Comments and Examples of NPV calculations

With applications to real life

Annuity Tables

There are tables that calculate the value of an annuity that pays $1 a year for n years at various interest rates. Table A.2 in your book (page 874-875) is one such table.

# of periods	6%	7%	8%	9%
11	7.8869	7.4987	7.1390	6.8052
12	8.3838	7.9427	7.5361	7.1607
13	8.8527	8.3577	7.9038	7.4869

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The notation for one of these entries is $A_{.06}^{12}$.

If we have 12 annual payments of $500, at an interest rate of 6%, the annuity is worth

$$500 \cdot A_{.06}^{12} = 500 \cdot 8.3838 = 4191.90$$

Limitations of the tables

- These tables usually only show integer interest rates.
- 1-20 periods are shown, but entries for 23 are missing.
- If you can use them, they'll save some calculation. However many situations will occur where the tables can't be used.

Future Value

One other concept that we will be using is **Future Value**. For an Annuity:

$$FV = (1+r)^T \times C\left[\frac{1}{r} - \frac{1}{r(1+r)^T}\right]$$

$$= C\left[\frac{(1+r)^T}{r} - \frac{1}{r}\right]$$

Infrequent Annuities

An annuity that pays every two years (or less)

Change time period.

Example: $100 every two years for 2T years with an interest rate of 8%.

Lottery Example

Suppose you win the lottery. It was advertised as a $100 Million lottery, but as payment options, you could accept a lump sum payment of $50 Million or 30 annual payments starting today of $3,333,333.33 (the payments add up to the advertised $100M).

Which option has a higher present value, if the interest rate is 8%?

Analysis:

The present value of the lottery annuity is given by:

$$PV = \$3{,}333{,}333.33 \cdot \left(1 + \left[\frac{1}{0.08} - \frac{1}{0.08 \cdot (1.08)^{29}}\right]\right)$$

$$= \$3{,}333{,}333.33 \cdot \left(1 + A_{0.08}^{29}\right)$$

$$= \$3{,}333{,}333.33 \cdot 12.1584\ldots$$

$$= \$40{,}528{,}019.99$$

Saving for Retirement

Two twenty year olds are saving for their retirement at age 50 (perhaps a little too optimistically). Julie decides to save $1 a year in her twenties, but tires of saving, and so does not save any money after she turns 30.

Peter does not save in his 20’s, but to try to make up for this oversight, he saves $1 each year in his 30’s and 40’s. Who has more money when they retire at 50?

Analysis for Julie

At the end of 10 years she will have

$$\$1 \cdot \left[\frac{(1.12)^{10}}{0.12} - \frac{1}{0.12} \right] = \$17.5487$$

in her account. Taking this forward another 20 years, she will have $169.28 in her account.

Analysis for Peter

He begins saving when he is 30, and saves for 20 years. The money in his account at the end of those 20 years is

$$\$1 \cdot \left[\frac{(1.12)^{20}}{0.12} - \frac{1}{0.12} \right] = \$72.05$$

This is a lot less than Julie’s $169.28, even though the sum of the payments is less.

Mortgage Payments

Given the length of the mortgage is 25 years, payments are made annually, the interest rate is 8% annually, and the total amount financed is $300,000 what will be the annual mortgage payments?

Mortgage Payments

Let the annual mortgage payment be *C*. Then we need to find an annuity with 25 payments *C* at an interest rate of 8% that has a NPV of $300,000.

$$\$300{,}000 = C \cdot A_{0.08}^{25} = C \cdot 10.675$$

Solving for *C*, we find *C* = $28, 100 per year.

The value of tax deferment

Let’s look at investing $2,000 a year in a tax-free IRA.

Assume the IRA yields 10% annually, and there are 35 years that $2,000 is contributed to the IRA. The future value is

$$FV = \frac{\$2,000}{0.1}\left[(1.1)^{35} - 1\right] = \$542,049$$

Collecting in retirement

Now, after building this nest egg, it will be used to support the retiree for 20 years. The tax rate on the payments will be assumed to be 30%. What are the yearly payments that the IRA will provide?

Collecting in retirement

$$\$542{,}049 = C \cdot \left[\frac{1}{0.1} - \frac{1}{0.1 \cdot (1.1)^{20}} \right]$$

And thus $C = \$63, 663$. But this is the pre-tax payment, so the actual payment to the retiree will be only 70% of this, or \$44,568

General Savings account

Assume the marginal tax rate is 50%. What changes?

- Initial Investment
- Return

Thus, the FV at retirement is

$$FV = \frac{\$1,000}{0.05}\left[(1.05)^{35} - 1\right] = \$90,320$$

Collecting at retirement II

Thus, the 20 year annuity from the savings account will be

$$\$90{,}320 = C \cdot \left[\frac{1}{0.07} - \frac{1}{0.07 \cdot (1.07)^{20}} \right]$$

So $C = \$8{,}526$.

No further taxes.

Comparing the two cases:

	Tax-Free IRA	Regular Account
Balance at Retirement	$542,049	$90,320
After-Tax Annuity	$44,568	$8,526

Lessons:

Two lessons can be taken away from these lectures:

- Begin saving for retirement early.
- Tax deferral increases the value of savings.