
CHAPTER 9

SOME LESSONS FROM CAPITAL MARKET HISTORY

Answers to Concepts Review and Critical Thinking Questions

1. They all wish they had! Since they didn't, it must have been the case that the stellar performance was not foreseeable, at least not by most.
2. As in the previous question, it's easy to see after the fact that the investment was terrible, but it probably wasn't so easy ahead of time.
3. No, stocks are riskier. Some investors are highly risk averse, and the extra possible return doesn't attract them relative to the extra risk.
4. Unlike gambling, the stock market is a positive sum game; everybody can win. Also, speculators provide liquidity to markets and thus help to promote efficiency.
5. T-bill rates were highest in the early eighties. This was during a period of high inflation and is consistent with the Fisher effect.
6. Before the fact, for most assets, the risk premium will be positive; investors demand compensation over and above the risk-free return to invest their money in the risky asset. After the fact, the observed risk premium can be negative if the asset's nominal return is unexpectedly low, the risk-free return is unexpectedly high, or if some combination of these two events occurs.
7. Yes, the stock prices are currently the same. Below is a diagram that depicts the stocks' price movements. Two years ago, each stock had the same price, P_0 . Over the first year, General Materials' stock price increased by 10 percent, or $(1.1) \times P_0$. Standard Fixtures' stock price declined by 10 percent, or $(0.9) \times P_0$. Over the second year, General Materials' stock price decreased by 10 percent, or $(0.9)(1.1) \times P_0$, while Standard Fixtures' stock price increased by 10 percent, or $(1.1)(0.9) \times P_0$. Today, each of the stocks is worth 99 percent of its original value.

	<u>2 years ago</u>	<u>1 year ago</u>	<u>Today</u>	
General Materials	P_0	$(1.1)P_0$	$(1.1)(0.9)P_0$	$= (0.99)P_0$
Standard Fixtures	P_0	$(0.9)P_0$	$(0.9)(1.1)P_0$	$= (0.99)P_0$

8. The stock prices are not the same. The return quoted for each stock is the arithmetic return, not the geometric return. The geometric return tells you the wealth increase from the beginning of the period to the end of the period, assuming the asset had the same return each year. As such, it is a better measure of ending wealth. To see this, assuming each stock had a beginning price of \$100 per share, the ending price for each stock would be:

Lake Minerals ending price = $\$100(1.10)(1.10) = \121.00

Small Town Furniture ending price = $\$100(1.25)(.95) = \118.75

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Whenever there is any variance in returns, the asset with the larger variance will always have the greater difference between the arithmetic and geometric return.

9. To calculate an arithmetic return, you simply sum the returns and divide by the number of returns. As such, arithmetic returns do not account for the effects of compounding. Geometric returns do account for the effects of compounding. As an investor, the more important return of an asset is the geometric return.
10. Risk premiums are about the same whether or not we account for inflation. The reason is that risk premiums are the difference between two returns, so inflation essentially nets out. Returns, risk premiums, and volatility would all be lower than we estimated because aftertax returns are smaller than pretax returns.

Solutions to Questions and Problems

NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The return of any asset is the increase in price, plus any dividends or cash flows, all divided by the initial price. The return of this stock is:

$$R = [(\$91 - 83) + 1.40] / \$83$$
$$R = .1133 \text{ or } 11.33\%$$

2. The dividend yield is the dividend divided by price at the beginning of the period, so:

$$\text{Dividend yield} = \$1.40 / \$83$$
$$\text{Dividend yield} = .0169 \text{ or } 1.69\%$$

And the capital gains yield is the increase in price divided by the initial price, so:

$$\text{Capital gains yield} = (\$91 - 83) / \$83$$
$$\text{Capital gains yield} = .0964 \text{ or } 9.64\%$$

3. Using the equation for total return, we find:

$$R = [(\$76 - 83) + 1.40] / \$83$$
$$R = -.0675 \text{ or } -6.75\%$$

And the dividend yield and capital gains yield are:

$$\text{Dividend yield} = \$1.40 / \$83$$
$$\text{Dividend yield} = .0169 \text{ or } 1.69\%$$

$$\text{Capital gains yield} = (\$76 - 83) / \$83$$

$$\text{Capital gains yield} = -.0843 \text{ or } -8.43\%$$

Here's a question for you: Can the dividend yield ever be negative? No, that would mean you were paying the company for the privilege of owning the stock. It has happened on bonds. Remember the Buffett bond's we discussed in the bond chapter.

4. The total dollar return is the change in price plus the coupon payment, so:

$$\text{Total dollar return} = \$1,074 - 1,120 + 90$$

$$\text{Total dollar return} = \$44$$

The total percentage return of the bond is:

$$R = [(\$1,074 - 1,120) + 90] / \$1,120$$

$$R = .0393 \text{ or } 3.93\%$$

Notice here that we could have simply used the total dollar return of \$44 in the numerator of this equation.

Using the Fisher equation, the real return was:

$$(1 + R) = (1 + r)(1 + h)$$

$$r = (1.0393 / 1.030) - 1$$

$$r = .0090 \text{ or } 0.90\%$$

5. The nominal return is the stated return, which is 12.40 percent. Using the Fisher equation, the real return was:

$$(1 + R) = (1 + r)(1 + h)$$

$$r = (1.1240)/(1.031) - 1$$

$$r = .0902 \text{ or } 9.02\%$$

6. Using the Fisher equation, the real returns for government and corporate bonds were:

$$(1 + R) = (1 + r)(1 + h)$$

$$r_G = 1.058/1.031 - 1$$

$$r_G = .0262 \text{ or } 2.62\%$$

$$r_C = 1.062/1.031 - 1$$

$$r_C = .0301 \text{ or } 3.01\%$$

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7. The average return is the sum of the returns, divided by the number of returns. The average return for each stock was:

$$\bar{X} = \left[\sum_{i=1}^N x_i \right] / N = \frac{[.11 + .06 - .08 + .28 + .13]}{5} = .1000 \text{ or } 10.00\%$$

$$\bar{Y} = \left[\sum_{i=1}^N y_i \right] / N = \frac{[.36 - .07 + .21 - .12 + .43]}{5} = .1620 \text{ or } 16.20\%$$

We calculate the variance of each stock as:

$$s_X^2 = \left[\sum_{i=1}^N (x_i - \bar{x})^2 \right] / (N - 1)$$

$$s_X^2 = \frac{1}{5-1} \left\{ (.11 - .100)^2 + (.06 - .100)^2 + (-.08 - .100)^2 + (.28 - .100)^2 + (.13 - .100)^2 \right\} = .016850$$

$$s_Y^2 = \frac{1}{5-1} \left\{ (.36 - .162)^2 + (-.07 - .162)^2 + (.21 - .162)^2 + (-.12 - .162)^2 + (.43 - .162)^2 \right\} = .061670$$

The standard deviation is the square root of the variance, so the standard deviation of each stock is:

$$s_X = (.016850)^{1/2}$$

$$s_X = .1298 \text{ or } 12.98\%$$

$$s_Y = (.061670)^{1/2}$$

$$s_Y = .2483 \text{ or } 24.83\%$$

8. We will calculate the sum of the returns for each asset and the observed risk premium first. Doing so, we get:

<u>Year</u>	<u>Large co. stock return</u>	<u>T-bill return</u>	<u>Risk premium</u>
1973	-14.69%	7.29%	-21.98%
1974	-26.47	7.99	-34.46
1975	37.23	5.87	31.36
1976	23.93	5.07	18.86
1977	-7.16	5.45	-12.61
1978	<u>6.57</u>	<u>7.64</u>	<u>-1.07</u>
	19.41	39.31	-19.90

- a. The average return for large company stocks over this period was:

$$\text{Large company stock average return} = 19.41\% / 6$$

$$\text{Large company stock average return} = 3.24\%$$

And the average return for T-bills over this period was:

$$\begin{aligned} \text{T-bills average return} &= 39.31\% / 6 \\ \text{T-bills average return} &= 6.55\% \end{aligned}$$

- b. Using the equation for variance, we find the variance for large company stocks over this period was:

$$\begin{aligned} \text{Variance} &= 1/5[(-.1469 - .0324)^2 + (-.2647 - .0324)^2 + (.3723 - .0324)^2 + (.2393 - .0324)^2 + \\ &\quad (-.0716 - .0324)^2 + (.0657 - .0324)^2] \\ \text{Variance} &= 0.058136 \end{aligned}$$

And the standard deviation for large company stocks over this period was:

$$\begin{aligned} \text{Standard deviation} &= (0.058136)^{1/2} \\ \text{Standard deviation} &= 0.2411 \text{ or } 24.11\% \end{aligned}$$

Using the equation for variance, we find the variance for T-bills over this period was:

$$\begin{aligned} \text{Variance} &= 1/5[(.0729 - .0655)^2 + (.0799 - .0655)^2 + (.0587 - .0655)^2 + (.0507 - .0655)^2 + \\ &\quad (.0545 - .0655)^2 + (.0764 - .0655)^2] \\ \text{Variance} &= 0.000153 \end{aligned}$$

And the standard deviation for T-bills over this period was:

$$\begin{aligned} \text{Standard deviation} &= (0.000153)^{1/2} \\ \text{Standard deviation} &= 0.0124 \text{ or } 1.24\% \end{aligned}$$

- c. The average observed risk premium over this period was:

$$\begin{aligned} \text{Average observed risk premium} &= -19.90\% / 6 \\ \text{Average observed risk premium} &= -3.32\% \end{aligned}$$

The variance of the observed risk premium was:

$$\begin{aligned} \text{Variance} &= 1/5[(-.2198 - .0332)^2 + (-.3446 - .0332)^2 + (.3136 - .0332)^2 + \\ &\quad (.1886 - .0332)^2 + (-.1261 - .0332)^2 + (-.0107 - .0332)^2] \\ \text{Variance} &= 0.062078 \end{aligned}$$

And the standard deviation of the observed risk premium was:

$$\begin{aligned} \text{Standard deviation} &= (0.062078)^{1/2} \\ \text{Standard deviation} &= 0.2492 \text{ or } 24.92\% \end{aligned}$$

9. a. To find the average return, we sum all the returns and divide by the number of returns, so:

$$\begin{aligned} \text{Arithmetic average return} &= (2.16 + .21 + .04 + .16 + .19)/5 \\ \text{Arithmetic average return} &= .5520 \text{ or } 55.20\% \end{aligned}$$

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b. Using the equation to calculate variance, we find:

$$\text{Variance} = 1/4[(2.16 - .552)^2 + (.21 - .552)^2 + (.04 - .552)^2 + (.16 - .552)^2 + (.19 - .552)^2]$$

$$\text{Variance} = 0.081237$$

So, the standard deviation is:

$$\text{Standard deviation} = (0.081237)^{1/2}$$

$$\text{Standard deviation} = 0.9013 \text{ or } 90.13\%$$

10. a. To calculate the average real return, we can use the average return of the asset and the average inflation rate in the Fisher equation. Doing so, we find:

$$(1 + R) = (1 + r)(1 + h)$$

$$\bar{r} = (1.5520/1.042) - 1$$

$$\bar{r} = .4894 \text{ or } 48.94\%$$

b. The average risk premium is simply the average return of the asset, minus the average risk-free rate, so, the average risk premium for this asset would be:

$$\overline{\text{RP}} = \bar{R} - \bar{R}_f$$

$$\overline{\text{RP}} = .5520 - .0510$$

$$\overline{\text{RP}} = .5010 \text{ or } 50.10\%$$

11. We can find the average real risk-free rate using the Fisher equation. The average real risk-free rate was:

$$(1 + R) = (1 + r)(1 + h)$$

$$\bar{r}_f = (1.051/1.042) - 1$$

$$\bar{r}_f = .0086 \text{ or } 0.86\%$$

And to calculate the average real risk premium, we can subtract the average risk-free rate from the average real return. So, the average real risk premium was:

$$\overline{\text{rp}} = \bar{r} - \bar{r}_f = 4.41\% - 0.86\%$$

$$\overline{\text{rp}} = 3.55\%$$

12. Apply the five-year holding-period return formula to calculate the total return of the stock over the five-year period, we find:

$$\text{5-year holding-period return} = [(1 + R_1)(1 + R_2)(1 + R_3)(1 + R_4)(1 + R_5)] - 1$$

$$\text{5-year holding-period return} = [(1 - .0491)(1 + .2167)(1 + .2257)(1 + .0619)(1 + .3185)] - 1$$

$$\text{5-year holding-period return} = 0.9855 \text{ or } 98.55\%$$

13. To find the return on the zero coupon bond, we first need to find the price of the bond today. Since one year has elapsed, the bond now has 19 years to maturity, so the price today is:

$$P_1 = \$1,000/1.10^{19}$$

$$P_1 = \$163.51$$

There are no intermediate cash flows on a zero coupon bond, so the return is the capital gains, or:

$$R = (\$163.51 - 152.37) / \$152.37$$

$$R = .0731 \text{ or } 7.31\%$$

14. The return of any asset is the increase in price, plus any dividends or cash flows, all divided by the initial price. This preferred stock paid a dividend of \$5, so the return for the year was:

$$R = (\$80.27 - 84.12 + 5.00) / \$84.12$$

$$R = .0137 \text{ or } 1.37\%$$

15. The return of any asset is the increase in price, plus any dividends or cash flows, all divided by the initial price. This stock paid no dividend, so the return was:

$$R = (\$42.02 - 38.65) / \$38.65$$

$$R = .0872 \text{ or } 8.72\%$$

This is the return for three months, so the APR is:

$$\text{APR} = 4(8.72\%)$$

$$\text{APR} = 34.88\%$$

And the EAR is:

$$\text{EAR} = (1 + .0872)^4 - 1$$

$$\text{EAR} = .3971 \text{ or } 39.71\%$$

16. To find the real return each year, we will use the Fisher equation, which is:

$$1 + R = (1 + r)(1 + h)$$

Using this relationship for each year, we find:

	<u>T-bills</u>	<u>Inflation</u>	<u>Real Return</u>
1926	0.0330	(0.0112)	0.0447
1927	0.0315	(0.0226)	0.0554
1928	0.0405	(0.0116)	0.0527
1929	0.0447	0.0058	0.0387
1930	0.0227	(0.0640)	0.0926
1931	0.0115	(0.0932)	0.1155
1932	0.0088	(0.1027)	0.1243

So, the average real return was:

$$\text{Average} = (.0447 + .0554 + .0527 + .0387 + .0926 + .1155 + .1243) / 7$$

$$\text{Average} = .0748 \text{ or } 7.48\%$$

Notice the real return was higher than the nominal return during this period because of deflation, or negative inflation.

17. Looking at the long-term corporate bond return history in Figure 9.2, we see that the mean return was 6.2 percent, with a standard deviation of 8.6 percent. The range of returns you would expect to see 68 percent of the time is the mean plus or minus 1 standard deviation, or:

$$R \in \mu \pm 1\sigma = 6.2\% \pm 8.6\% = -2.40\% \text{ to } 14.80\%$$

The range of returns you would expect to see 95 percent of the time is the mean plus or minus 2 standard deviations, or:

$$R \in \mu \pm 2\sigma = 6.2\% \pm 2(8.6\%) = -11.00\% \text{ to } 23.40\%$$

18. Looking at the large-company stock return history in Figure 9.2, we see that the mean return was 12.4 percent, with a standard deviation of 20.3 percent. The range of returns you would expect to see 68 percent of the time is the mean plus or minus 1 standard deviation, or:

$$R \in \mu \pm 1\sigma = 12.4\% \pm 20.3\% = -7.90\% \text{ to } 32.70\%$$

The range of returns you would expect to see 95 percent of the time is the mean plus or minus 2 standard deviations, or:

$$R \in \mu \pm 2\sigma = 12.4\% \pm 2(20.3\%) = -28.20\% \text{ to } 53.00\%$$

19. To find the best forecast, we apply Blume's formula as follows:

$$R(5) = \frac{5-1}{29} \times 10.7\% + \frac{30-5}{29} \times 12.8\% = 12.51\%$$

$$R(10) = \frac{10-1}{29} \times 10.7\% + \frac{30-10}{29} \times 12.8\% = 12.15\%$$

$$R(20) = \frac{20-1}{29} \times 10.7\% + \frac{30-20}{29} \times 12.8\% = 11.42\%$$

20. The best forecast for a one year return is the arithmetic average, which is 12.4 percent. The geometric average, found in Table 9.3 is 10.4 percent. To find the best forecast for other periods, we apply Blume's formula as follows:

$$R(5) = \frac{5-1}{80-1} \times 10.4\% + \frac{80-5}{80-1} \times 12.4\% = 12.30\%$$

$$R(20) = \frac{20-1}{80-1} \times 10.4\% + \frac{80-20}{80-1} \times 12.4\% = 11.92\%$$

$$R(30) = \frac{30-1}{80-1} \times 10.4\% + \frac{80-30}{80-1} \times 12.4\% = 11.67\%$$

Intermediate

21. Here we know the average stock return, and four of the five returns used to compute the average return. We can work the average return equation backward to find the missing return. The average return is calculated as:

$$.55 = .08 - .13 - .07 + .29 + R$$

$$R = .38 \text{ or } 38\%$$

The missing return has to be 38 percent. Now we can use the equation for the variance to find:

$$\text{Variance} = 1/4[(.08 - .11)^2 + (-.13 - .11)^2 + (-.07 - .11)^2 + (.29 - .11)^2 + (.38 - .11)^2]$$

$$\text{Variance} = 0.049050$$

And the standard deviation is:

$$\text{Standard deviation} = (0.049050)^{1/2}$$

$$\text{Standard deviation} = 0.2215 \text{ or } 22.15\%$$

22. The arithmetic average return is the sum of the known returns divided by the number of returns, so:

$$\text{Arithmetic average return} = (.29 + .14 + .23 - .08 + .09 - .14) / 6$$

$$\text{Arithmetic average return} = .0883 \text{ or } 8.83\%$$

Using the equation for the geometric return, we find:

$$\text{Geometric average return} = [(1 + R_1) \times (1 + R_2) \times \dots \times (1 + R_T)]^{1/T} - 1$$

$$\text{Geometric average return} = [(1 + .29)(1 + .14)(1 + .23)(1 - .08)(1 + .09)(1 - .14)]^{(1/6)} - 1$$

$$\text{Geometric average return} = .0769 \text{ or } 7.69\%$$

Remember, the geometric average return will always be less than the arithmetic average return if the returns have any variation.

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23. To calculate the arithmetic and geometric average returns, we must first calculate the return for each year. The return for each year is:

$$R_1 = (\$49.07 - 43.12 + 0.55) / \$43.12 = .1507 \text{ or } 15.07\%$$

$$R_2 = (\$51.19 - 49.07 + 0.60) / \$49.07 = .0554 \text{ or } 5.54\%$$

$$R_3 = (\$47.24 - 51.19 + 0.63) / \$51.19 = -.0649 \text{ or } -6.49\%$$

$$R_4 = (\$56.09 - 47.24 + 0.72) / \$47.24 = .2026 \text{ or } 20.26\%$$

$$R_5 = (\$67.21 - 56.09 + 0.81) / \$56.09 = .2127 \text{ or } 21.27\%$$

The arithmetic average return was:

$$R_A = (0.1507 + 0.0554 - 0.0649 + 0.2026 + 0.2127) / 5$$

$$R_A = 0.1113 \text{ or } 11.13\%$$

And the geometric average return was:

$$R_G = [(1 + .1507)(1 + .0554)(1 - .0649)(1 + .2026)(1 + .2127)]^{1/5} - 1$$

$$R_G = 0.1062 \text{ or } 10.62\%$$

24. To find the real return we need to use the Fisher equation. Re-writing the Fisher equation to solve for the real return, we get:

$$r = [(1 + R) / (1 + h)] - 1$$

So, the real return each year was:

<u>Year</u>	<u>T-bill return</u>	<u>Inflation</u>	<u>Real return</u>
1973	0.0729	0.0871	-0.0131
1974	0.0799	0.1234	-0.0387
1975	0.0587	0.0694	-0.0100
1976	0.0507	0.0486	0.0020
1977	0.0545	0.0670	-0.0117
1978	0.0764	0.0902	-0.0127
1979	0.1056	0.1329	-0.0241
1980	0.1210	0.1252	-0.0037
	0.6197	0.7438	-0.1120

- a. The average return for T-bills over this period was:

$$\text{Average return} = 0.619 / 8$$

$$\text{Average return} = .0775 \text{ or } 7.75\%$$

And the average inflation rate was:

$$\text{Average inflation} = 0.7438 / 8$$

$$\text{Average inflation} = .0930 \text{ or } 9.30\%$$

- b. Using the equation for variance, we find the variance for T-bills over this period was:

$$\begin{aligned} \text{Variance} &= 1/7[(.0729 - .0775)^2 + (.0799 - .0775)^2 + (.0587 - .0775)^2 + (.0507 - .0775)^2 + \\ &\quad (.0545 - .0775)^2 + (.0764 - .0775)^2 + (.1056 - .0775)^2 + (.1210 - .0775)^2] \\ \text{Variance} &= 0.000616 \end{aligned}$$

And the standard deviation for T-bills was:

$$\begin{aligned} \text{Standard deviation} &= (0.000616)^{1/2} \\ \text{Standard deviation} &= 0.0248 \text{ or } 2.48\% \end{aligned}$$

The variance of inflation over this period was:

$$\begin{aligned} \text{Variance} &= 1/7[(.0871 - .0930)^2 + (.1234 - .0930)^2 + (.0694 - .0930)^2 + (.0486 - .0930)^2 + \\ &\quad (.0670 - .0930)^2 + (.0902 - .0930)^2 + (.1329 - .0930)^2 + (.1252 - .0930)^2] \\ \text{Variance} &= 0.000971 \end{aligned}$$

And the standard deviation of inflation was:

$$\begin{aligned} \text{Standard deviation} &= (0.000971)^{1/2} \\ \text{Standard deviation} &= 0.0312 \text{ or } 3.12\% \end{aligned}$$

- c. The average observed real return over this period was:

$$\begin{aligned} \text{Average observed real return} &= -.1122 / 8 \\ \text{Average observed real return} &= -.0140 \text{ or } -1.40\% \end{aligned}$$

- d. The statement that T-bills have no risk refers to the fact that there is only an extremely small chance of the government defaulting, so there is little default risk. Since T-bills are short term, there is also very limited interest rate risk. However, as this example shows, there is inflation risk, i.e. the purchasing power of the investment can actually decline over time even if the investor is earning a positive return.

25. To find the return on the coupon bond, we first need to find the price of the bond today. Since one year has elapsed, the bond now has six years to maturity, so the price today is:

$$\begin{aligned} P_1 &= \$80(\text{PVIFA}_{7\%,6}) + \$1,000/1.07^6 \\ P_1 &= \$1,047.67 \end{aligned}$$

You received the coupon payments on the bond, so the nominal return was:

$$\begin{aligned} R &= (\$1,047.67 - 1,028.50 + 80) / \$1,028.50 \\ R &= .0964 \text{ or } 9.64\% \end{aligned}$$

And using the Fisher equation to find the real return, we get:

$$\begin{aligned} r &= (1.0964 / 1.048) - 1 \\ r &= .0462 \text{ or } 4.62\% \end{aligned}$$

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26. Looking at the long-term government bond return history in Table 9.2, we see that the mean return was 5.8 percent, with a standard deviation of 9.3 percent. In the normal probability distribution, approximately 2/3 of the observations are within one standard deviation of the mean. This means that 1/3 of the observations are outside one standard deviation away from the mean. Or:

$$\Pr(R < -3.5 \text{ or } R > 15.1) \approx 1/3$$

But we are only interested in one tail here, that is, returns less than -3.5 percent, so:

$$\Pr(R < -3.5) \approx 1/6$$

You can use the z-statistic and the cumulative normal distribution table to find the answer as well. Doing so, we find:

$$z = (X - \mu)/\sigma$$

$$z = (-3.5\% - 5.8)/9.3\% = -1.00$$

Looking at the z-table, this gives a probability of 15.87%, or:

$$\Pr(R < -3.5) \approx .1587 \text{ or } 15.87\%$$

The range of returns you would expect to see 95 percent of the time is the mean plus or minus 2 standard deviations, or:

$$95\% \text{ level: } R \in \mu \pm 2\sigma = 5.8\% \pm 2(9.3\%) = -12.80\% \text{ to } 24.40\%$$

The range of returns you would expect to see 99 percent of the time is the mean plus or minus 3 standard deviations, or:

$$99\% \text{ level: } R \in \mu \pm 3\sigma = 5.8\% \pm 3(9.3\%) = -22.10\% \text{ to } 33.70\%$$

27. The mean return for small company stocks was 17.5 percent, with a standard deviation of 33.1 percent. Doubling your money is a 100% return, so if the return distribution is normal, we can use the z-statistic. So:

$$z = (X - \mu)/\sigma$$

$$z = (100\% - 17.5\%)/33.1\% = 2.492 \text{ standard deviations above the mean}$$

This corresponds to a probability of $\approx 0.634\%$, or less than once every 100 years. Tripling your money would be:

$$z = (200\% - 17.5\%)/33.1\% = 5.514 \text{ standard deviations above the mean.}$$

This corresponds to a probability of (much) less than 0.5%, or once every 200 years. The actual answer is $\approx .00000176\%$, or about once every 1 million years.

28. It is impossible to lose more than 100 percent of your investment. Therefore, return distributions are truncated on the lower tail at -100 percent.

Challenge

29. Using the z-statistic, we find:

$$z = (X - \mu)/\sigma$$

$$z = (0\% - 12.4\%)/20.3\% = -0.6108$$

$$\Pr(R \leq 0) \approx 27.07\%$$

30. For each of the questions asked here, we need to use the z-statistic, which is:

$$z = (X - \mu)/\sigma$$

a. $z_1 = (10\% - 6.2\%)/8.6\% = 0.4419$

This z-statistic gives us the probability that the return is less than 10 percent, but we are looking for the probability the return is greater than 10 percent. Given that the total probability is 100 percent (or 1), the probability of a return greater than 10 percent is 1 minus the probability of a return less than 10 percent. Using the cumulative normal distribution table, we get:

$$\Pr(R \geq 10\%) = 1 - \Pr(R \leq 10\%) = 1 - .6707 \approx 32.93\%$$

For a return less than 0 percent:

$$z_2 = (0\% - 6.2\%)/8.6 = -0.7209$$

$$\Pr(R < 10\%) = 1 - \Pr(R > 0\%) = 1 - .7645 \approx 23.55\%$$

- b. The probability that T-bill returns will be greater than 10 percent is:

$$z_3 = (10\% - 3.8\%)/3.1\% = 2$$

$$\Pr(R \geq 10\%) = 1 - \Pr(R \leq 10\%) = 1 - .9772 \approx 2.28\%$$

And the probability that T-bill returns will be less than 0 percent is:

$$z_4 = (0\% - 3.8\%)/3.1\% = -1.2258$$

$$\Pr(R \leq 0) \approx 11.01\%$$

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- c. The probability that the return on long-term corporate bonds will be less than -4.18 percent is:

$$z_5 = (-4.18\% - 6.2\%) / 8.6\% = -1.20698$$

$$\Pr(R \leq -4.18\%) \approx 11.37\%$$

And the probability that T-bill returns will be greater than 10.32 percent is:

$$z_6 = (10.32\% - 3.8\%) / 3.1\% = 2.1032$$

$$\Pr(R \geq 10.38\%) = 1 - \Pr(R \leq 10.38\%) = 1 - .9823 \approx 1.77\%$$