

1. (30 pts) Two firms, Blue and Yellow will merge to form a new firm called Green. Firm Blue is all equity financed with a value of \$120 and an equity beta of 2.00. Firm Yellow is also equity financed, its stock has a beta of 0.8 and a value of \$280. The merged firm Green will have debt in its capital structure and maintain a debt-to-equity ratio of $\frac{1}{2}$. Green's debt has a beta of 0.2. There are no taxes, and no synergies are created by merging the two firms. The risk free rate is 6% and the expected market risk premium is 9%.

- What is Green's asset beta?
- What is Green's equity beta?
- What is Green's WACC? What is the WACC if the tax rate was $T_C=34\%$?
- What is Green's required return to equity? To debt?

a). Total value = $\$120 + 280 = 400$

5pts $\beta_G = \frac{120}{400} \beta_B + \frac{280}{400} \beta_Y = .3 \cdot 2 + .7 \cdot .8 = 1.16$

b). ~~$\beta_A = 1.16$~~ $\beta_A = 1.16$ $\frac{D}{E} = \frac{1}{2} \Rightarrow 2D = E$ $\frac{E}{D+E} = \frac{2D}{3D} = \frac{2}{3}$
 $\therefore \frac{D}{D+E} = \frac{1}{3}$

5pts $1.16 = \frac{D}{D+E} \beta_D + \frac{E}{D+E} \beta_E = \frac{1}{3} \cdot (0.2) + \frac{2}{3} \beta_E$

$\Rightarrow \beta_E = \frac{1.16 - \frac{1}{3}(0.2)}{\frac{2}{3}} = 1.64$

c). SML: $\bar{R}_i = 6\% + \beta(9\%)$

Equity: $R_e = 6\% + 1.64 \cdot 9\% = 20.76\%$ (5pts)

Debt: $R_d = 6\% + .2 \cdot 9\% = 7.8\%$ (5pts)

d). WACC: $\frac{E}{D+E} r_e + \frac{D}{D+E} r_d = \frac{2}{3}(20.76) + \frac{1}{3}(7.8) = 16.44\%$ (5pts)

w/Tax: $\frac{E}{D+E} r_e + \frac{D}{D+E} r_d (1 - T_C) = \frac{2}{3}(20.76) + \frac{1 - T_C}{3}(7.8) = 15.55\%$ (5pts)

2. (25 pts) The market portfolio consists of two stocks, U and V. Stock U is worth \$60 and stock V is worth \$40. The expected returns on U and V are 7.90% and 18.15% respectively. The standard deviation of U is 18% and the standard deviation of V is 42%. The correlation coefficient between them is 0.5.

- 7 a. Calculate the return and standard deviation of the market portfolio
 7 b. If the risk-free rate is 4%, calculate the beta for U and V
 6 c. What is the correlation of U with the market? What is V's correlation with the market?
 5 d. If a well diversified portfolio had a SD of 20%, what would its expected return be?

a). $r_m = \frac{60}{100} r_u + \frac{40}{100} r_v$ (weighted avg returns)

3pts $= (.6)(7.9) + (.4)(18.15) = 12.7\%$

4pts $SD = \sqrt{(.6)^2(.18)^2 + (.4)^2(.42)^2 + 2(.6)(.4)(.18)(.42)(.5)} = \sqrt{.058032} = .2409 = 24.09\%$

b). SML: $4\% + \beta(R_m - 4\%) = 4\% + \beta(8\%)$ (1pt)
 -1 if forgetting this -1 for no \sqrt

3pts U: $7.9 = 4\% + \beta_u(8\%) \Rightarrow \beta_u = \frac{7.9 - 4}{8} = .4875$

3pts V: $18.15 = 4\% + \beta_v(8\%) \Rightarrow \beta_v = 1.769$

c). $\beta_u = \frac{\rho_{u,m} \sigma_m \sigma_u}{\sigma_u^2} = \frac{\rho_{u,m} \sigma_m \sigma_u}{\sigma_u^2} = \frac{\rho_{u,m} \sigma_m}{\sigma_u} \Rightarrow \rho_{u,m} = \beta_u \left(\frac{\sigma_m}{\sigma_u} \right)$

3pts $\rho_{u,m} = .488 \left(\frac{24.09}{18.00} \right) = .653$

3pts $\rho_{v,m} = 1.769 \left(\frac{24.09}{42.00} \right) = 1.015$

d). CML: $\sigma_p = X_m \sigma_m \Rightarrow X_m = \frac{20}{24.09} = 83\%$ (27% rf)

5pts $R_p = X_m R_m + X_{rf} R_{rf} = .83(.12) + (.17)(.04) = 10.64\%$

3. (25 pts) There are three possible states in the economy, and they are not equally likely:

	Probability	Asset 1	Asset 2	Asset 3
Recession	.2	-4%	5%	6%
Normal	.5	9%	5%	3%
Boom	.3	25%	5%	8%

- 6 a. What are the expected returns for each asset? *2 each*
 9 b. What are the standard deviations of each asset? *3 each*
 5 c. What are the covariances of each pair of assets (1+2, 2+3, 1+3)?
 5 d. What are the correlation coefficients of each pair of assets?

a) $R_1 = .2(-4) + .5(9) + .3(25) = 12.8\%$ $R_2 = 5\%$
 $R_3 = .2(6) + .5(3) + .3(8) = 5.1\%$

b) $\sigma_1 = \sqrt{(-4-12.8)^2(.2) + (9-12.8)^2(.5) + (25-12.8)^2(.3)} = \sqrt{108.32} = 10.41\%$
 $\sigma_2 = 0$ $\sigma_3 = \sqrt{(6-5.1)^2(.2) + (3-5.1)^2(.5) + (8-5.1)^2(.3)} = 2.23\%$

c) $\sigma_{12} = 0$ $\sigma_{23} = 0$

③ $\sigma_{13} = \sqrt{(-4-12.8)(6-5.1)(.2) + (9-12.8)(3-5.1)(.5) + (25-12.8)(8-5.1)(.3)}$
 $= \sqrt{11.58} = 3.4$ 11.58

d) $\rho_{12} = 0$ $\rho_{23} = 0$ $\rho_{13} = \frac{\sigma_{13}}{\sigma_1 \sigma_3} = \frac{11.58}{(10.41)(2.23)} = .4988$

4 (20 pts) The risk-free rate is 4.5%. Security A has a standard deviation of 22% and an expected return of 14%. The market portfolio has a standard deviation of 16%, and the correlation coefficient between the market portfolio and Security A is 0.75.

- What is Security A's Beta?
- What is the Expected Market Risk Premium? *analysts expect a*
- Now suppose Stock B has a beta of 1.2 and an ~~expected~~ return of 16%. Is Stock B overpriced, underpriced, or correctly priced? *according to the analysts?*

5 a). $\beta_A = \frac{\sigma_{AM}}{\sigma_M^2} = \frac{\rho_{AM} \sigma_A \sigma_M}{\sigma_M^2} = \frac{(0.75)(22)(16)}{(16)^2} = 1.03$

5 b). SML: $R_i = 4.5\% + \beta(EMRP)$. For A,
 $14\% = 4.5\% + (1.03)(EMRP) \Rightarrow EMRP = \frac{14 - 4.5}{1.03} = 9.223$

c). By CAPM:

$$R_B = 4.5 + 1.2(EMRP) = \boxed{15.57\%} \quad 5$$

B would be correctly priced if the return were 15.57%.

Since it returns higher, it is underpriced 5