1. To buy a house, you need to borrow $300,000. The bank offers you a 30 year loan at a stated annual interest rate of 6% (compounded monthly). You will make fixed monthly payments **starting one month from today** until the entire debt is repaid.

a. What is your monthly mortgage payment?

b. After five years (and sixty mortgage payments), what is the remaining balance on your loan?

Answer:

a. First note that the monthly interest rate is .06/12=.005

$$\begin{aligned} 300000 &= X \cdot A_{.005}^{360} \\ X &= \frac{300000}{166.8} = 1798 \end{aligned}$$

b. The present value of those five years of mortgage payments is an annuity. You've made 60 payments of $1,798:

$$\begin{aligned} PV(payments) &= 1798 \cdot A_{.005}^{60} \\ &= 1798 \cdot 51.72 \\ &= 92992 \end{aligned}$$

The present value of your remaining balance is simply

$$\begin{aligned} PV(house) - PV(payments) &= 300000 - 92992 \\ &= 207008 \end{aligned}$$

The remaining balance after five years is then

$$207008 \cdot 1.005^{60} = 279222$$

2. **You are a financial advisor who believes strongly in the CAPM**. A new client, John, shows you his current portfolio of investments. You calculate the following:

	Recession	Normal	Expansion
(prob)	20%	50%	30%
market portfolio return	-4%	10%	20%
John's portfolio return	-12%	12%	30%

Suppose the risk free rate is 5%.

a. What are the expected returns and the standard deviations for the market and John's portfolios, respectively? ($E[R_M]$, $E[R_{John}]$, σ_M, σ_{John})

b. You tell John he could lower the risk (standard deviation) of his portfolio without lowering the expected return. Explain how.

c. You tell John he alternatively could raise the expected return of his portfolio without raising the risk. Explain how.

A numerical result is required for each part.

Answer:

a.

	Recession	Normal	Expansion	E[R]	σ
(prob)	20%	50%	30%	-	-
market portfolio return	-4%	10%	20%	10.2	8.32
John's portfolio return	-12%	12%	30%	12.6	14.56

b.

$$\begin{aligned} 12.6 &= 5 + \frac{\sigma}{8.32}(10.2 - 5) \\ \sigma &= \frac{12.6 - 5}{10.2 - 5}(8.32) \\ &= 12.16 \end{aligned}$$

c.

$$\begin{aligned} E[r] &= 5 + \frac{14.56}{8.32}(10.2 - 5) \\ &= 14.1 \end{aligned}$$

3. Nantucket Airlines is currently an all equity company with 1 million shares outstanding. It has earnings of \$15 million per year in perpetuity. All earnings are distributed to shareholders as dividends.

The cost of equity capital is 15% and the corporate tax rate is 30%.

a. What is the current price per share of the company?

b. The company issues \$30 million worth of bonds at a cost of 8% and use the proceeds to retire \$30 million of the stock. What will be the new price per share of the company? How many shares are retired? What will be the return on the levered equity?

Answer

a.

$$\begin{aligned} V_U &= \frac{\$15}{.15}(1-0.30) = \$70 \\ \text{price}_{stock} &= \$70 \text{ million } / \text{ 1 million } = \$70/\text{share} \end{aligned}$$

b.

$$\begin{aligned} V_L &= V_U + T_C B = \$70 + .30(\$30) = \$79 \\ \text{price}_{stock} &= \$79/1 = \$79/\text{share} \end{aligned}$$

The company retires \$30 million / \$79 = 379,746.84 shares, leaving 1,000,000 - 379,746.84 = 620253.16 shares outstanding.

Of the now \$79 million company, \$30 million is debt and the remaining \$49 million is equity.

$$\begin{aligned} r_S &= r_0 + \frac{B}{S} \times (1-T_C)(r_0 - r_B) \\ &= .15 + \frac{30}{49}(.70)(.15-.08) \\ &= 0.18 \end{aligned}$$

4. Company XYZ has a market value of \$80 million dollars. Its debt-equity ratio is 1/3 and there are 1 million shares of stock outstanding. XYZ's current cost of levered equity is 12% and their cost of debt is 8%.

Tomorrow, XYZ will announce a new project. The project will cost \$5 million and return \$1.1 million every year, starting one year from now. Assume that there are **no corporate taxes**.

a. What will be the price of the stock immediately after the project is announced (assuming markets are semi-strong efficient)?

b. If the firm issues debt to cover the project cost, what will be the return on XYZ stock after the new debt has been issued?

Answer:

a. Initially, we have

$$\begin{aligned}
V_{equity} &= \frac{3}{4} \cdot 80 = \$60 \text{ million} \\
V_{debt} &= \frac{1}{4} \cdot 80 = \$20 \text{ million} \\
\text{price}_{stock} &= \$60 \text{ million}/1 \text{ million} = \$60 \\
r_0 &= \frac{1}{4} \cdot .08 + \frac{3}{4} \cdot .12 = .11
\end{aligned}$$

Immediately after the announcement, the NPV of XYZ rises by

$$NPV = -5 + \frac{1.1}{.11} = 5$$

This \$5 million which will be split up between the 1 million shares, meaning each share rises by \$5. The new share price is then \$60 + \$5 = \$65 and $V_{equity} = 60 + 5 = \$65$ million.

b. After the debt has been issued, $V_{debt} = 20 + 5 = \$25$ million. In (a) we found $V_{equity} = \$65$ million. So we can find the new required return on equity by

$$\begin{aligned}
r_s &= r_0 + \frac{B}{S} \times (r_0 - r_B) \\
&= .11 + \frac{25}{65} \cdot (.11 - .08) \\
&= .12154
\end{aligned}$$