Cultural Evolution for Families and Tribes

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The idea of this paper:

- Suppose that:
  - individuals acquire their behavior by copying others and that relatively “successful” behaviors are copied more often.
  - individuals frequently encounter others who have the same role models as themselves.
- In these circumstances we are likely to see the emergence of partially “altruistic” behavior.
A Reduced-Form Model

- In a population, individuals meet others according to some encounter rule.
- On encounter, they play a two-player, two-strategy symmetric game.
- Individuals are of two types, one for each strategy.
- The strategy type with the higher expected payoff reproduces at the faster rate.
Some Notation

A Two-Player Symmetric Game

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<th>Player 2</th>
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<tr>
<td></td>
<td>C</td>
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<tr>
<td>Player 1</td>
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<tr>
<td>C</td>
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- If Game is Prisoners’ Dilemma, $T > R > P > S$. 
Algebra of Encounters

- Let $x$ be proportion of C-strategists in population.
- Let $p$ be the probability that a C-strategist encounters a C-strategist.
- Let $q$ be the probability that a D-strategist encounters a C-strategist.
- Parity requirement—Number of C’s who encounter D’s equals number of D’s who encounter C’s.
  
  That is:  $x(1-p)=(1-x)q$
An Index of Assortativity

- Define the difference between the probability that a C-strategist meets a C-strategist and the probability a D-strategist meets a C-strategist to be \( a = p - q \).

- Then \( a = (1 - q) - (1 - p) \) is also the difference between the probability that a D-strategist meets a D-strategist and the probability that a C-strategist meets a D-strategist.

- Thus for either type, \( a \) is the difference between the probability of meeting your own type and the probability of meeting the other type.
Relating p, q and x

- From the parity equation \( x(1 - p) = (1 - x)q \), it follows that \( a = p - q \) if and only if:

  \[
  p = x + a(1 - x) \quad \text{and} \\
  q = x(1 - a)
  \]

- An interpretation for \( a \) where \( 0 < a < 1 \). Suppose there are purely random initial meetings. Meetings between two persons of the same type result in a match. When opposite types meet, the fraction \( 1 - a \) form matches with the individual they met. The remainder are matched with their own type.
Difference in Payoffs Between Types

- Expected payoffs are respectively:
  - C-strategist:
    \[ pR + (1 - p)S = aR + (1 - a)S + x(1 - a)(R - S) \]
  - D-strategist:
    \[ qT + (1 - q)P = P + x(1 - a)(T - P) \]

- **Theorem:** The difference between the expected payoff of a C-strategist and that of a D-strategist is
  \[ \delta(x) = aR + (1 - a)S - P + x(1 - a)[(R + P) - (S + T)] \]
A Special Case—Additive Benefits and Costs

- Let \( c \) be the cost of helping the other guy and \( b \) be the benefit received from being helped.

- Payoffs are \( T = b, \ R = b - c, \ P = 0, \ S = -c \)

- Then \( T + S = R + P \), so the difference between payoffs of C and D strategist is simply \( aR + (1 - a)S - P = ab - c \).

- In this case, C-strategists always prevail if \( ab > c \) and D-strategists if \( ab < c \).

- In biology, this is *Hamilton’s Rule*, where the coefficient of relatedness is \( a \).
General Payoffs with constant $a$

- In general if $a$ is constant, $\delta(x)$ changes linearly with $x$
- As $x$ ranges from 0 to 1, $\delta(x)$ ranges from

$$
\delta(0) = [aR + (1 - a)S] - P \quad \text{to}
$$

$$
\delta(1) = R - [aP + (1 - a)T].
$$

- Now there are four possible combinations of signs for $\delta(0)$ and $\delta(1)$, leading to four qualitatively distinct possible phase diagrams.
Unique Equilibrium with $x=1$

\[ aR + (1-a)S > P \quad \text{and} \quad R > aP + (1-a)T \]
\[ \delta(0) > 0 \quad \text{and} \quad \delta(1) > 0 \]
Unique Equilibrium with $x=0$

\[ aR + (1 - a)S < P \quad \text{and} \quad R < aP + (1 - a)T \]
\[ \delta(0) > 0 \quad \text{and} \quad \delta(1) > 0 \]

\[ \delta(x) \]

\[ \begin{array}{c}
0 \\
\hline
\hline
1
\end{array} \]

\[ x \]
Unique Polymorphic Equilibrium

\[ aR + (1 - a)S < P \quad \text{and} \quad R < aP + (1 - a)T \]
\[ \delta(0) > 0 \quad \text{and} \quad \delta(1) < 0 \]
\[ \delta(x) \]
Two Stable Equilibria; \( x = 0, \ x = 1 \)

\[
aR + (1 - a)S < P \quad \text{and} \quad R > aP + (1 - a)T
\]

\[
\delta(0) < 0 \quad \text{and} \quad \delta(1) > 0
\]
Games Between Sexual haploid siblings

- Behavior of interest is strategy played in a two player symmetric game with a sibling.
- Children have two parents, inherit their strategy from one parent selected at random.
- Parents mate randomly. Population proportion of C-strategists (in games with sibs) is $x$. 
Hamilton’s Inclusive Fitness

- With probability 1/2, your sibling will inherit its behavior from the same parent that you got yours from. In this case sibling will be same as you.

- With probability 1/2, sibling inherits its behavior from the other parent. In this case, probability is $x$ that sibling is a C-strategist.

- The probability that a random sibling of a C-strategist is a C-strategist is $p = 1/2 + x/2$.

- The probability that a random sibling of a D-strategist is a D-strategist is $q = x/2$.

- Therefore the index of assortiveness $a = p - q = 1/2$. 
Assortative Mating

- Generalize previous example to assume that mating is assortative and that the index of assortativeness in mating is $m$.

- A C-strategist child has at least 1 C-strategist parent. The other parent will also be a C-strategist with probability $x + m(1 - x)$.

- Probability that random sib of C-strategist child is a C-strategist is
  \[
  1/2 + (x + m(1 - x))/2 = 1/2 + m/2 + (1 - m)x/2.
  \]

- Probability that a random sib of a D-strategist child is a C-strategist is $(1 - m)x/2$.

- Index of assortativity between children is $p - q = (1 + m)/2$. 
What about the Milkman?

- In model of previous section, suppose that with probability $v$ an individual inherits behavior from a parent and with probability $1 - v$ inherits behavior from a random member of society (the milkman? a teacher? an older kid?)

- Then $p = v[1/2 + (x + m(1 - x))/2] + (1 - v)x$.

- and $q = v(1 - m)x/2 + (1 - v)x$.

- Thus $a = p - q = v(1 + m)/2$. 
Randomly Formed Tribes

- A population lives in tribes of $N$ members. Tribes form by random draws from the population of young adults.

- Children live with their parents’ tribe until reaching adulthood.

- Children choose a strategy for dealing with other tribe members by copying adults.

- The probability that a child selects a strategy is equal to the proportion of adult tribe members who use that strategy.

- Then $a = 1/N$. Not much cooperation for $N$ equal to 20 or 30.
Persistent Tribes with Inbreeding

- Tribes consist of $N$ members. Fraction $v$ of young adults leave home for another tribe and are replaced by random draws from the overall population.

- Children copy a randomly selected tribe member.

- Now even if two children have different role models, their role models may share common role models or their role models role models may share common role models, etc.

- Let $k$ be the probability that two individuals of the same generation in the same tribe are descended from a common ancestor. We can calculate $k$ either by an infinite series, or by a simple “dynamic programming” argument.
Calculating relatedness

- Two individuals of the same tribe and generation share a common parent only if neither of these individuals is an immigrant. Neither is an immigrant with probability \((1 - v)^2\).

- Therefore the probability that they share a common parent is \((1 - v)^2/N\).

- If they do not share a common cultural parent and neither is an immigrant, then each has a different parent from the same tribe. The probability that this happens is

\[
(1 - v)^2 \frac{N - 1}{N}.
\]
• Given that the parents of the two individuals belong to the same tribe, the probability that these parents share a common ancestor is also $k$.

• The probability that two individuals have a common ancestor equals the probability that they have a common parent plus the probability that they have different parents but their parents have a common ancestor.

• Therefore

$$k = (1 - v)^2 \left( \frac{1}{N} + \frac{N - 1}{N} k. \right)$$

• Solving this equation:

$$k = \frac{1}{N - (N - 1)(1 - v)^2} = \frac{1}{N (1 - (1 - v)^2) + (1 - v)^2}.$$
Numerical Examples

- Suppose tribes are of size $N = 30$ and the probability of immigration (emigration) is $v = .10$. Then

$$k \approx \frac{1}{.19N + .81} \approx 1/6.$$  

- If $N = 30$ and $V = .20$, then

$$k \approx \frac{1}{.36N + .64} \approx 1/11.$$  

- If $N = 30$ and $v = .05$, then

$$k \approx \frac{1}{.1N + .9} \approx 1/4.$$
Extensions

- Model with two tribes. Migration between tribes is fairly common. Migration to and from outside world is much less common. This can be solved as two equations in two unknowns: 1) probability that two born in same tribe have common a ancestor. 2) probability that two born in opposite tribes have a common ancestor.

- Extension to a string of tribes along a coastline. Kimura’s ”stepping stone model”