Answers to Problem 19.3 in Kreps

Here is my proposed answer.

a) If Entrepreneur bears all the risk and tries hard, her utility will be

$$.1(\sqrt{100,000,000} - 500) + .9(0 - 500) = .1(10,000) - 500 = 500$$

If she doesn't try, it will be

$$.02(\sqrt{100,000,000} + .98(0) = 200.$$

So she will try hard.

b) If the VC can contract with the entrepreneur about her effort level, the efficient contract would be one where she tries hard and the VC bears all the risk. (Remember, the VC is risk neutral, the entrepreneur is risk averse.) The entrepreneur would get an upfront payment of B and nothing more, but would contract to work hard. The VC would get the entire $100,000,000 if the project succeeds. If she takes this contract her utility will be

$$\sqrt{B} - 500$$

She will accept this contract only if it is at least as good for her as bearing all the risk herself. That is, only if

$$\sqrt{B} - 500 \geq 500$$

or equivalently

$$\sqrt{B} \geq 1000.$$

The lowest amount that the the VC can offer the entrepreneur and get a deal is therefore $1000^2 = 1,000,000$. The expected return to the VC is $.1 \times 100,000,000 = \$10,000,000$.

c) Suppose that the VC cannot write a contract for effort level, but can offer to pay an amount B upfront and to let the entrepreneur keep the amount $100,000,000 - X$ of her profits if the venture is successful. What values of B and X will he choose to make sure she is (just barely) willing to take the contract and that she will do high effort. If the project succeeds, her income will be $B + 100,000,000 - X$ and if it fails her income will be B. Her expected utility if she tries hard will be

$$.1\sqrt{B + 100,000,000 - X} + .9\sqrt{B} - 500$$

and if she doesn't try hard, it will be

$$.02\sqrt{B + 100,000,000 - X} + .98\sqrt{B}.$$

The most profitable deal for VC is one in which the entrepreneur is just barely willing to try hard and just barely willing to take the deal.

Let us define $z_1 = \sqrt{B + 100,000,000 - X}$ and $z_2 = \sqrt{B}$. Then the entrepreneur will be just on each margin if

$$.1z_1 + .9z_2 - 500 = .02z_1 + .98z_2 = 500.$$

This is two equations in two unknowns, z_1 and z_2. Solve these equations for z_1 and z_2. Then square z_1 and z_2 to find B and $B + 100,000,000 - X$. Then finally solve for X. When I did this I found $z_2 = 375$ and $z_1 = 6625$. Then $B = 375^2 = 140,625$ and $B + 100,000,000 - X = 43,890,625$. So $X = 56,250,000$. The expected profit for the VC would then be $.1X - B = 5,625,000 - 140,625$.

I won't promise that I didn't make an algebraic mistake somewhere. Let me know if you think I did.