Comments and Examples of NPV calculations

Annuity Tables

There are tables that calculate the value of an annuity that pays $1 a year for n years at various interest rates. Table A.2 in your book (page 874-875) is one such table. A sampling of the table is below:

# of periods	6%	7%	8%	9%
11	7.8869	7.4987	7.1390	6.8052
12	8.3838	7.9427	7.5361	7.1607
13	8.8527	8.3577	7.9038	7.4869

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The notation for one of these entries is $A_{.06}^{12}$.

That is the first entry in the '12 period' row. The 12 means 12 payments, and the 0.06 is the interest rate.

If we have 12 annual payments of $500, at an interest rate of 6%, the annuity is worth

$$500 \cdot A_{.06}^{12} = 500 \cdot 8.3838 = 4191.90$$

Future Value

One other concept that we will be using is **Future Value**. It is the value of a cash flow or flows at some date in the future. For instance, the future value of an annuity at the end of the *T* payments will be:

$$FV = (1+r)^T \times C\left[\frac{1}{r} - \frac{1}{r(1+r)^T}\right]$$

$$= C\left[\frac{(1+r)^T}{r} - \frac{1}{r}\right]$$

This gives the value of the annuity at year *T*, in year *T* dollars.

Infrequent Annuities

This works with perpetuities as well. If we have an annuity that pays every two years, we can still use our formula to obtain the value of the annuity

The way to do this is to treat the time between each annuity payment as one period of time, and calculate the interest rate appropriately.

Example: $100 every two years for 2T years with an interest rate of 8%. This has the same NPV as a yearly annuity of $100 for T years with an interest rate of 16.64%.

Lottery Example

Suppose you win the lottery. It was advertised as a $100 Million lottery, but as payment options, you could accept a lump sum payment of $50 Million or 30 annual payments starting today of $3,333,333.33 (the payments add up to the advertised $100M).

Which option has a higher present value, if the interest rate is 8%?

Analysis:

The present value of the lottery annuity is given by:

$$PV = \$3{,}333{,}333.33 \cdot \left(1 + \left[\frac{1}{0.08} - \frac{1}{0.08 \cdot (1.08)^{29}} \right] \right)$$

$$= \$3{,}333{,}333.33 \cdot \left(1 + A_{0.08}^{29} \right)$$

$$= \$3{,}333{,}333.33 \cdot 12.1584...$$

$$= \$40{,}528{,}019.99$$

This is less than the lump sum payment, so we should choose to accept the lump sum payment, and not the annuity (most lotteries are set up this way)

Saving for Retirement

Two twenty year olds are saving for their retirement at age 50 (perhaps a little too optimistically). Julie decides to save $1 a year in her twenties, but tires of saving, and so does not save any money after she turns 30.

Peter does not save in his 20’s, but to try to make up for this oversight, he saves $1 each year in his 30’s and 40’s. Who has more money when they retire at 50?

Assume the interest rate r is 12%

Analysis for Julie

At the end of 10 years (when she is 30, and made her last payment) she will have

$$\$1\cdot\left[\frac{(1.12)^{10}}{0.12}-\frac{1}{0.12}\right]=\$17.5487$$

in her account. Taking this forward another 20 years (multiplying by 1.1220), she will have $169.28 in her account.

Analysis for Peter

He begins saving when he is 30, and saves for 20 years. The money in his account at the end of those 20 years is

$$\$1 \cdot \left[\frac{(1.12)^{20}}{0.12} - \frac{1}{0.12} \right] = \$72.05$$

This is a lot less than Julie’s $169.28, even though the sum of the payments is less.

Mortgage Payments

Given the length of a loan is 25 years, payments are made annually, the interest rate is 8% annually, and the total amount financed is $300,000 what will be the annual mortgage payments?

Let the annual mortgage payment be *C*. Then we need to find an annuity with 25 payments *C* at an interest rate of 8% that has a NPV of $300,000. Using the annuity tables, we have

$$\$300{,}000 = C \cdot A_{0.08}^{25} = C \cdot 10.675$$

Solving for *C*, we find *C* = $28, 100 per year.

The value of tax deferment

Let’s look at investing $2,000 a year in a tax-free IRA.

Assume the IRA yields 10% annually, and there are 35 years that $2,000 is contributed to the IRA. How much money will be in the IRA at the end of the 35 years? The future value of the payments will be

$$FV = \frac{\$2,000}{0.1}\left[(1.1)^{35} - 1\right] = \$542,049$$

Collecting in retirement

Now, after building this nest egg, it will be used to support the retiree for 20 years. The tax rate on the payments will be assumed to be 30%. What are the yearly payments that the IRA will provide?

$$\$542{,}049 = C \cdot \left[\frac{1}{0.1} - \frac{1}{0.1 \cdot (1.1)^{20}} \right]$$

And thus C = \$63, 663. But this is the pre-tax payment, so the actual payment to the retiree will be only 70% of this, or \$44,568

General Savings account

Assume the marginal tax rate is 50%. What would change? We still set aside $2,000 in pre-tax income. After taxes, the initial $2,000 is only valued at $1,000.

The effective interest rate will be 5%. This is because half of the interest is lost to taxes. Thus, the future value at retirement of this savings is

$$FV = \frac{\$1,000}{0.05}\left[(1.05)^{35} - 1\right] = \$90,320$$

Collecting at retirement II

The after tax interest rate on our savings account is 7%. (30% of the interest is taxed away). Thus, the 20 year annuity from the savings account will be

$$\$90{,}320 = C \cdot \left[\frac{1}{0.07} - \frac{1}{0.07 \cdot (1.07)^{20}} \right]$$

Solving for *C*, we obtain *C* = \$8, 526, which is the actual annuity, since the principal and interest have already been taxed—there are no more additional taxes.

Comparing the two cases:

	Tax-Free IRA	Regular Account
Balance at Retirement	$542,049	$90,320
After-Tax Annuity	$44,568	$8,526