the portfolio. The β of the risk-free asset is zero, and the weight of the risk-free asset is one minus the weight of the stock, the β of the portfolio is:

$$\beta_p = w_W(1.3) + (1 - w_W)(0) = 1.3w_W$$

So, to find the β of the portfolio for any weight of the stock, we simply multiply the weight of the stock times its β .

Even though we are solving for the β and expected return of a portfolio of one stock and the risk-free asset for different portfolio weights, we are really solving for the SML. Any combination of this stock, and the risk-free asset will fall on the SML. For that matter, a portfolio of any stock and the risk-free asset, or any portfolio of stocks, will fall on the SML. We know the slope of the SML line is the market risk premium, so using the CAPM and the information concerning this stock, the market risk premium is:

$$E(R_W) = .16 = .05 + \text{MRP}(1.30)$$

$$\text{MRP} = .11/1.3 = .0846 \text{ or } 8.46\%$$

So, now we know the CAPM equation for any stock is:

$$E(R_p) = .05 + .0846\beta_p$$

The slope of the SML is equal to the market risk premium, which is 0.0846. Using these equations to fill in the table, we get the following results:

w_W	$E(R_p)$	β_p
0%	.0500	0
25	.0775	0.325
50	.1050	0.650
75	.1325	0.975
100	.1600	1.300
125	.1875	1.625
150	.2150	1.950

19. There are two ways to correctly answer this question. We will work through both. First, we can use the CAPM. Substituting in the value we are given for each stock, we find:

$$E(R_Y) = .055 + .075(1.50) = .1675 \text{ or } 16.75\%$$

It is given in the problem that the expected return of Stock Y is 17 percent, but according to the CAPM, the return of the stock based on its level of risk, the expected return should be 16.75 percent. This means the stock return is too high, given its level of risk. Stock Y plots above the SML and is undervalued. In other words, its price must increase to reduce the expected return to 16.75 percent. For Stock Z, we find:

$$E(R_Z) = .055 + .075(0.80) = .1150 \text{ or } 11.50\%$$

The return given for Stock Z is 10.5 percent, but according to the CAPM the expected return of the stock should be 11.50 percent based on its level of risk. Stock Z plots below the SML and is overvalued. In other words, its price must decrease to increase the expected return to 11.50 percent.

We can also answer this question using the reward-to-risk ratio. All assets must have the same reward-to-risk ratio, that is, every asset must have the same ratio of the asset risk premium to its beta. This follows from the linearity of the SML in Figure 11.11. The reward-to-risk ratio is the risk premium of the asset divided by its β . This is also known as the Treynor ratio or Treynor index. We are given the market risk premium, and we know the β of the market is one, so the reward-to-risk ratio for the market is 0.075, or 7.5 percent. Calculating the reward-to-risk ratio for Stock Y, we find:

$$\text{Reward-to-risk ratio Y} = (.17 - .055) / 1.50 = .0767$$

The reward-to-risk ratio for Stock Y is too high, which means the stock plots above the SML, and the stock is undervalued. Its price must increase until its reward-to-risk ratio is equal to the market reward-to-risk ratio. For Stock Z, we find:

$$\text{Reward-to-risk ratio Z} = (.105 - .055) / .80 = .0625$$

The reward-to-risk ratio for Stock Z is too low, which means the stock plots below the SML, and the stock is overvalued. Its price must decrease until its reward-to-risk ratio is equal to the market reward-to-risk ratio.

20. We need to set the reward-to-risk ratios of the two assets equal to each other (see the previous problem), which is:

$$(.17 - R_f)/1.50 = (.105 - R_f)/0.80$$

We can cross multiply to get:

$$0.80(.17 - R_f) = 1.50(.105 - R_f)$$

Solving for the risk-free rate, we find:

$$0.136 - 0.80R_f = 0.1575 - 1.50R_f$$

$$R_f = .0307 \text{ or } 3.07\%$$

Intermediate

21. For a portfolio that is equally invested in large-company stocks and long-term bonds:

$$\text{Return} = (12.4\% + 5.8\%)/2 = 9.1\%$$

For a portfolio that is equally invested in small stocks and Treasury bills:

$$\text{Return} = (17.5\% + 3.8\%)/2 = 10.65\%$$

22. We know that the reward-to-risk ratios for all assets must be equal (See Question 19). This can be expressed as:

$$[E(R_A) - R_f]/\beta_A = [E(R_B) - R_f]/\beta_B$$

The numerator of each equation is the risk premium of the asset, so:

$$RP_A/\beta_A = RP_B/\beta_B$$

We can rearrange this equation to get:

$$\beta_B/\beta_A = RP_B/RP_A$$

If the reward-to-risk ratios are the same, the ratio of the betas of the assets is equal to the ratio of the risk premiums of the assets.

23. *a.* We need to find the return of the portfolio in each state of the economy. To do this, we will multiply the return of each asset by its portfolio weight and then sum the products to get the portfolio return in each state of the economy. Doing so, we get:

$$\text{Boom: } E(R_p) = .4(.20) + .4(.35) + .2(.60) = .3400 \text{ or } 34.00\%$$

$$\text{Normal: } E(R_p) = .4(.15) + .4(.12) + .2(.05) = .1180 \text{ or } 11.80\%$$

$$\text{Bust: } E(R_p) = .4(.01) + .4(-.25) + .2(-.50) = -.1960 \text{ or } -19.60\%$$

And the expected return of the portfolio is:

$$E(R_p) = .4(.34) + .4(.118) + .2(-.196) = .1440 \text{ or } 14.40\%$$

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, than add all of these up. The result is the variance. So, the variance and standard deviation of the portfolio is:

$$\sigma_p^2 = .4(.34 - .1440)^2 + .4(.118 - .1440)^2 + .2(-.196 - .1440)^2$$

$$\sigma_p^2 = .03876$$

$$\sigma_p = (.03876)^{1/2} = .1969 \text{ or } 19.69\%$$

- b. The risk premium is the return of a risky asset, minus the risk-free rate. T-bills are often used as the risk-free rate, so:

$$RP_i = E(R_p) - R_f = .1440 - .038 = .1060 \text{ or } 10.60\%$$

- c. The approximate expected real return is the expected nominal return minus the inflation rate, so:

$$\text{Approximate expected real return} = .1440 - .035 = .1090 \text{ or } 10.90\%$$

To find the exact real return, we will use the Fisher equation. Doing so, we get:

$$\begin{aligned} 1 + E(R_i) &= (1 + h)[1 + e(r_i)] \\ 1.1440 &= (1.0350)[1 + e(r_i)] \\ e(r_i) &= (1.1440/1.035) - 1 = .1053 \text{ or } 10.53\% \end{aligned}$$

The approximate real risk premium is the expected return minus the inflation rate, so:

$$\text{Approximate expected real risk premium} = .1060 - .035 = .0710 \text{ or } 7.10\%$$

To find the exact expected real risk premium we use the Fisher effect. Doing so, we find:

$$\text{Exact expected real risk premium} = (1.1060/1.035) - 1 = .0686 \text{ or } 6.86\%$$

24. We know the total portfolio value and the investment of two stocks in the portfolio, so we can find the weight of these two stocks. The weights of Stock A and Stock B are:

$$w_A = \$200,000 / \$1,000,000 = .20$$

$$w_B = \$250,000 / \$1,000,000 = .25$$

Since the portfolio is as risky as the market, the β of the portfolio must be equal to one. We also know the β of the risk-free asset is zero. We can use the equation for the β of a portfolio to find the weight of the third stock. Doing so, we find:

$$\beta_p = 1.0 = w_A(.8) + w_B(1.3) + w_C(1.5) + w_{RF}(0)$$

Solving for the weight of Stock C, we find:

$$w_C = .343333$$

So, the dollar investment in Stock C must be:

$$\text{Invest in Stock C} = .343333(\$1,000,000) = \$343,333$$

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We also know the total portfolio weight must be one, so the weight of the risk-free asset must be one minus the asset weight we know, or:

$$\begin{aligned}1 &= w_A + w_B + w_C + w_{Rf} \\1 &= .20 + .25 + .34333 + w_{Rf} \\w_{Rf} &= .206667\end{aligned}$$

So, the dollar investment in the risk-free asset must be:

$$\text{Invest in risk-free asset} = .206667(\$1,000,000) = \$206,667$$

- 25.** We are given the expected return and β of a portfolio and the expected return and β of assets in the portfolio. We know the β of the risk-free asset is zero. We also know the sum of the weights of each asset must be equal to one. So, the weight of the risk-free asset is one minus the weight of Stock X and the weight of Stock Y. Using this relationship, we can express the expected return of the portfolio as:

$$E(R_p) = .135 = w_X(.31) + w_Y(.20) + (1 - w_X - w_Y)(.07)$$

And the β of the portfolio is:

$$\beta_p = .7 = w_X(1.8) + w_Y(1.3) + (1 - w_X - w_Y)(0)$$

We have two equations and two unknowns. Solving these equations, we find that:

$$\begin{aligned}w_X &= -0.0833333 \\w_Y &= 0.6538462 \\w_{Rf} &= 0.4298472\end{aligned}$$

The amount to invest in Stock X is:

$$\text{Investment in stock X} = -0.0833333(\$100,000) = -\$8,333.33$$

A negative portfolio weight means that you short sell the stock. If you are not familiar with short selling, it means you borrow a stock today and sell it. You must then purchase the stock at a later date to repay the borrowed stock. If you short sell a stock, you make a profit if the stock decreases in value.

- 26.** The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of each stock is:

$$E(R_A) = .33(.063) + .33(.105) + .33(.156) = .1080 \text{ or } 10.80\%$$

$$E(R_B) = .33(-.037) + .33(.064) + .33(.253) = .0933 \text{ or } 9.33\%$$

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the variance and standard deviation of Stock A are:

$$\sigma^2 = .33(.063 - .1080)^2 + .33(.105 - .1080)^2 + .33(.156 - .1080)^2 = .00145$$

$$\sigma = (.00145)^{1/2} = .0380 \text{ or } 3.80\%$$

And the standard deviation of Stock B is:

$$\sigma^2 = .33(-.037 - .0933)^2 + .33(.064 - .0933)^2 + .33(.253 - .0933)^2 = .01445$$

$$\sigma = (.01445)^{1/2} = .1202 \text{ or } 12.02\%$$

To find the covariance, we multiply each possible state times the product of each assets' deviation from the mean in that state. The sum of these products is the covariance. So, the covariance is:

$$\begin{aligned} \text{Cov}(A,B) &= .33(.063 - .1080)(-.037 - .0933) + .33(.105 - .1080)(.064 - .0933) \\ &\quad + .33(.156 - .1080)(.253 - .0933) \end{aligned}$$

$$\text{Cov}(A,B) = .004539$$

And the correlation is:

$$\rho_{A,B} = \text{Cov}(A,B) / \sigma_A \sigma_B$$

$$\rho_{A,B} = .004539 / (.0380)(.1202)$$

$$\rho_{A,B} = .9931$$

27. The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of each stock is:

$$E(R_A) = .25(-.020) + .60(.092) + .15(.154) = .0733 \text{ or } 7.33\%$$

$$E(R_B) = .25(.050) + .60(.062) + .15(.074) = .0608 \text{ or } 6.08\%$$

To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the variance and standard deviation of Stock A are:

$$\sigma_A^2 = .25(-.020 - .0733)^2 + .60(.092 - .0733)^2 + .15(.154 - .0733)^2 = .00336$$

$$\sigma_A = (.00336)^{1/2} = .0580 \text{ or } 5.80\%$$

And the standard deviation of Stock B is:

$$\sigma_B^2 = .25(.050 - .0608)^2 + .60(.062 - .0608)^2 + .15(.074 - .0608)^2 = .00006$$

$$\sigma_B = (.00006)^{1/2} = .0075 \text{ or } 0.75\%$$

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To find the covariance, we multiply each possible state times the product of each assets' deviation from the mean in that state. The sum of these products is the covariance. So, the covariance is:

$$\text{Cov}(A,B) = .25(-.020 - .0733)(.050 - .0608) + .60(.092 - .0733)(.062 - .0608) \\ + .15(.154 - .0733)(.074 - .0608)$$

$$\text{Cov}(A,B) = .000425$$

And the correlation is:

$$\rho_{A,B} = \text{Cov}(A,B) / \sigma_A \sigma_B$$

$$\rho_{A,B} = .000425 / (.0580)(.0075)$$

$$\rho_{A,B} = .9783$$

28. a. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so:

$$E(R_P) = w_F E(R_F) + w_G E(R_G)$$

$$E(R_P) = .30(.12) + .70(.18)$$

$$E(R_P) = .1620 \text{ or } 16.20\%$$

- b. The variance of a portfolio of two assets can be expressed as:

$$\sigma_P^2 = w_F^2 \sigma_F^2 + w_G^2 \sigma_G^2 + 2w_F w_G \sigma_F \sigma_G \rho_{F,G}$$

$$\sigma_P^2 = .30^2(.34^2) + .70^2(.50^2) + 2(.30)(.70)(.34)(.50)(.20)$$

$$\sigma_P^2 = .14718$$

So, the standard deviation is:

$$\sigma = (.14718)^{1/2} = .3836 \text{ or } 38.36\%$$

29. a. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so:

$$E(R_P) = w_A E(R_A) + w_B E(R_B)$$

$$E(R_P) = .40(.15) + .60(.25)$$

$$E(R_P) = .2100 \text{ or } 21.00\%$$

The variance of a portfolio of two assets can be expressed as:

$$\sigma_P^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \sigma_A \sigma_B \rho_{A,B}$$

$$\sigma_P^2 = .40^2(.40^2) + .60^2(.65^2) + 2(.40)(.60)(.40)(.65)(.50)$$

$$\sigma_P^2 = .24010$$

So, the standard deviation is:

$$\sigma = (.24010)^{1/2} = .4900 \text{ or } 49.00\%$$

- b. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so:

$$\begin{aligned} E(R_P) &= w_A E(R_A) + w_B E(R_B) \\ E(R_P) &= .40(.15) + .60(.25) \\ E(R_P) &= .2100 \text{ or } 21.00\% \end{aligned}$$

The variance of a portfolio of two assets can be expressed as:

$$\begin{aligned} \sigma_P^2 &= w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \sigma_A \sigma_B \rho_{A,B} \\ \sigma_P^2 &= .40^2(.40^2) + .60^2(.65^2) + 2(.40)(.60)(.40)(.65)(-.50) \\ \sigma_P^2 &= .11530 \end{aligned}$$

So, the standard deviation is:

$$\sigma = (.11530)^{1/2} = .3396 \text{ or } 33.96\%$$

- c. As Stock A and Stock B become less correlated, or more negatively correlated, the standard deviation of the portfolio decreases.

30. a. (i) We can use the equation to calculate beta, we find:

$$\begin{aligned} \beta_I &= (\rho_{I,M})(\sigma_I) / \sigma_M \\ 0.9 &= (\rho_{I,M})(0.38) / 0.20 \\ \rho_{I,M} &= 0.47 \end{aligned}$$

- (ii) Using the equation to calculate beta, we find:

$$\begin{aligned} \beta_I &= (\rho_{I,M})(\sigma_I) / \sigma_M \\ 1.1 &= (.40)(\sigma_I) / 0.20 \\ \sigma_I &= 0.55 \end{aligned}$$

- (iii) Using the equation to calculate beta, we find:

$$\begin{aligned} \beta_I &= (\rho_{I,M})(\sigma_I) / \sigma_M \\ \beta_I &= (.35)(.65) / 0.20 \\ \beta_I &= 1.14 \end{aligned}$$

- (iv) The market has a correlation of 1 with itself.

- (v) The beta of the market is 1.

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- (vi) The risk-free asset has zero standard deviation.
- (vii) The risk-free asset has zero correlation with the market portfolio.
- (viii) The beta of the risk-free asset is 0.

b. Using the CAPM to find the expected return of the stock, we find:

Firm A:

$$\begin{aligned}E(R_A) &= R_f + \beta_A[E(R_M) - R_f] \\E(R_A) &= 0.05 + 0.9(0.15 - 0.05) \\E(R_A) &= .1400 \text{ or } 14.00\%\end{aligned}$$

According to the CAPM, the expected return on Firm A's stock should be 14 percent. However, the expected return on Firm A's stock given in the table is only 13 percent. Therefore, Firm A's stock is overpriced, and you should sell it.

Firm B:

$$\begin{aligned}E(R_B) &= R_f + \beta_B[E(R_M) - R_f] \\E(R_B) &= 0.05 + 1.1(0.15 - 0.05) \\E(R_B) &= .1600 \text{ or } 16.00\%\end{aligned}$$

According to the CAPM, the expected return on Firm B's stock should be 16 percent. The expected return on Firm B's stock given in the table is also 16 percent. Therefore, Firm B's stock is correctly priced.

Firm C:

$$\begin{aligned}E(R_C) &= R_f + \beta_C[E(R_M) - R_f] \\E(R_C) &= 0.05 + 1.14(0.15 - 0.05) \\E(R_C) &= .1638 \text{ or } 16.38\%\end{aligned}$$

According to the CAPM, the expected return on Firm C's stock should be 16.38 percent. However, the expected return on Firm C's stock given in the table is 20 percent. Therefore, Firm C's stock is underpriced, and you should buy it.

31. Because a well-diversified portfolio has no unsystematic risk, this portfolio should lie on the Capital Market Line (CML). The slope of the CML equals:

$$\begin{aligned}\text{Slope}_{\text{CML}} &= [E(R_M) - R_f] / \sigma_M \\ \text{Slope}_{\text{CML}} &= (0.12 - 0.05) / 0.10 \\ \text{Slope}_{\text{CML}} &= 0.70\end{aligned}$$

a. The expected return on the portfolio equals:

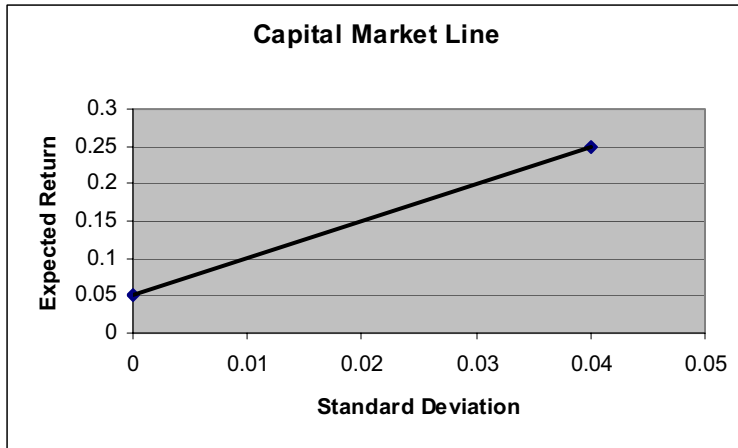
$$\begin{aligned}E(R_P) &= R_f + \text{Slope}_{\text{CML}}(\sigma_P) \\ E(R_P) &= .05 + .70(.07) \\ E(R_P) &= .0990 \text{ or } 9.90\%\end{aligned}$$

b. The expected return on the portfolio equals:

$$E(R_P) = R_f + \text{Slope}_{\text{CML}}(\sigma_P)$$

$$.20 = .05 + .70(\sigma_P)$$

$$\sigma_P = .2143 \text{ or } 21.43\%$$



32. First, we can calculate the standard deviation of the market portfolio using the Capital Market Line (CML). We know that the risk-free rate asset has a return of 5 percent and a standard deviation of zero and the portfolio has an expected return of 14 percent and a standard deviation of 18 percent. These two points must lie on the Capital Market Line. The slope of the Capital Market Line equals:

$$\text{Slope}_{\text{CML}} = \text{Rise} / \text{Run}$$

$$\text{Slope}_{\text{CML}} = \text{Increase in expected return} / \text{Increase in standard deviation}$$

$$\text{Slope}_{\text{CML}} = (.12 - .05) / (.18 - 0)$$

$$\text{Slope}_{\text{CML}} = .39$$

According to the Capital Market Line:

$$E(R_I) = R_f + \text{Slope}_{\text{CML}}(\sigma_I)$$

Since we know the expected return on the market portfolio, the risk-free rate, and the slope of the Capital Market Line, we can solve for the standard deviation of the market portfolio which is:

$$E(R_M) = R_f + \text{Slope}_{\text{CML}}(\sigma_M)$$

$$.12 = .05 + (.39)(\sigma_M)$$

$$\sigma_M = (.12 - .05) / .39$$

$$\sigma_M = .1800 \text{ or } 18.00\%$$

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Next, we can use the standard deviation of the market portfolio to solve for the beta of a security using the beta equation. Doing so, we find the beta of the security is:

$$\begin{aligned}\beta_I &= (\rho_{I,M})(\sigma_I) / \sigma_M \\ \beta_I &= (.45)(.40) / .1800 \\ \beta_I &= 1.00\end{aligned}$$

Now we can use the beta of the security in the CAPM to find its expected return, which is:

$$\begin{aligned}E(R_I) &= R_f + \beta_I[E(R_M) - R_f] \\ E(R_I) &= 0.05 + 1.00(.14 - 0.05) \\ E(R_I) &= .1400 \text{ or } 14.00\%\end{aligned}$$

- 33.** First, we need to find the standard deviation of the market and the portfolio, which are:

$$\begin{aligned}\sigma_M &= (.0498)^{1/2} \\ \sigma_M &= .2232 \text{ or } 22.32\%\end{aligned}$$

$$\begin{aligned}\sigma_Z &= (.1783)^{1/2} \\ \sigma_Z &= .4223 \text{ or } 42.23\%\end{aligned}$$

Now we can use the equation for beta to find the beta of the portfolio, which is:

$$\begin{aligned}\beta_Z &= (\rho_{Z,M})(\sigma_Z) / \sigma_M \\ \beta_Z &= (.45)(.4223) / .2232 \\ \beta_Z &= .85\end{aligned}$$

Now, we can use the CAPM to find the expected return of the portfolio, which is:

$$\begin{aligned}E(R_Z) &= R_f + \beta_Z[E(R_M) - R_f] \\ E(R_Z) &= .063 + .85(.148 - .063) \\ E(R_Z) &= .1354 \text{ or } 13.54\%\end{aligned}$$

- 34.** The amount of systematic risk is measured by the β of an asset. Since we know the market risk premium and the risk-free rate, if we know the expected return of the asset we can use the CAPM to solve for the β of the asset. The expected return of Stock I is:

$$E(R_I) = .15(.09) + .70(.42) + .15(.26) = .3465 \text{ or } 34.65\%$$

Using the CAPM to find the β of Stock I, we find:

$$\begin{aligned}.3465 &= .04 + .10\beta_I \\ \beta_I &= 3.07\end{aligned}$$

The total risk of the asset is measured by its standard deviation, so we need to calculate the standard deviation of Stock I. Beginning with the calculation of the stock's variance, we find:

$$\begin{aligned}\sigma_I^2 &= .15(.09 - .3465)^2 + .70(.42 - .3465)^2 + .15(.26 - .3465)^2 \\ \sigma_I^2 &= .01477\end{aligned}$$

$$\sigma_I = (.01477)^{1/2} = .1215 \text{ or } 12.15\%$$

Using the same procedure for Stock II, we find the expected return to be:

$$E(R_{II}) = .15(-.30) + .70(.12) + .15(.44) = .1050$$

Using the CAPM to find the β of Stock II, we find:

$$\begin{aligned}.1050 &= .04 + .10\beta_{II} \\ \beta_{II} &= 0.65\end{aligned}$$

And the standard deviation of Stock II is:

$$\begin{aligned}\sigma_{II}^2 &= .15(-.30 - .105)^2 + .70(.12 - .105)^2 + .15(.44 - .105)^2 \\ \sigma_{II}^2 &= .04160\end{aligned}$$

$$\sigma_{II} = (.04160)^{1/2} = .2039 \text{ or } 20.39\%$$

Although Stock II has more total risk than I, it has much less systematic risk, since its beta is much smaller than I's. Thus, I has more systematic risk, and II has more unsystematic and more total risk. Since unsystematic risk can be diversified away, I is actually the "riskier" stock despite the lack of volatility in its returns. Stock I will have a higher risk premium and a greater expected return.

35. Here we have the expected return and beta for two assets. We can express the returns of the two assets using CAPM. Now we have two equations and two unknowns. Going back to Algebra, we can solve the two equations. We will solve the equation for Pete Corp. to find the risk-free rate, and solve the equation for Repete Co. to find the expected return of the market. We next substitute the expected return of the market into the equation for Pete Corp., and then solve for the risk-free rate. Now that we have the risk-free rate, we can substitute this into either original CAPM expression and solve for expected return of the market. Doing so, we get:

$$\begin{aligned}E(R_{\text{Pete Corp.}}) &= .23 = R_f + 1.3(R_M - R_f); & E(R_{\text{Repete Co.}}) &= .13 = R_f + .6(R_M - R_f) \\ .23 &= R_f + 1.3R_M - 1.3R_f = 1.3R_M - .3R_f; & .13 &= R_f + .6(R_M - R_f) = R_f + .6R_M - .6R_f \\ R_f &= (1.3R_M - .23)/.3 & R_M &= (.13 - .4R_f)/.6 \\ & & R_M &= .217 - .667R_f\end{aligned}$$

$$\begin{aligned}R_f &= [1.3(.217 - .667R_f) - .23]/.3 \\ 1.167R_f &= .0521 \\ R_f &= .0443 \text{ or } 4.43\%\end{aligned}$$

$$\begin{aligned}.23 &= .0443 + 1.3(R_M - .0443) \\ R_M &= .1871 \text{ or } 18.71\%\end{aligned}$$

$$\begin{aligned}.13 &= .0443 + .6(R_M - .0443) \\ R_M &= .1871 \text{ or } 18.71\%\end{aligned}$$

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36. a. The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. To calculate the standard deviation, we first need to calculate the variance. To find the variance, we find the squared deviations from the expected return. We then multiply each possible squared deviation by its probability, and then add all of these up. The result is the variance. So, the expected return and standard deviation of each stock are:

Asset 1:

$$E(R_1) = .10(.25) + .40(.20) + .40(.15) + .10(.10) = .1750 \text{ or } 17.50\%$$

$$\sigma_1^2 = .10(.25 - .1750)^2 + .40(.20 - .1750)^2 + .40(.15 - .1750)^2 + .10(.10 - .1750)^2 = .00163$$

$$\sigma_1 = (.00163)^{1/2} = .0403 \text{ or } 4.03\%$$

Asset 2:

$$E(R_2) = .10(.25) + .40(.15) + .40(.20) + .10(.10) = .1750 \text{ or } 17.50\%$$

$$\sigma_2^2 = .10(.25 - .1750)^2 + .40(.15 - .1750)^2 + .40(.20 - .1750)^2 + .10(.10 - .1750)^2 = .00163$$

$$\sigma_2 = (.00163)^{1/2} = .0403 \text{ or } 4.03\%$$

Asset 3:

$$E(R_3) = .10(.10) + .40(.15) + .40(.20) + .10(.25) = .1750 \text{ or } 17.50\%$$

$$\sigma_3^2 = .10(.10 - .1750)^2 + .40(.15 - .1750)^2 + .40(.20 - .1750)^2 + .10(.25 - .1750)^2 = .00163$$

$$\sigma_3 = (.00163)^{1/2} = .0403 \text{ or } 4.03\%$$

- b. To find the covariance, we multiply each possible state times the product of each assets' deviation from the mean in that state. The sum of these products is the covariance. The correlation is the covariance divided by the product of the two standard deviations. So, the covariance and correlation between each possible set of assets are:

Asset 1 and Asset 2:

$$\begin{aligned} \text{Cov}(1,2) = & .10(.25 - .1750)(.25 - .1750) + .40(.20 - .1750)(.15 - .1750) \\ & + .40(.15 - .1750)(.20 - .1750) + .10(.10 - .1750)(.10 - .1750) \end{aligned}$$

$$\text{Cov}(1,2) = .000625$$

$$\rho_{1,2} = \text{Cov}(1,2) / \sigma_1 \sigma_2$$

$$\rho_{1,2} = .000625 / (.0403)(.0403)$$

$$\rho_{1,2} = .3846$$

Asset 1 and Asset 3:

$$\begin{aligned}\text{Cov}(1,3) &= .10(.25 - .1750)(.10 - .1750) + .40(.20 - .1750)(.15 - .1750) \\ &\quad + .40(.15 - .1750)(.20 - .1750) + .10(.10 - .1750)(.25 - .1750) \\ \text{Cov}(1,3) &= -.001625\end{aligned}$$

$$\begin{aligned}\rho_{1,3} &= \text{Cov}(1,3) / \sigma_1 \sigma_3 \\ \rho_{1,3} &= -.001625 / (.0403)(.0403) \\ \rho_{1,3} &= -1\end{aligned}$$

Asset 2 and Asset 3:

$$\begin{aligned}\text{Cov}(2,3) &= .10(.25 - .1750)(.10 - .1750) + .40(.15 - .1750)(.15 - .1750) \\ &\quad + .40(.20 - .1750)(.20 - .1750) + .10(.10 - .1750)(.25 - .1750) \\ \text{Cov}(2,3) &= -.000625\end{aligned}$$

$$\begin{aligned}\rho_{2,3} &= \text{Cov}(2,3) / \sigma_2 \sigma_3 \\ \rho_{2,3} &= -.000625 / (.0403)(.0403) \\ \rho_{2,3} &= -.3846\end{aligned}$$

- c. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 1 and Asset 2:

$$\begin{aligned}E(R_p) &= w_1E(R_1) + w_2E(R_2) \\ E(R_p) &= .50(.1750) + .50(.1750) \\ E(R_p) &= .1750 \text{ or } 17.50\%\end{aligned}$$

The variance of a portfolio of two assets can be expressed as:

$$\begin{aligned}\sigma_p^2 &= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1w_2\sigma_1\sigma_2\rho_{1,2} \\ \sigma_p^2 &= .50^2(.0403^2) + .50^2(.0403^2) + 2(.50)(.50)(.0403)(.0403)(.3846) \\ \sigma_p^2 &= .001125\end{aligned}$$

And the standard deviation of the portfolio is:

$$\begin{aligned}\sigma_p &= (.001125)^{1/2} \\ \sigma_p &= .0335 \text{ or } 3.35\%\end{aligned}$$

- d. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 1 and Asset 3:

$$\begin{aligned}E(R_p) &= w_1E(R_1) + w_3E(R_3) \\ E(R_p) &= .50(.1750) + .50(.1750) \\ E(R_p) &= .1750 \text{ or } 17.50\%\end{aligned}$$

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The variance of a portfolio of two assets can be expressed as:

$$\begin{aligned}\sigma_p^2 &= w_1^2 \sigma_1^2 + w_3^2 \sigma_3^2 + 2w_1w_3\sigma_1\sigma_3\rho_{1,3} \\ \sigma_p^2 &= .50^2(.0403^2) + .50^2(.0403^2) + 2(.50)(.50)(.0403)(.0403)(-1) \\ \sigma_p^2 &= .000000\end{aligned}$$

Since the variance is zero, the standard deviation is also zero.

- e. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so, for a portfolio of Asset 1 and Asset 3:

$$\begin{aligned}E(R_p) &= w_2E(R_2) + w_3E(R_3) \\ E(R_p) &= .50(.1750) + .50(.1750) \\ E(R_p) &= .1750 \text{ or } 17.50\%\end{aligned}$$

The variance of a portfolio of two assets can be expressed as:

$$\begin{aligned}\sigma_p^2 &= w_2^2 \sigma_2^2 + w_3^2 \sigma_3^2 + 2w_2w_3\sigma_2\sigma_3\rho_{1,3} \\ \sigma_p^2 &= .50^2(.0403^2) + .50^2(.0403^2) + 2(.50)(.50)(.0403)(.0403)(-.3846) \\ \sigma_p^2 &= .000500\end{aligned}$$

And the standard deviation of the portfolio is:

$$\begin{aligned}\sigma_p &= (.000500)^{1/2} \\ \sigma_p &= .0224 \text{ or } 2.24\%\end{aligned}$$

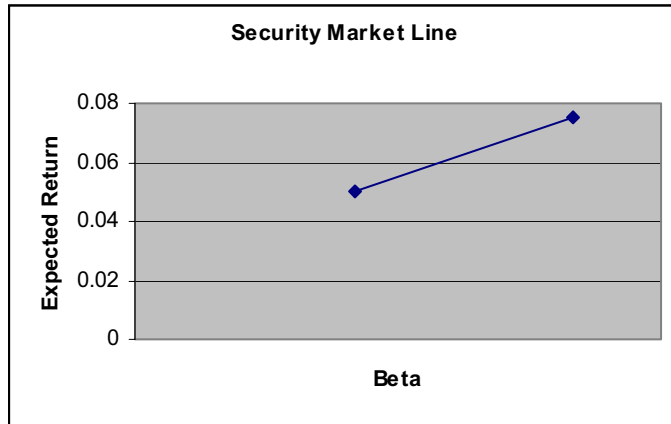
- f. As long as the correlation between the returns on two securities is below 1, there is a benefit to diversification. A portfolio with negatively correlated stocks can achieve greater risk reduction than a portfolio with positively correlated stocks, holding the expected return on each stock constant. Applying proper weights on perfectly negatively correlated stocks can reduce portfolio variance to 0.

37. a. The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return of each stock is:

$$E(R_A) = .25(-.10) + .50(.10) + .25(.20) = .0750 \text{ or } 7.50\%$$

$$E(R_B) = .25(-.30) + .50(.05) + .25(.40) = .0500 \text{ or } 5.00\%$$

- b. We can use the expected returns we calculated to find the slope of the Security Market Line. We know that the beta of Stock A is .25 greater than the beta of Stock B. Therefore, as beta increases by .25, the expected return on a security increases by .025 (= .075 – .05). The slope of



the security market line (SML) equals:

$$\begin{aligned} \text{Slope}_{\text{SML}} &= \text{Rise} / \text{Run} \\ \text{Slope}_{\text{SML}} &= \text{Increase in expected return} / \text{Increase in beta} \\ \text{Slope}_{\text{SML}} &= (.075 - .05) / .25 \\ \text{Slope}_{\text{SML}} &= .1000 \text{ or } 10\% \end{aligned}$$

Since the market's beta is 1 and the risk-free rate has a beta of zero, the slope of the Security Market Line equals the expected market risk premium. So, the expected market risk premium must be 10 percent.

38. a. A typical, risk-averse investor seeks high returns and low risks. For a risk-averse investor holding a well-diversified portfolio, beta is the appropriate measure of the risk of an individual security. To assess the two stocks, we need to find the expected return and beta of each of the two securities.

Stock A:

Since Stock A pays no dividends, the return on Stock A is simply: $(P_1 - P_0) / P_0$. So, the return for each state of the economy is:

$$\begin{aligned} R_{\text{Recession}} &= (\$40 - 50) / \$50 = -.20 \text{ or } 20\% \\ R_{\text{Normal}} &= (\$55 - 50) / \$50 = .10 \text{ or } 10\% \\ R_{\text{Expanding}} &= (\$60 - 50) / \$50 = .20 \text{ or } 20\% \end{aligned}$$

The expected return of an asset is the sum of the probability of each return occurring times the probability of that return occurring. So, the expected return the stock is:

$$E(R_A) = .10(-.20) + .80(.10) + .10(.20) = .0800 \text{ or } 8.00\%$$

And the variance of the stock is:

$$\begin{aligned} \sigma_A^2 &= .10(-0.20 - 0.08)^2 + .80(.10 - .08)^2 + .10(.20 - .08)^2 \\ \sigma_A^2 &= 0.0096 \end{aligned}$$

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Which means the standard deviation is:

$$\begin{aligned}\sigma_A &= (0.0096)^{1/2} \\ \sigma_A &= .098 \text{ or } 9.8\%\end{aligned}$$

Now we can calculate the stock's beta, which is:

$$\begin{aligned}\beta_A &= (\rho_{A,M})(\sigma_A) / \sigma_M \\ \beta_A &= (.80)(.098) / .10 \\ \beta_A &= .784\end{aligned}$$

For Stock B, we can directly calculate the beta from the information provided. So, the beta for Stock B is:

Stock B:

$$\begin{aligned}\beta_B &= (\rho_{B,M})(\sigma_B) / \sigma_M \\ \beta_B &= (.20)(.12) / .10 \\ \beta_B &= .240\end{aligned}$$

The expected return on Stock B is higher than the expected return on Stock A. The risk of Stock B, as measured by its beta, is lower than the risk of Stock A. Thus, a typical risk-averse investor holding a well-diversified portfolio will prefer Stock B. Note, this situation implies that at least one of the stocks is mispriced since the higher risk (beta) stock has a lower return than the lower risk (beta) stock.

- b. The expected return of the portfolio is the sum of the weight of each asset times the expected return of each asset, so:

$$\begin{aligned}E(R_P) &= w_A E(R_A) + w_B E(R_B) \\ E(R_P) &= .70(.08) + .30(.09) \\ E(R_P) &= .083 \text{ or } 8.30\%\end{aligned}$$

To find the standard deviation of the portfolio, we first need to calculate the variance. The variance of the portfolio is:

$$\begin{aligned}\sigma_P^2 &= w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \sigma_A \sigma_B \rho_{A,B} \\ \sigma_P^2 &= (.70)^2 (.098)^2 + (.30)^2 (.12)^2 + 2(.70)(.30)(.098)(.12)(.60) \\ \sigma_P^2 &= .00896\end{aligned}$$

And the standard deviation of the portfolio is:

$$\begin{aligned}\sigma_P &= (0.00896)^{1/2} \\ \sigma_P &= .0947 \text{ or } 9.47\%\end{aligned}$$

- c. The beta of a portfolio is the weighted average of the betas of its individual securities. So the beta of the portfolio is:

$$\beta_P = .70(.784) + .30(0.24)$$

$$\beta_P = .621$$

39. a. The variance of a portfolio of two assets equals:

$$\sigma_P^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \sigma_A \sigma_B \text{Cov}(A,B)$$

Since the weights of the assets must sum to one, we can write the variance of the portfolio as:

$$\sigma_P^2 = w_A^2 \sigma_A^2 + (1 - w_A)^2 \sigma_B^2 + 2w_A(1 - w_A)\sigma_A \sigma_B \text{Cov}(A,B)$$

To find the minimum for any function, we find the derivative and set the derivative equal to zero. Finding the derivative of the variance function, setting the derivative equal to zero, and solving for the weight of Asset A, we find:

$$w_A = [\sigma_B^2 - \text{Cov}(A,B)] / [\sigma_A^2 + \sigma_B^2 - 2\text{Cov}(A,B)]$$

Using this expression, we find the weight of Asset A must be:

$$w_A = (.20^2 - .001) / [.10^2 + .20^2 - 2(.001)]$$

$$w_A = .8125$$

This implies the weight of Stock B is:

$$w_B = 1 - w_A$$

$$w_B = 1 - .8125$$

$$w_B = .1875$$

- b. Using the weights calculated in part a, determine the expected return of the portfolio, we find:

$$E(R_P) = w_A E(R_A) + w_B E(R_B)$$

$$E(R_P) = .8125(.05) + .1875(0.10)$$

$$E(R_P) = 0.0594$$

- c. Using the derivative from part a, with the new covariance, the weight of each stock in the minimum variance portfolio is:

$$w_A = [\sigma_B^2 + \text{Cov}(A,B)] / [\sigma_A^2 + \sigma_B^2 - 2\text{Cov}(A,B)]$$

$$w_A = (.10^2 + -.02) / [.10^2 + .20^2 - 2(-.02)]$$

$$w_A = .6667$$

This implies the weight of Stock B is:

$$w_B = 1 - w_A$$

$$w_B = 1 - .6667$$

$$w_B = .3333$$

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d. The variance of the portfolio with the weights on part c is:

$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_A w_B \sigma_A \sigma_B \text{Cov}(A,B)$$

$$\sigma_p^2 = (.6667)^2 (.10)^2 + (.3333)^2 (.20)^2 + 2(.6667)(.3333)(.10)(.20)(-.02)$$

$$\sigma_p^2 = .0000$$

Because the stocks have a perfect negative correlation (-1), we can find a portfolio of the two stocks with a zero variance.