**Vector space (aka Linear space) and convex combinations**

- A collection of objects called vectors, which can be added together or multiplied by scalars.— Euclidean $n$-space is an example
- If $S$ is a vector space, a convex combination of two elements $x \in S$ and $y \in S$ of a linear space is an element $\lambda x + (1 - \lambda)y$ of $S$ where $\lambda \in [0, 1]$.
- A set $A$ is said to be a convex set if for all $x$ and $y$ in $A$, every convex combination of $x$ and $y$ is also in $A$. 
Convex preferences

- The preference relation $\succeq$ on $X$ is defined to be convex if for all $x$ and $y$ in $X$, if $x \succeq y$, then $\lambda x + (1 - \lambda)y \succeq y$ for all $\lambda \in [0, 1]$.
- This is equivalent to the statement that: For all $y \in X$, $\succeq (y)$ is a convex set.
Strictly convex preferences

- The preference relation $\succeq$ is *strictly convex* if for all $x$ and $y$ in $X$, if $x \neq y$ and $x \succeq y$, then $\lambda x + (1 - \lambda)y \succ y$ for all $\lambda \in [0, 1]$. (Every convex combination of two distinct vectors is strictly preferred to at least one of them.)

- The preference relation $\succeq$ is *semi-strictly convex* if for all $x$ and $y$ in $X$, if $x \succ y$, then $\lambda x + (1 - \lambda)y \succ y$ for all $\lambda \in [0, 1)$.

- Find an example of preferences that are semi-strictly convex, but not strictly convex.
Quasi-concave functions

- A function $f: A \to \mathbb{R}$ is quasi-concave if for $x, y \in A$, $f(x) \geq f(y)$ implies that $f(\lambda x + (1 - \lambda)y) \geq f(y)$ for all $\lambda \in [0, 1]$.
- A function $f: A \to \mathbb{R}$ is strictly quasi-concave if for $x, y \in A$, $f(x) \geq f(y)$ and $x \neq y$ implies that $f(\lambda x + (1 - \lambda)y) > f(y)$ for all $\lambda \in [0, 1)$.
- A function $f: A \to \mathbb{R}$ is semi-strictly quasi-concave if for $x, y \in A$, if $f(x) > f(y)$ implies that $f(\lambda x + (1 - \lambda)y) > f(y)$ for all $\lambda \in [0, 1)$. 
Concave functions

- A function $f : A \to \mathbb{R}$ is concave if for $x, y \in A$, $f(\lambda x + (1 - \lambda)y) \geq \lambda f(x) + (1 - \lambda)f(y)$ for all $\lambda \in [0, 1]$.
- A concave function must be quasi-concave, but the converse is not true... Prove this.
- Rainshed property: The set $\{(x, y) | f(x) \leq y\}$ is convex.
- What do concave functions of single variable look like? What do quasi-concave functions of a single variable look like?